

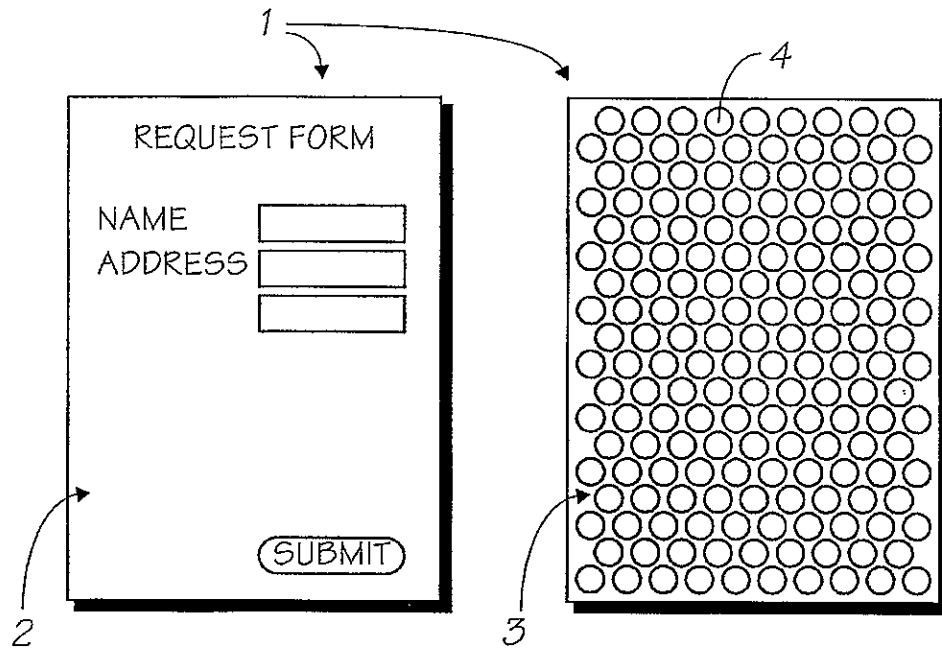
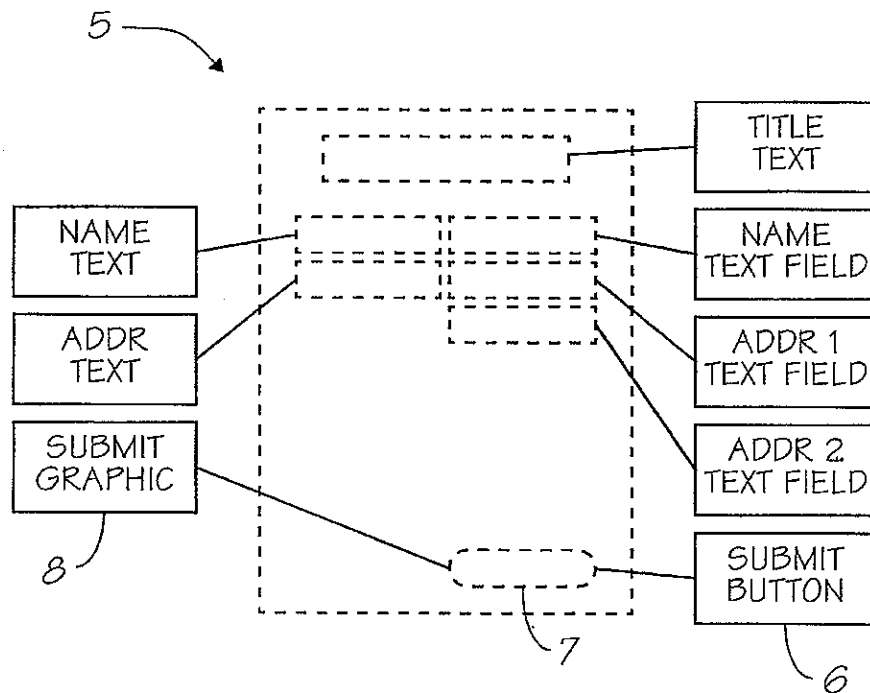


FIG. 1

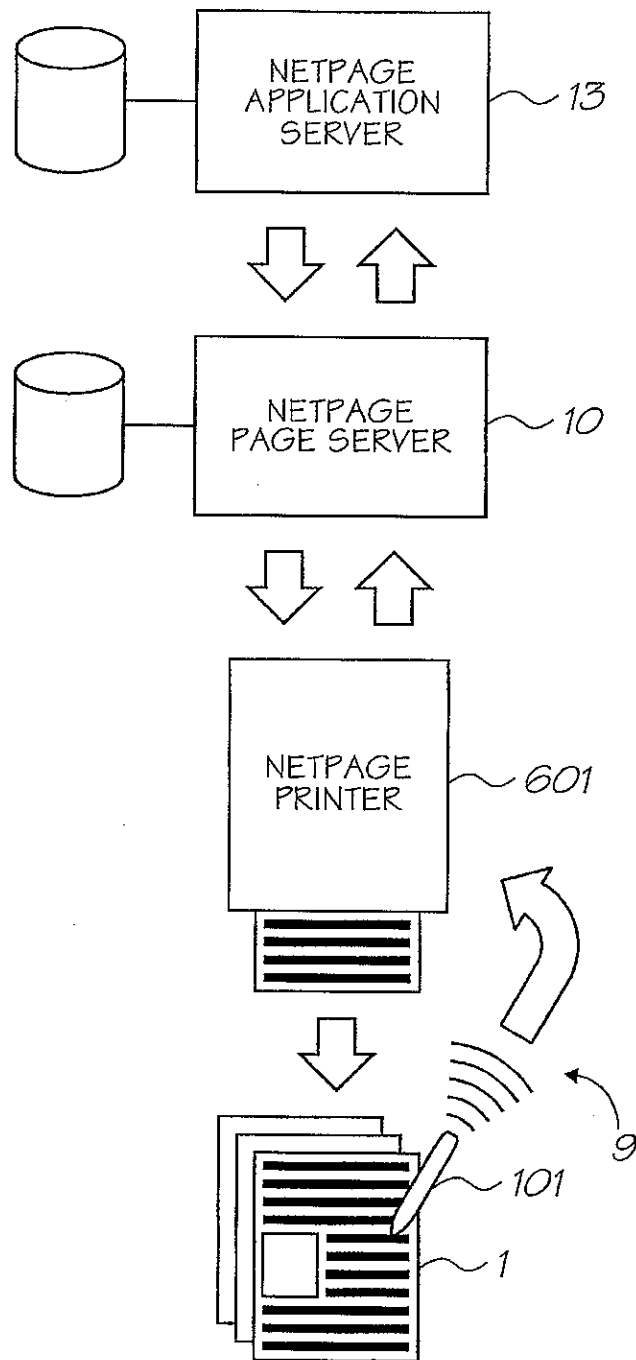


FIG. 2

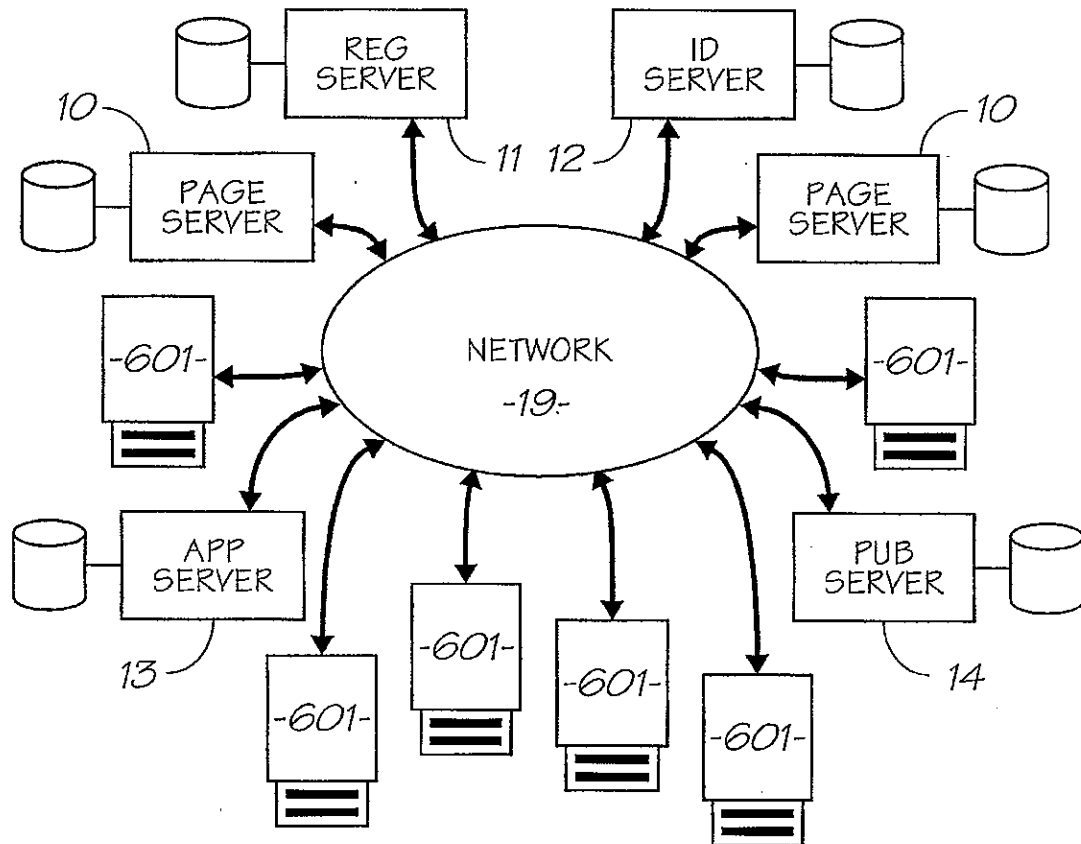


FIG. 3

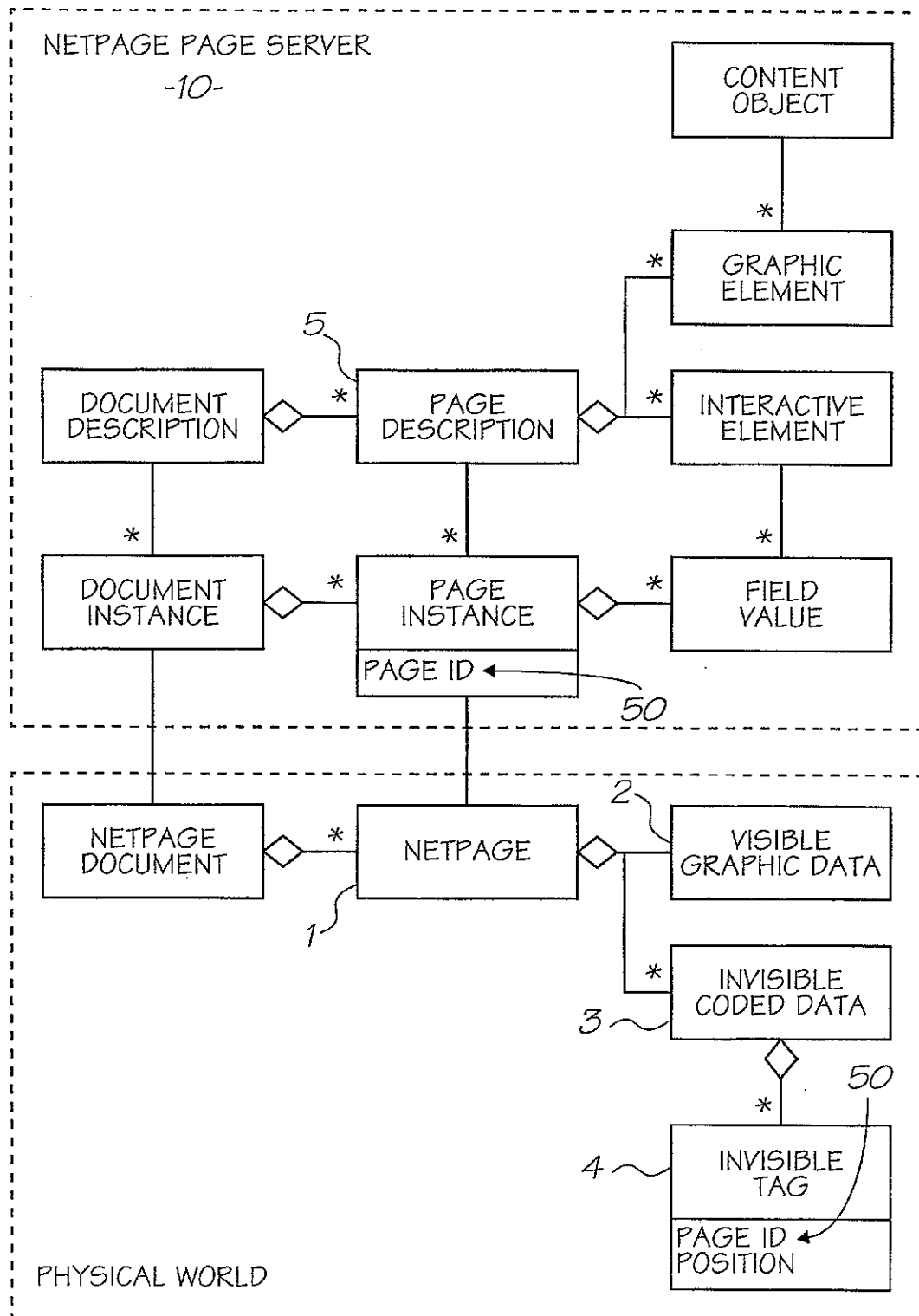


FIG. 4

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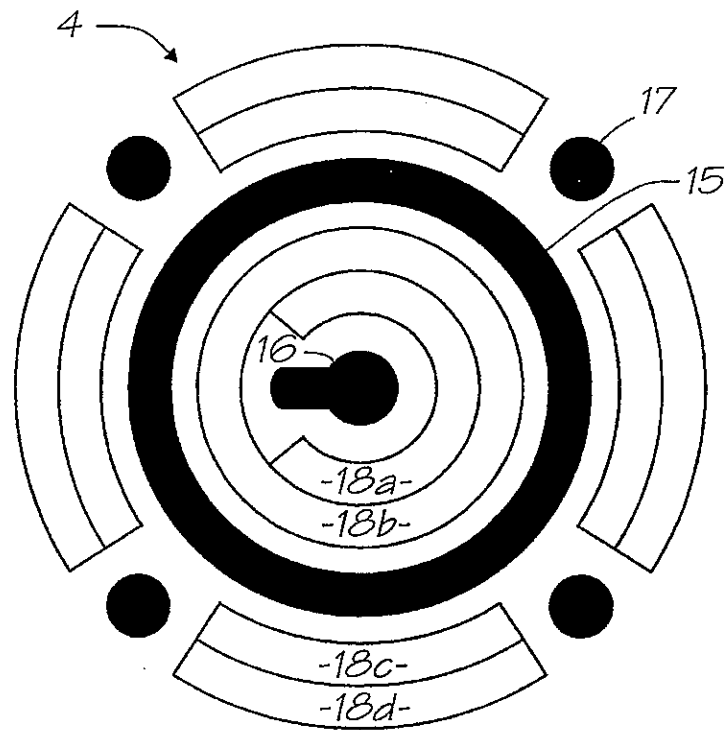


FIG. 5

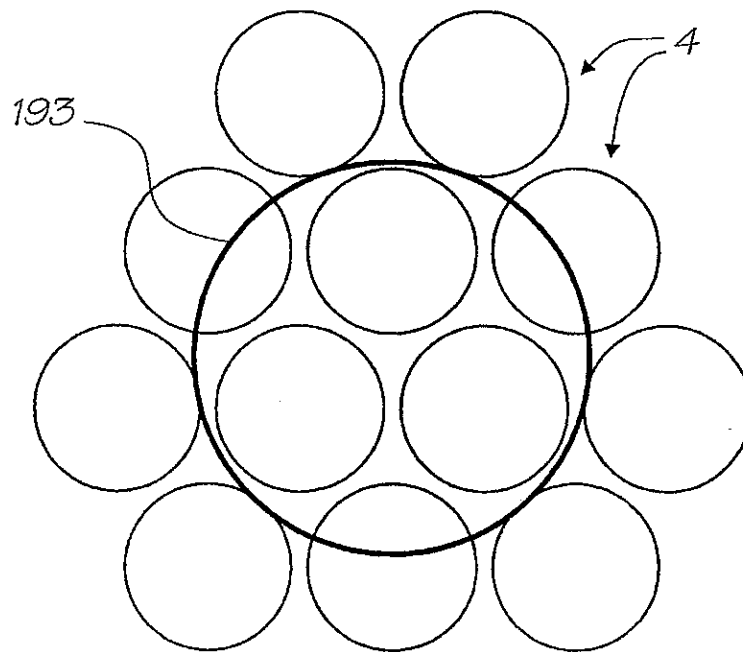


FIG. 6

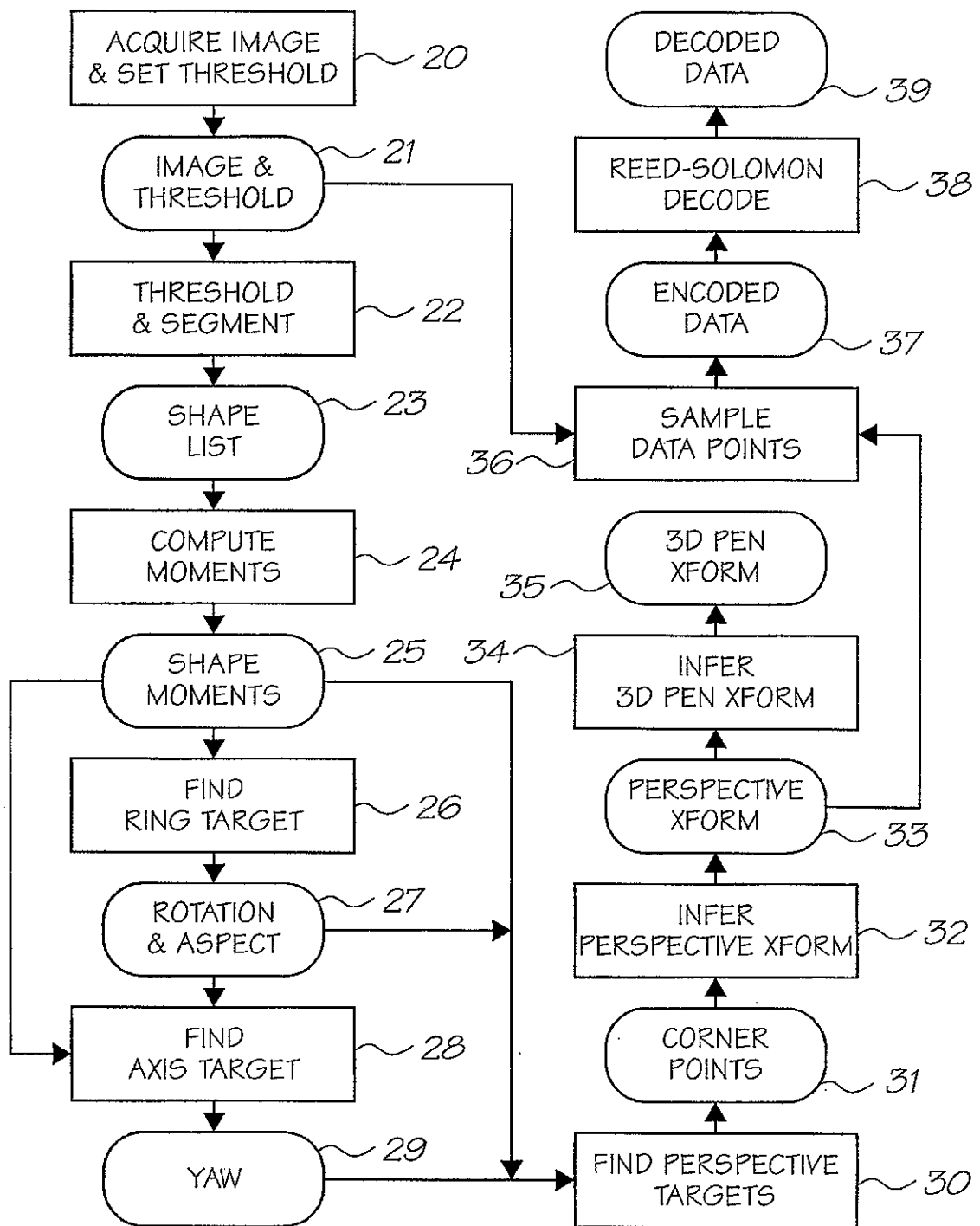


FIG. 7

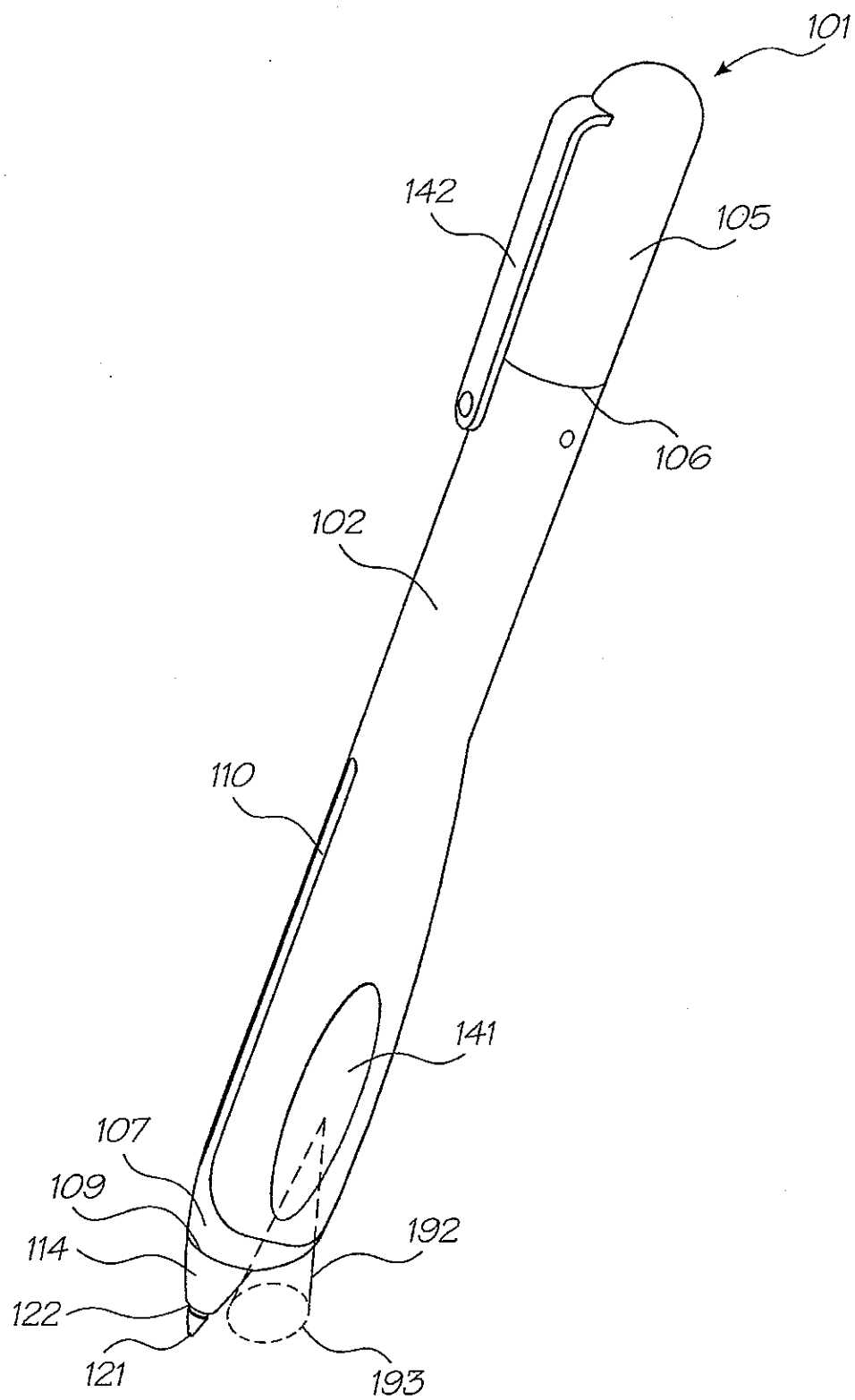


FIG. 8

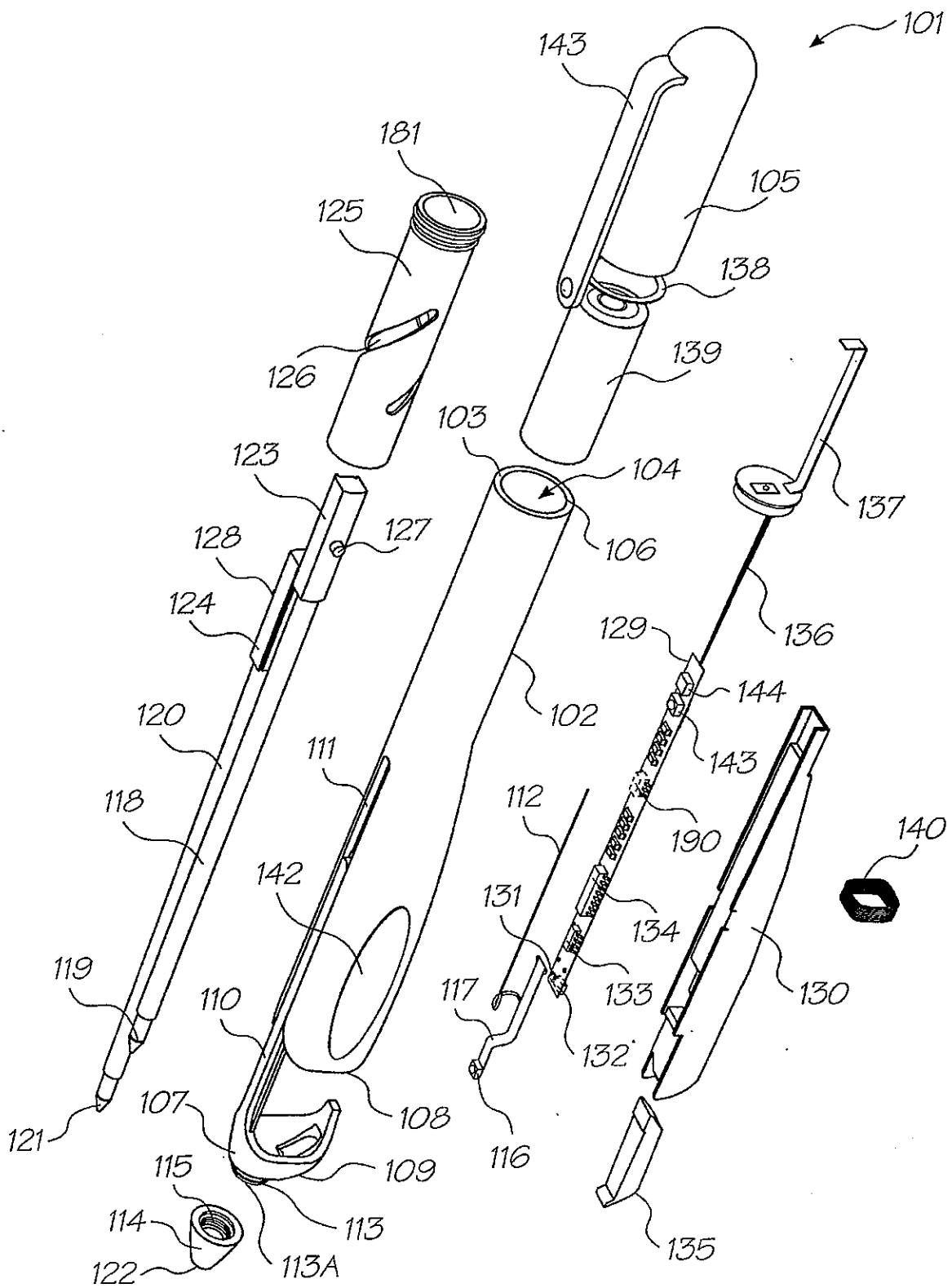


FIG. 9

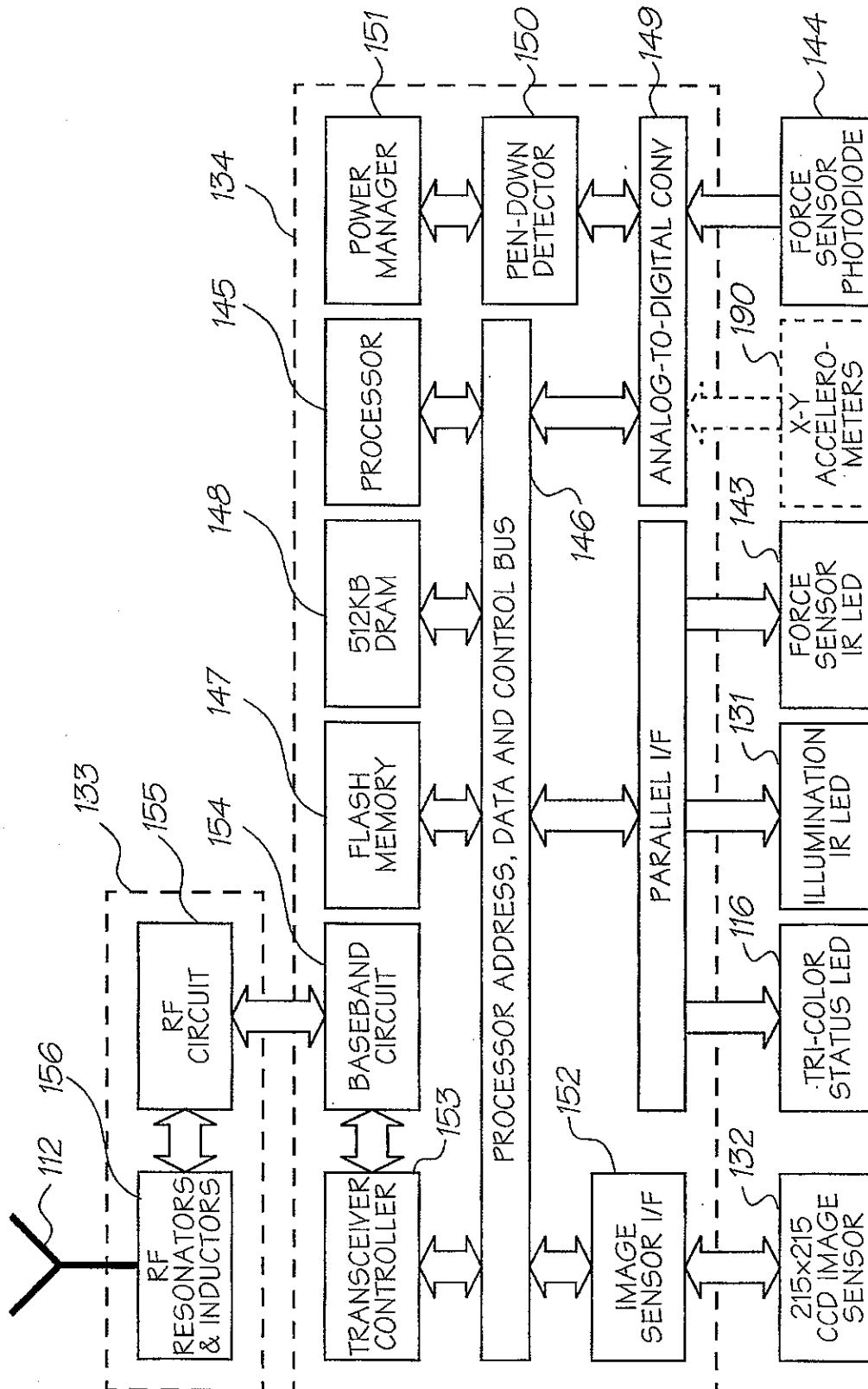


FIG. 10

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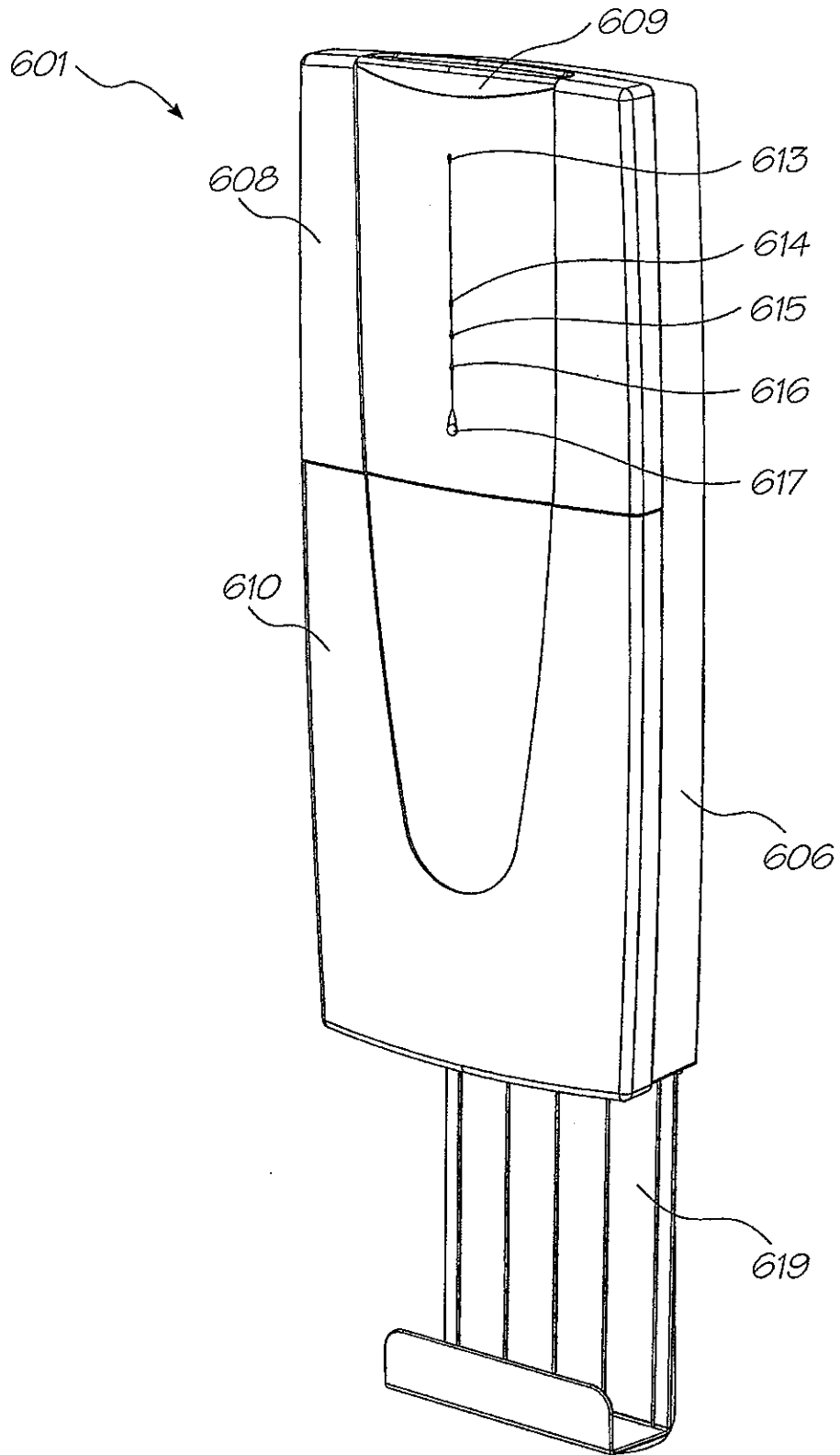


FIG. 11

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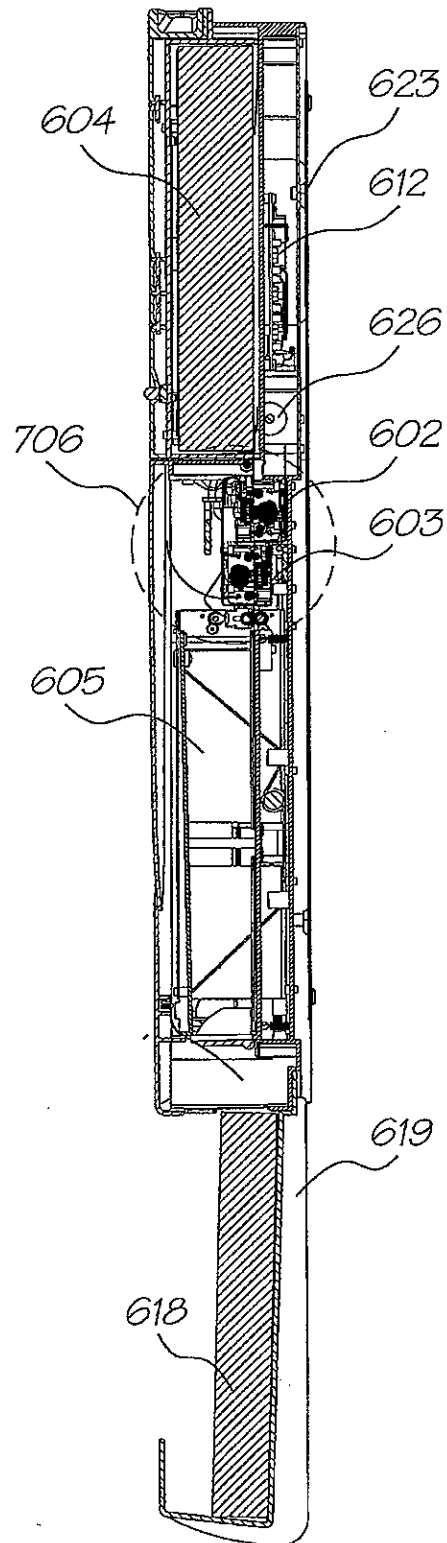


FIG. 12

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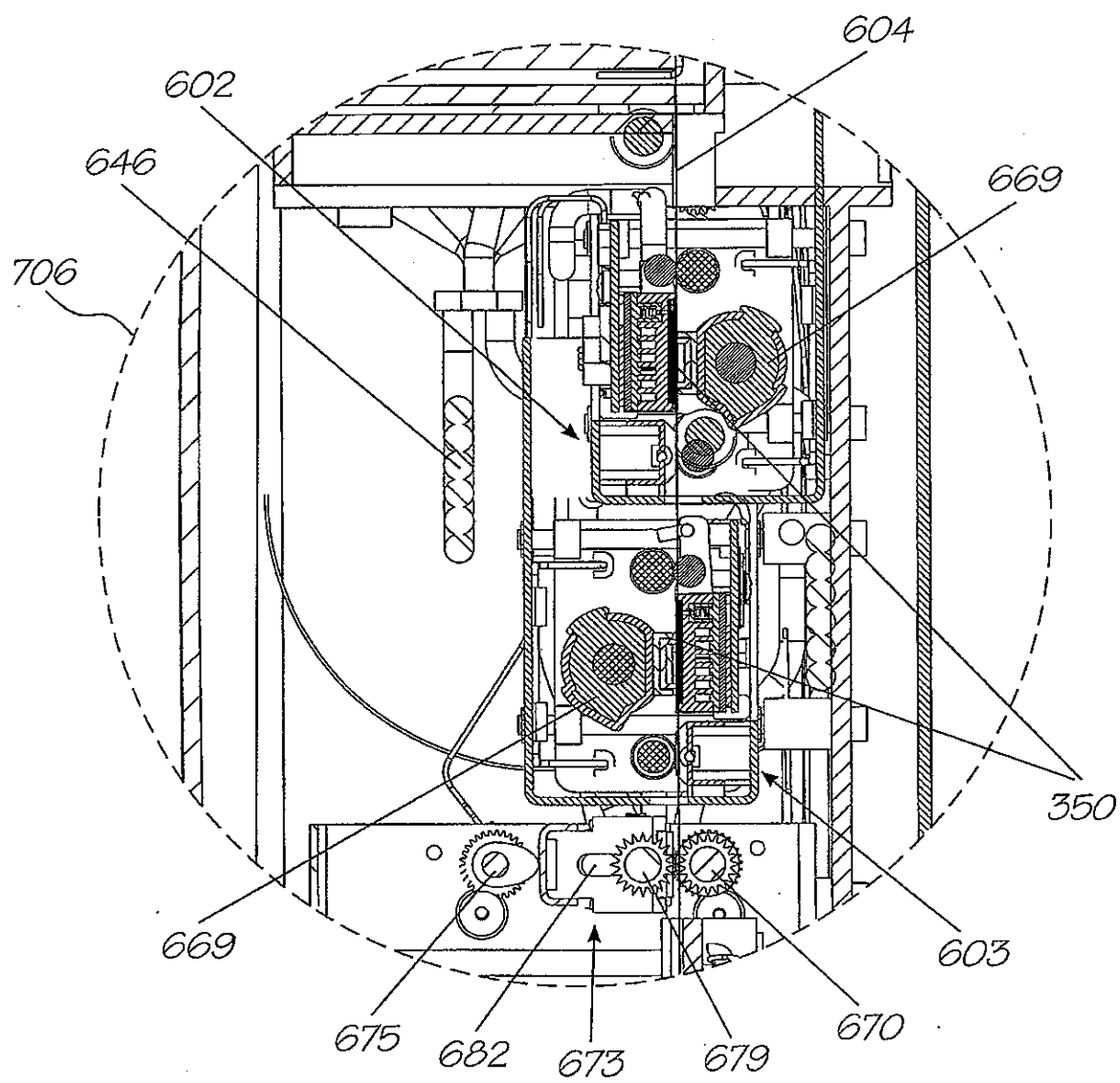


FIG. 12a

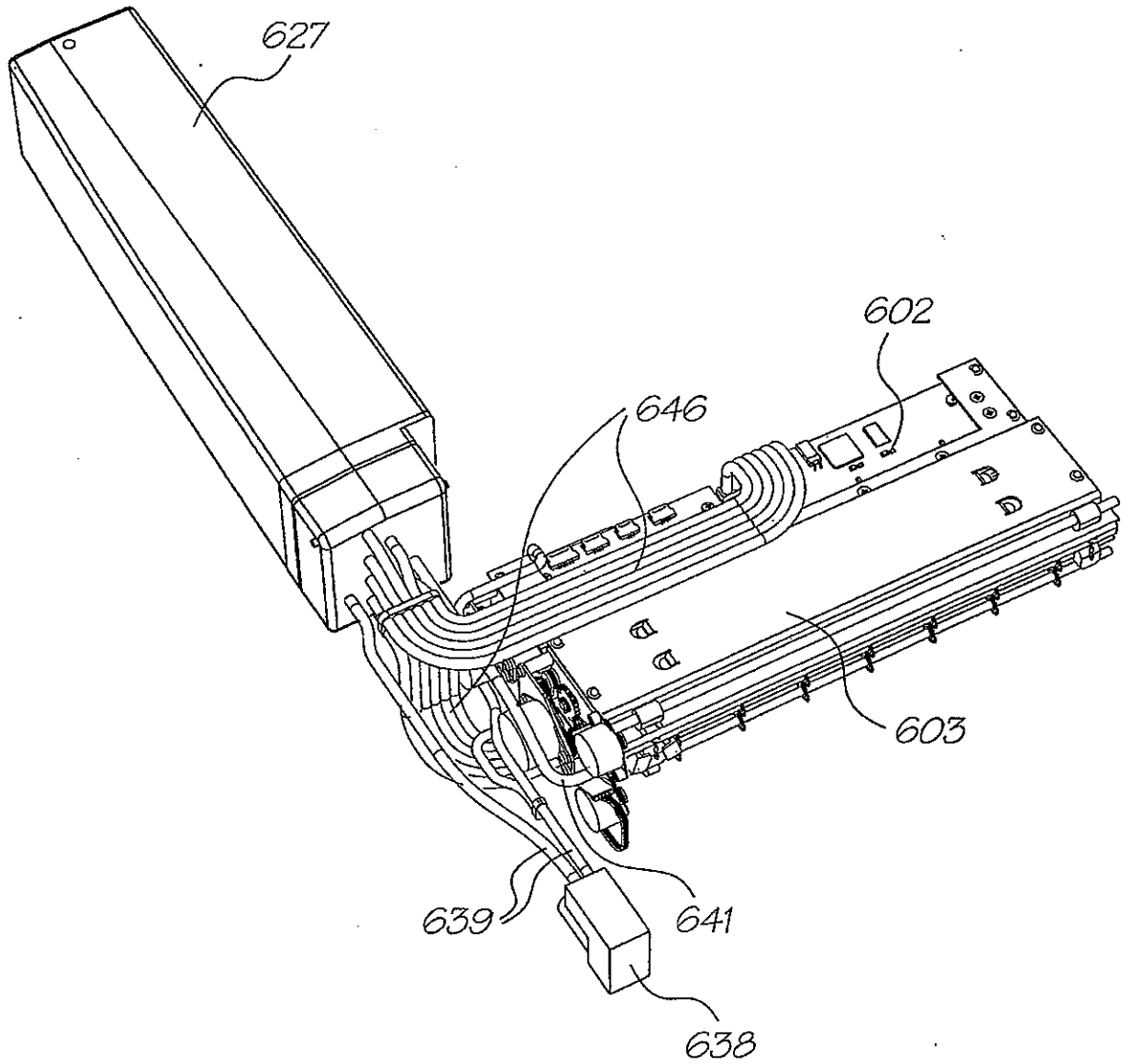


FIG. 13

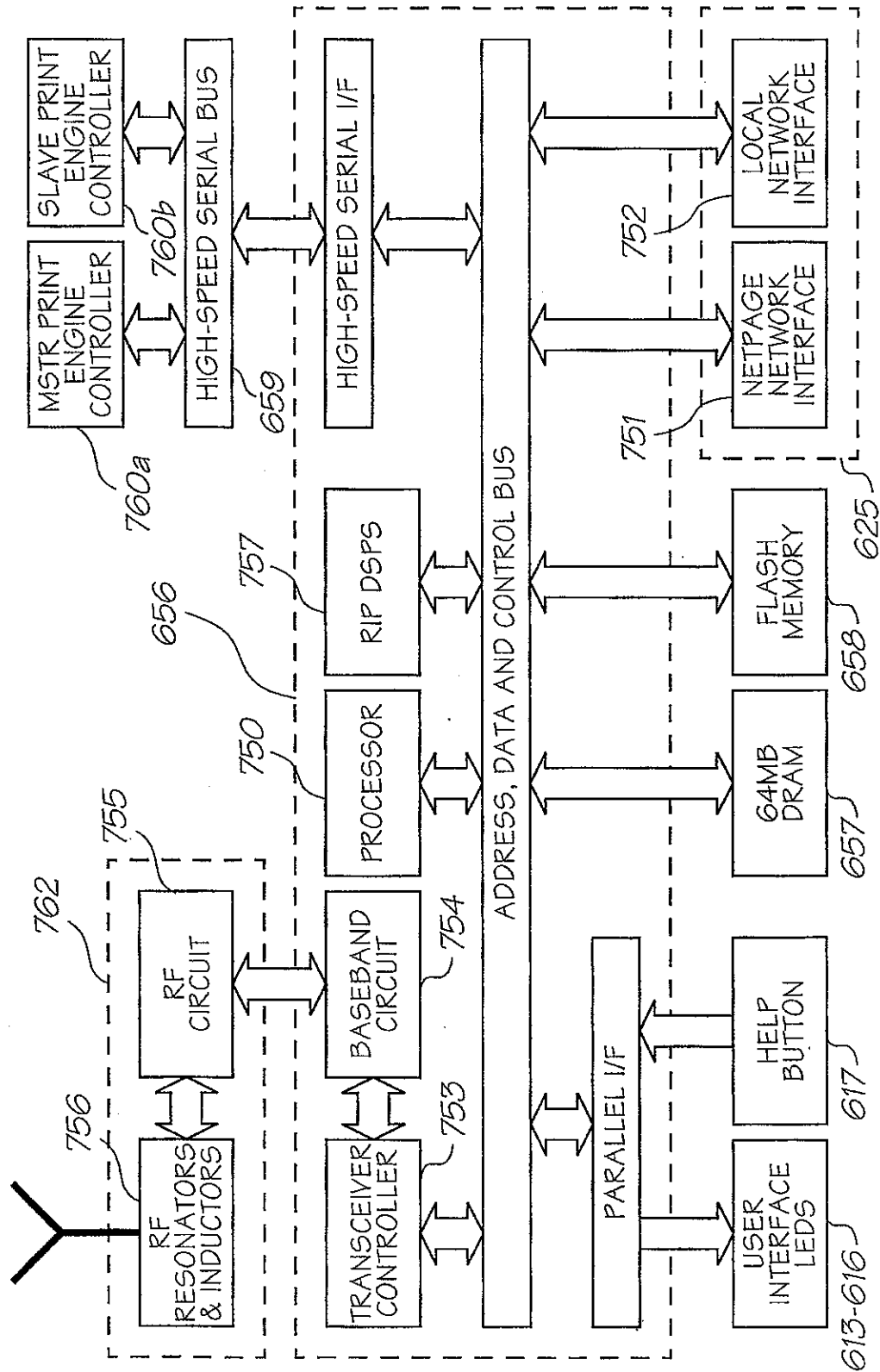


FIG. 14

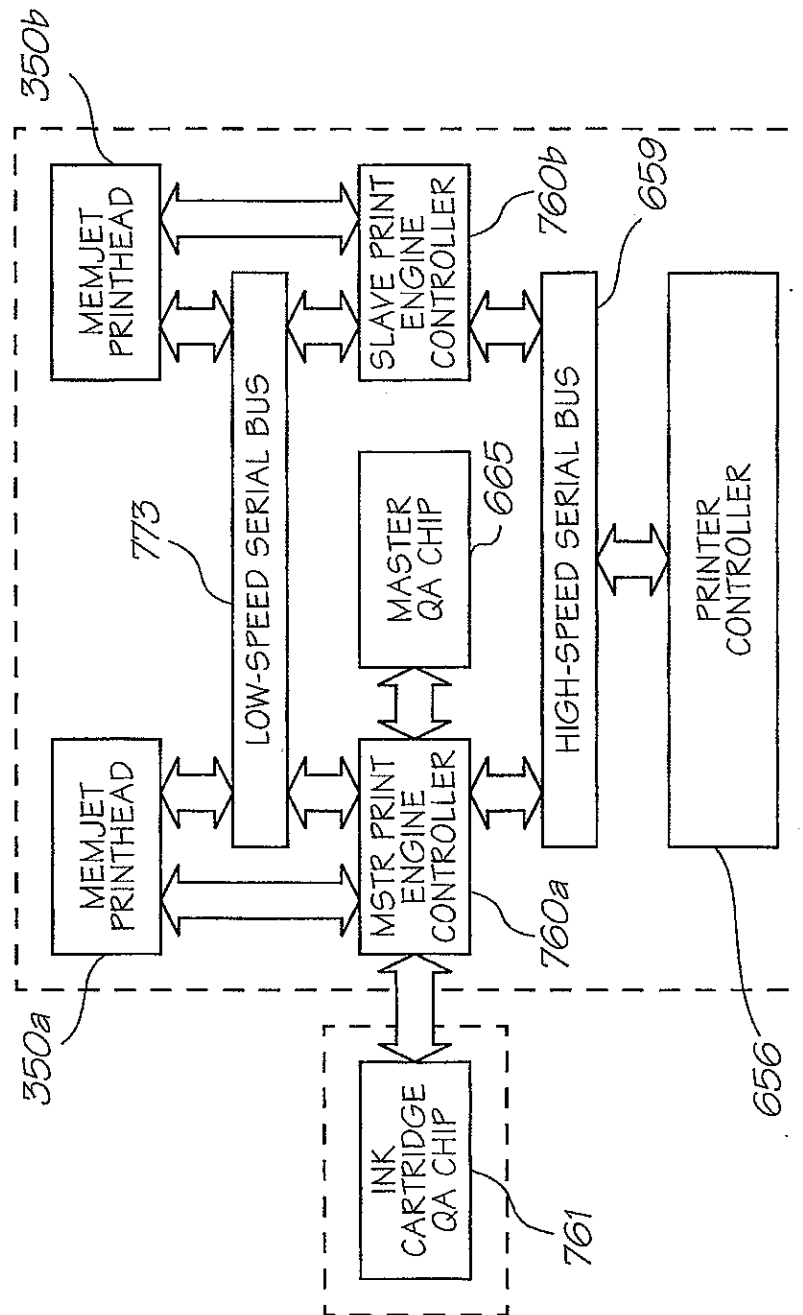


FIG. 15

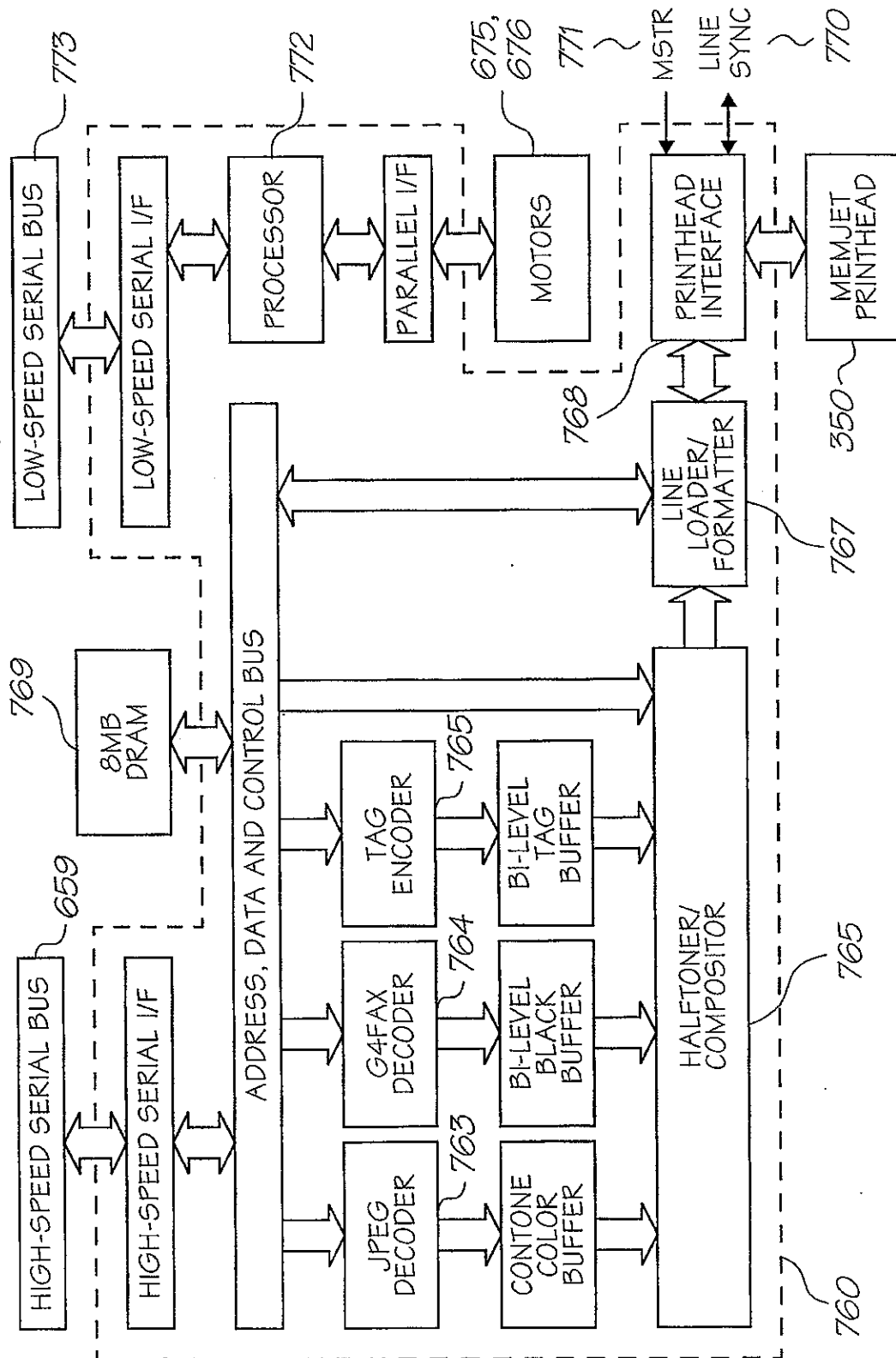


FIG. 16

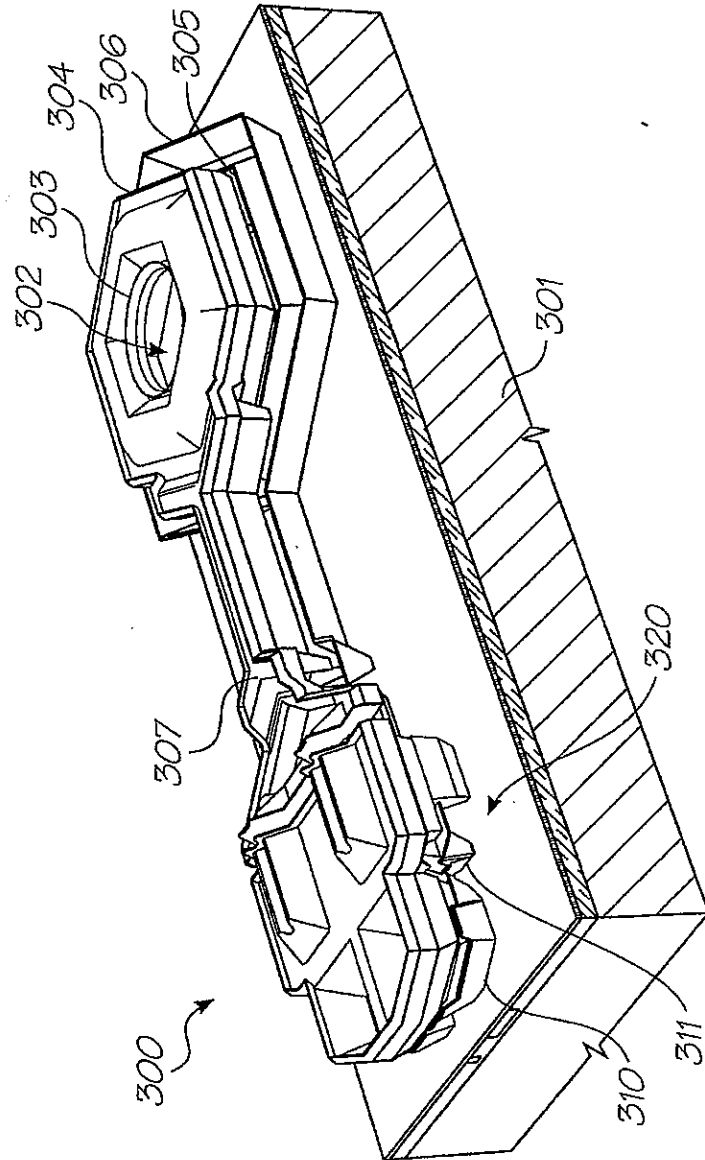


FIG. 17

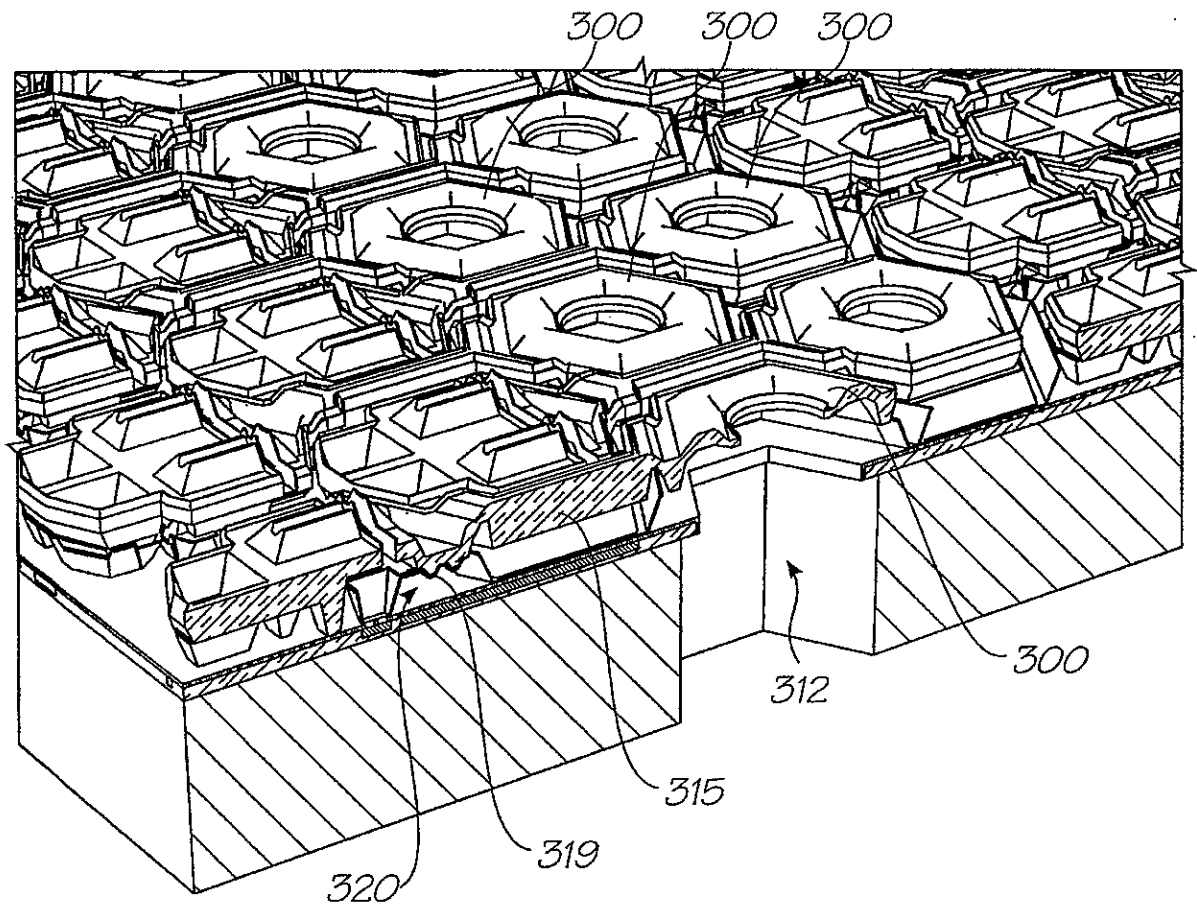


FIG. 18

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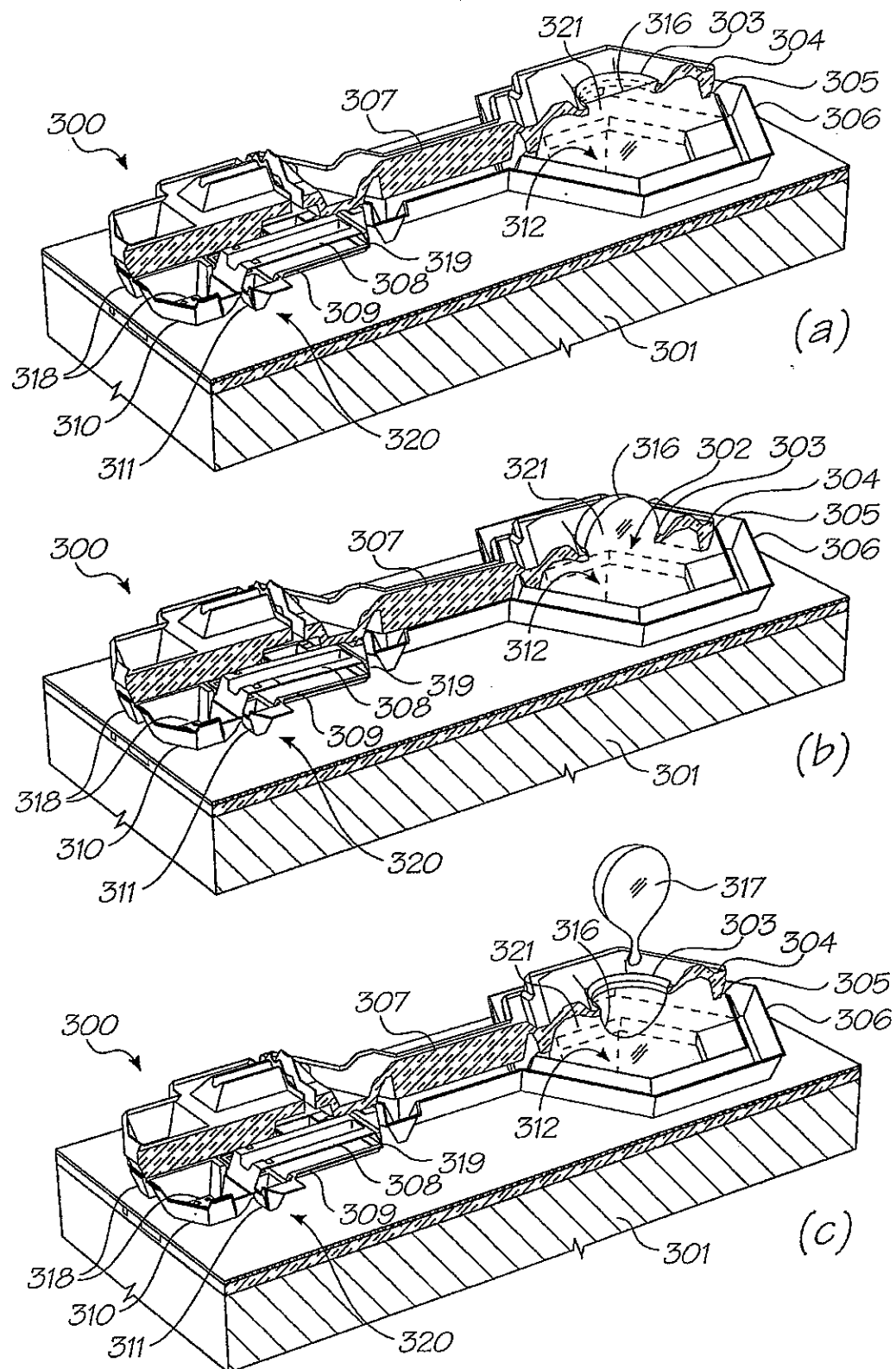


FIG. 19

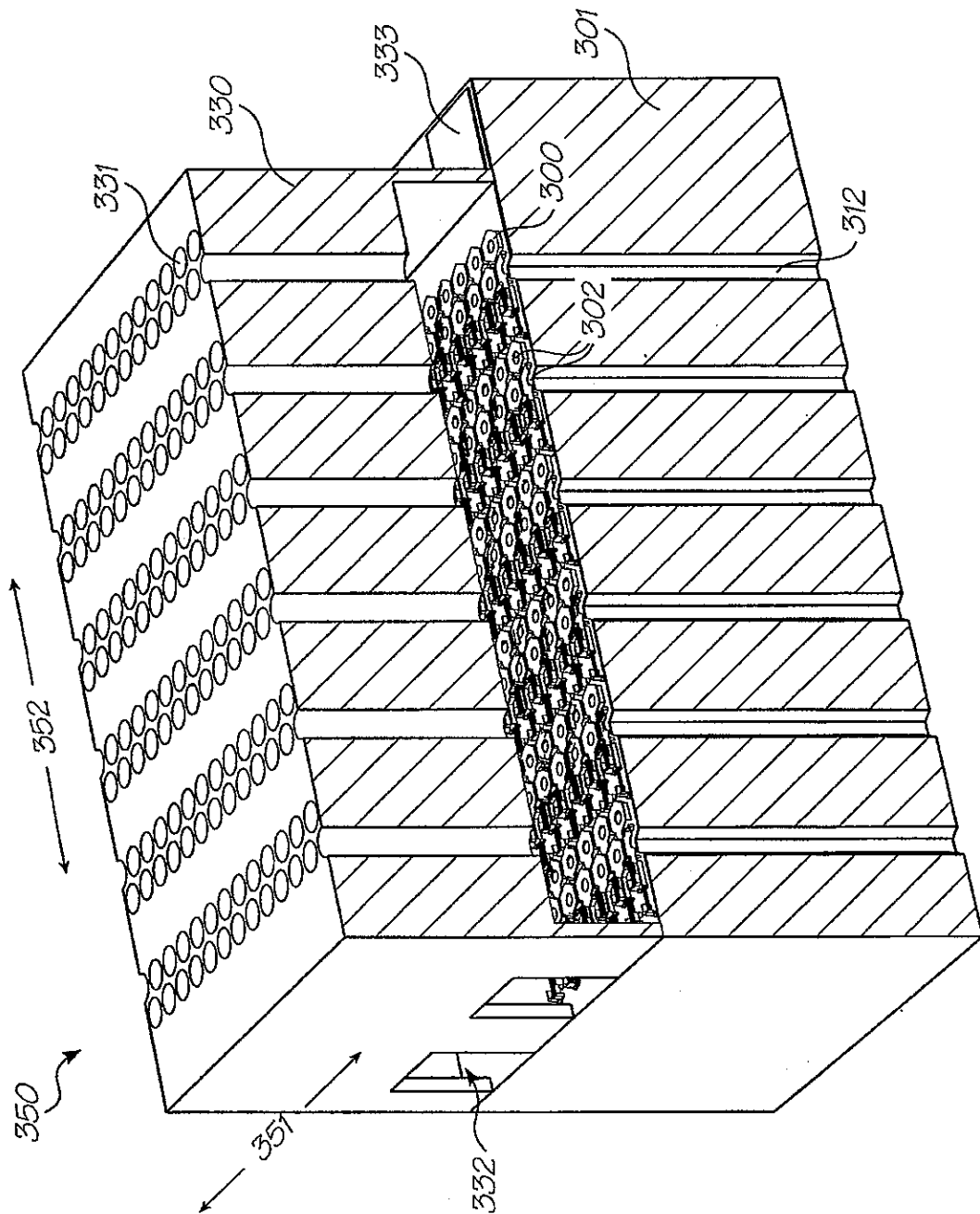


FIG. 20

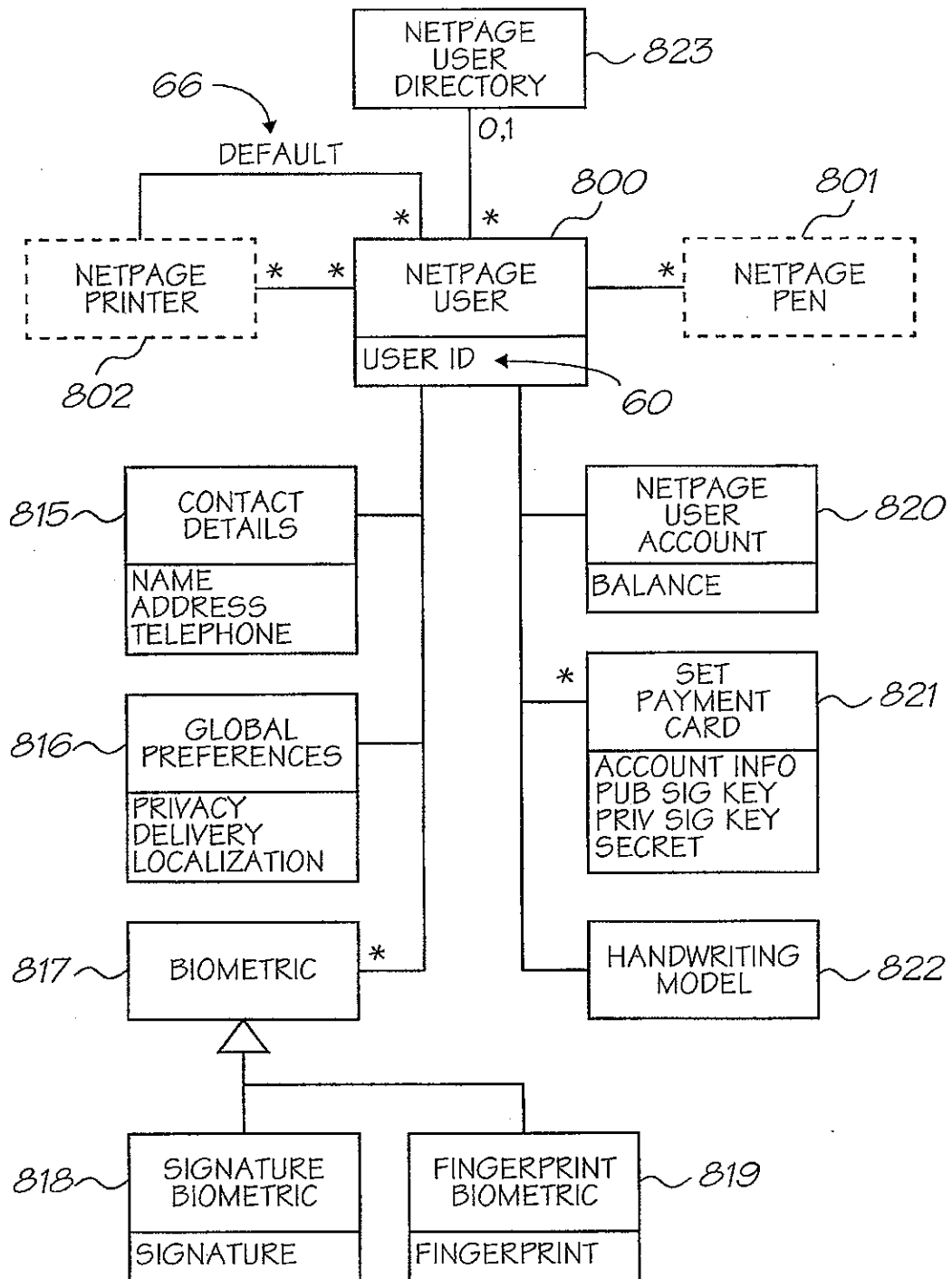


FIG. 21

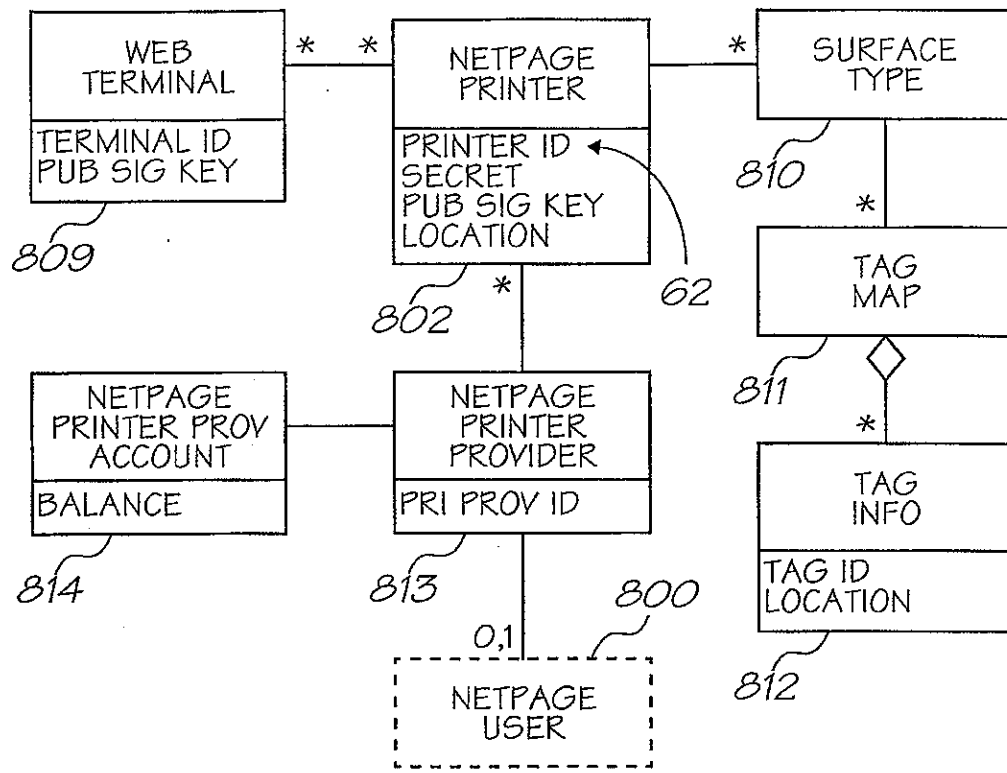


FIG. 22

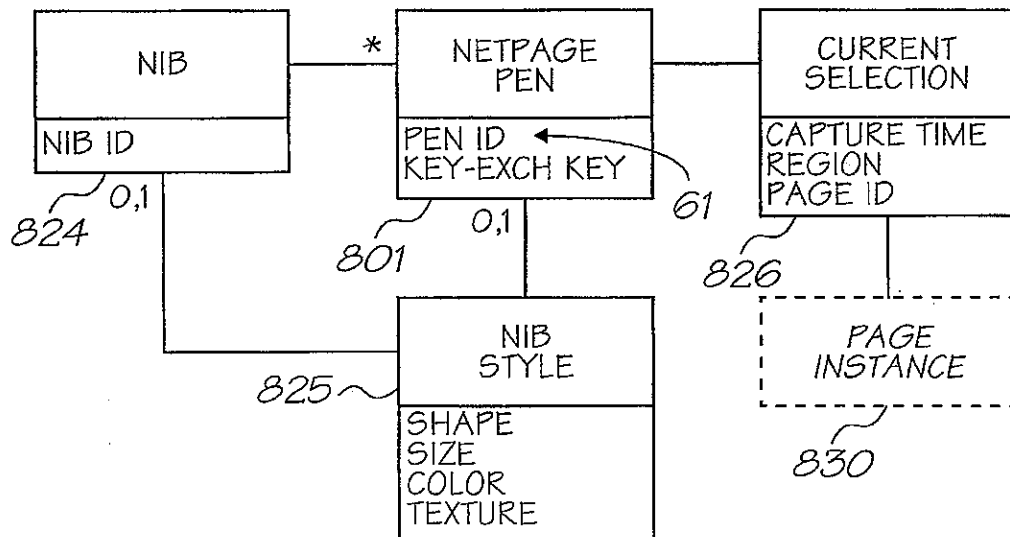


FIG. 23

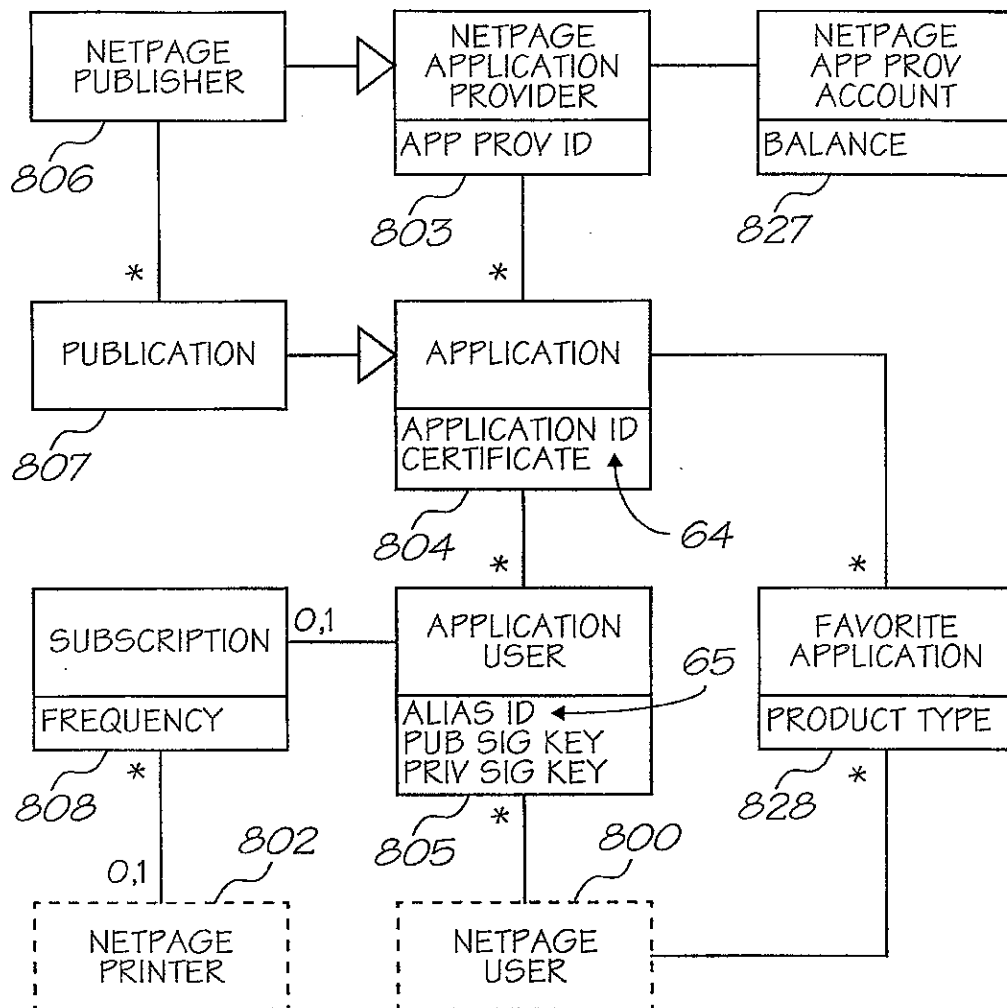


FIG. 24

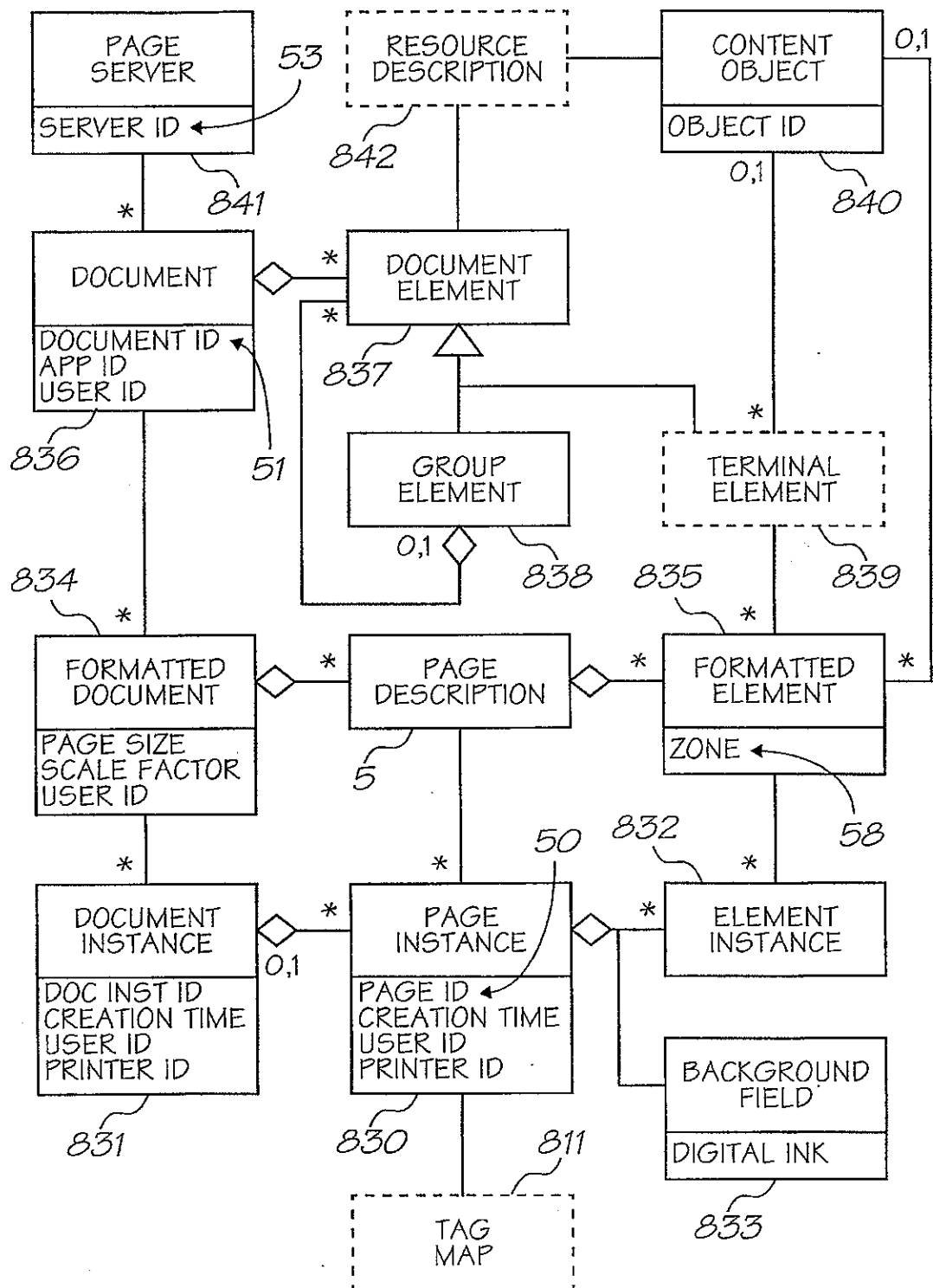


FIG. 25

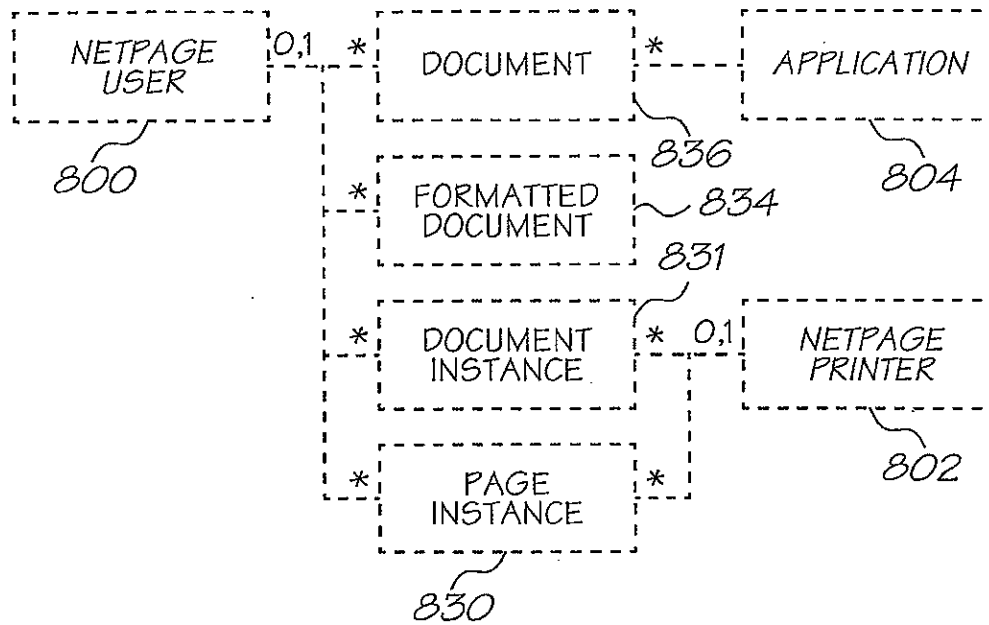


FIG. 26

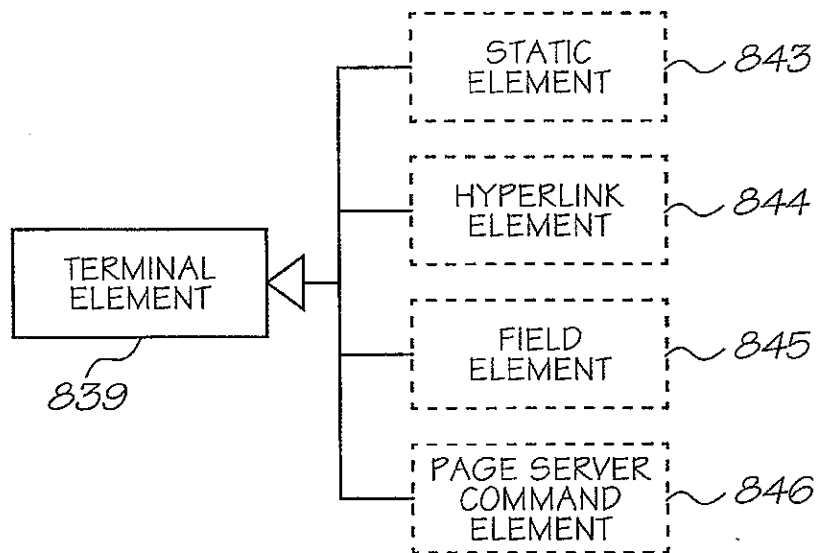


FIG. 27

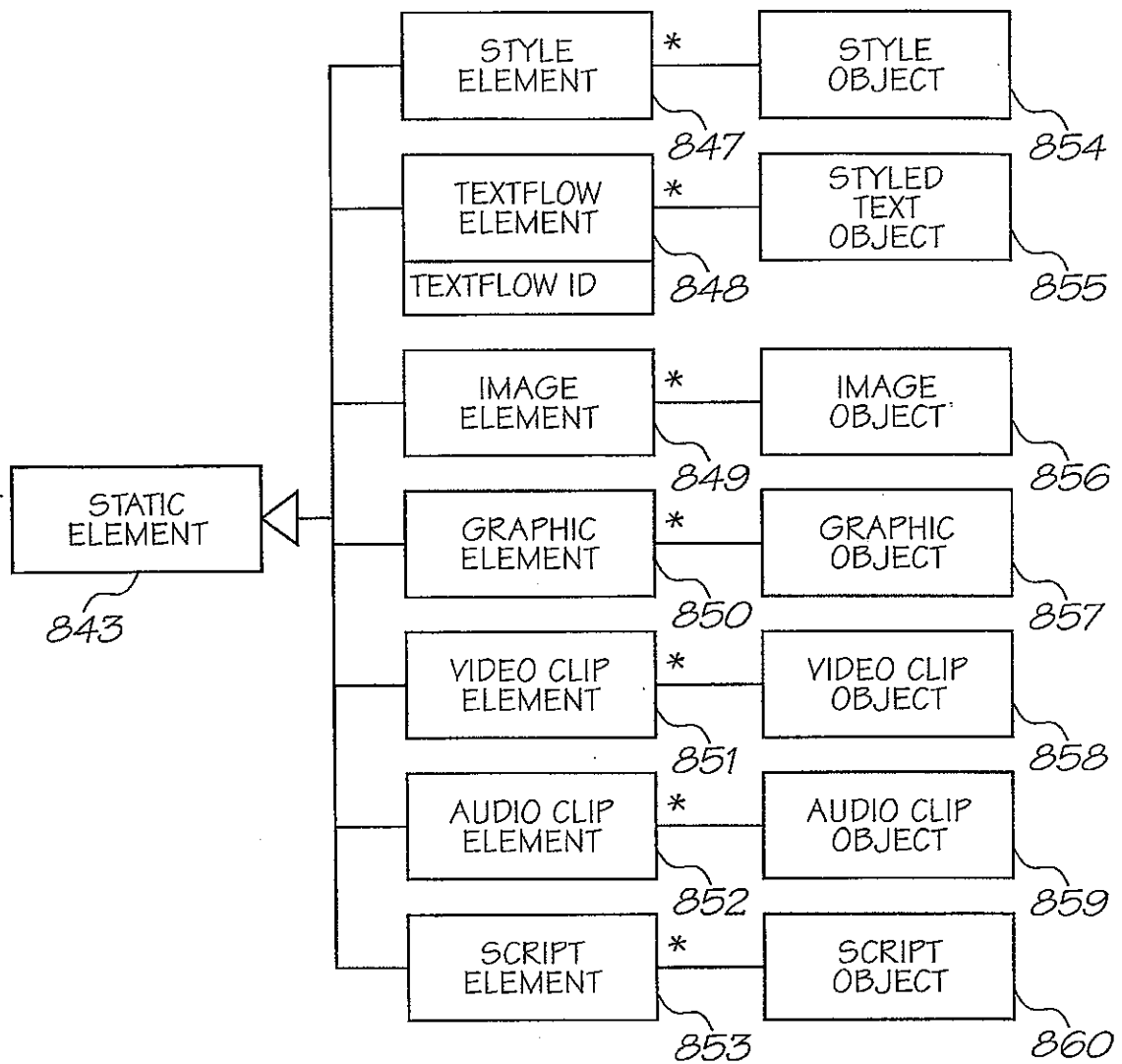


FIG. 28

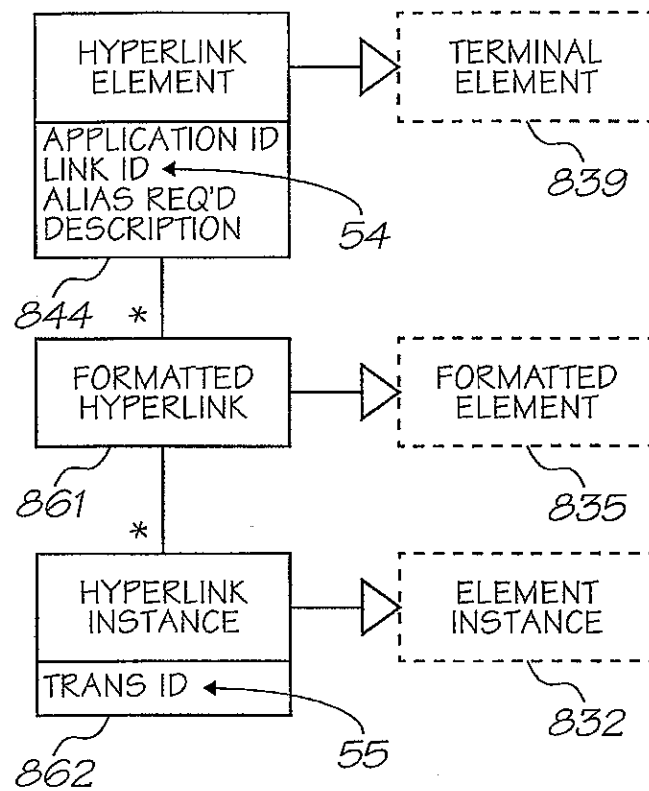


FIG. 29

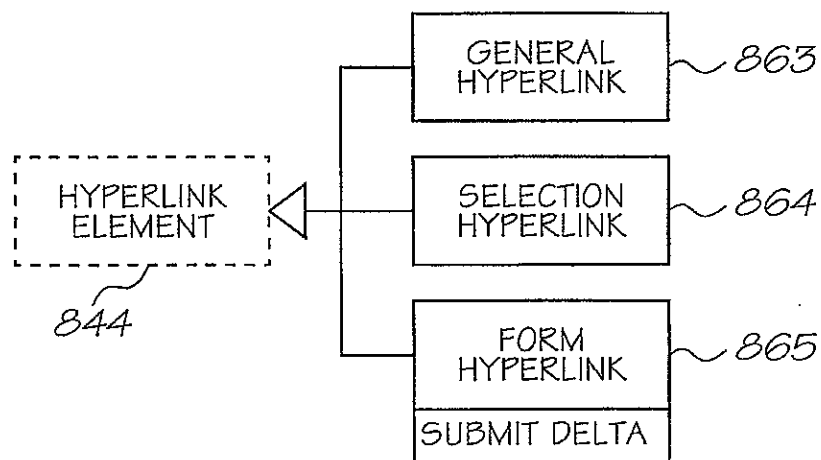


FIG. 30

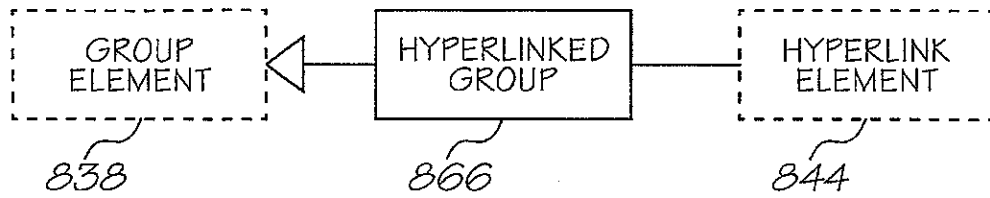


FIG. 31

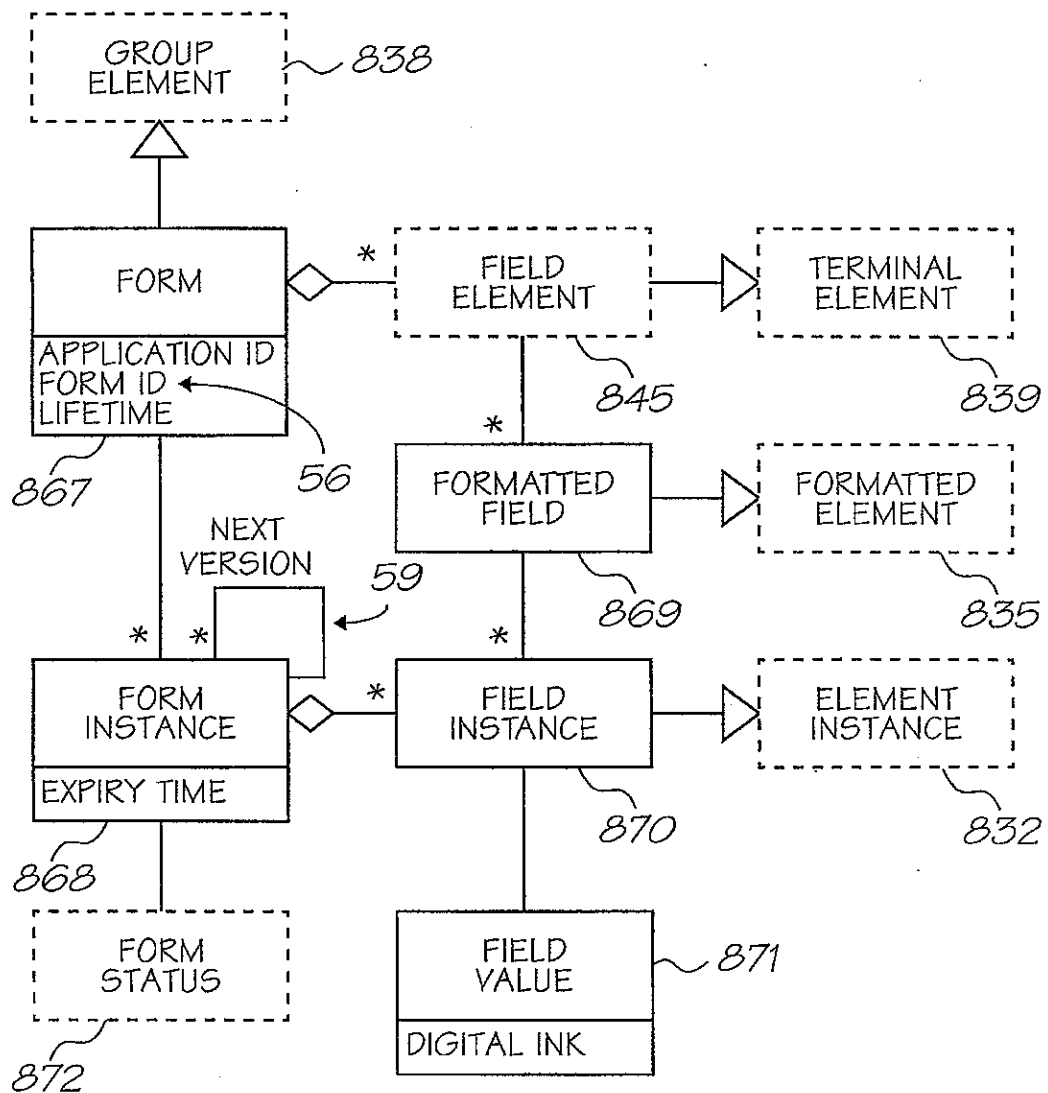


FIG. 32

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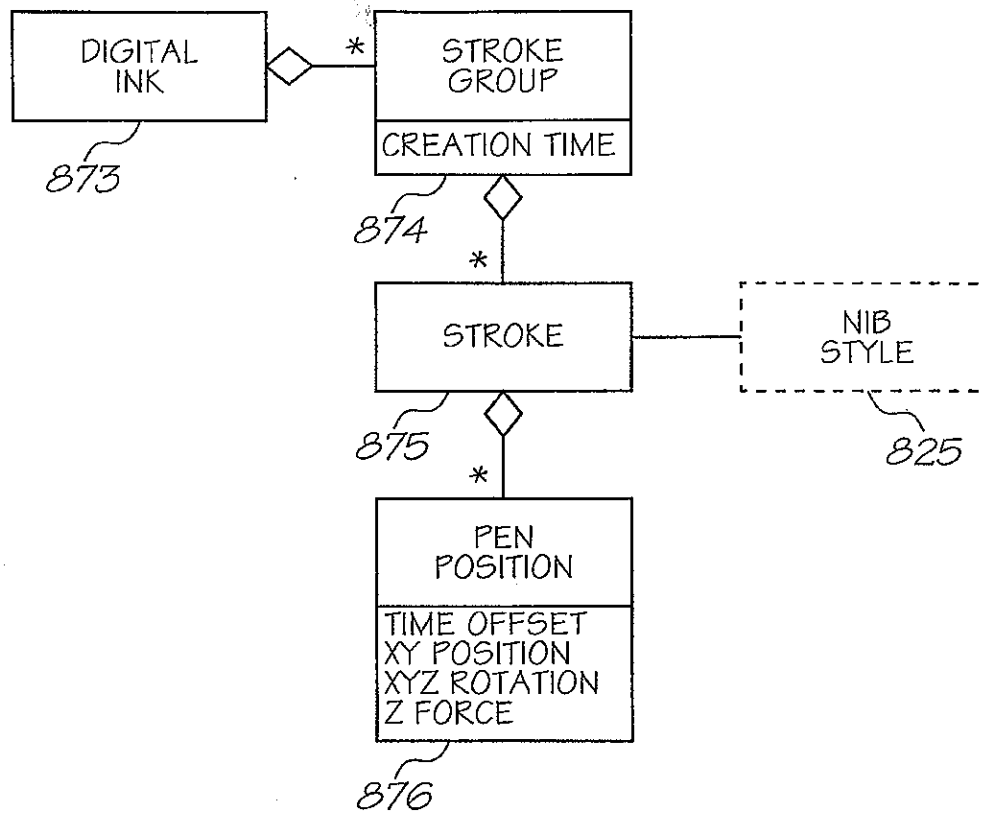


FIG. 33

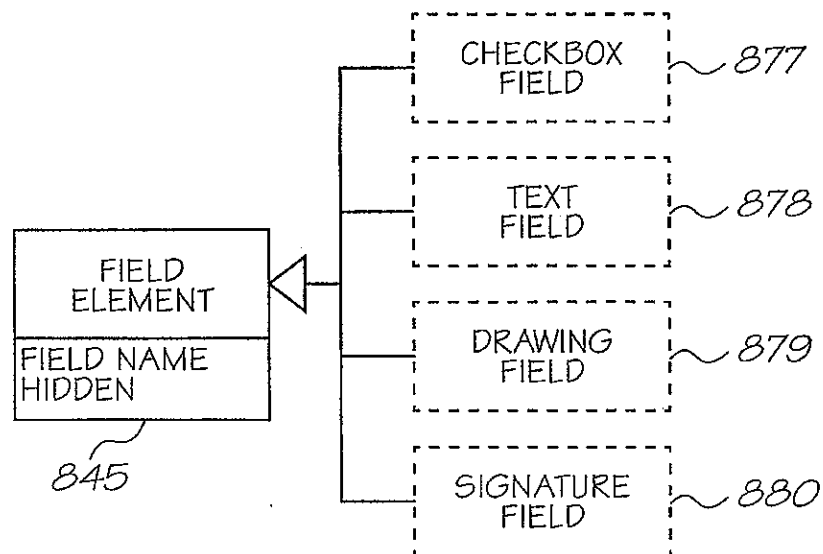


FIG. 34

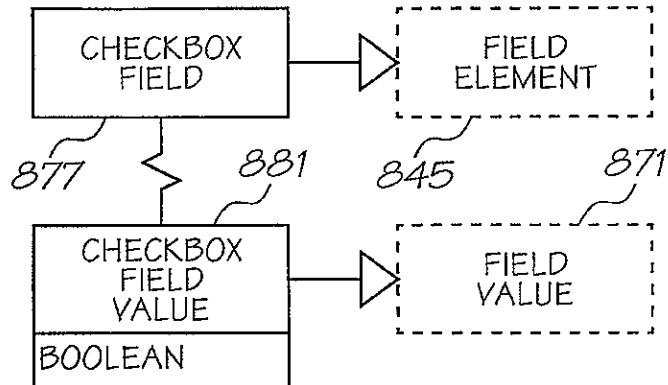


FIG. 35

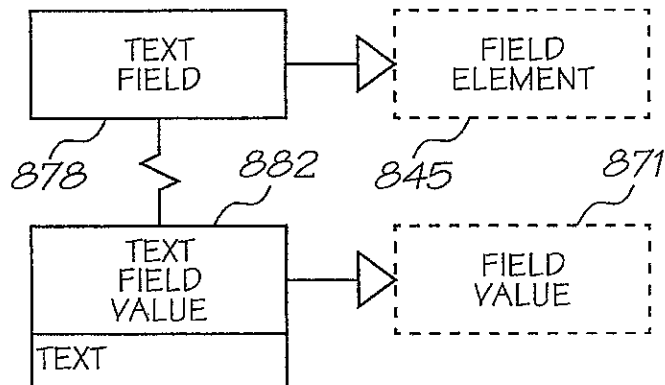


FIG. 36

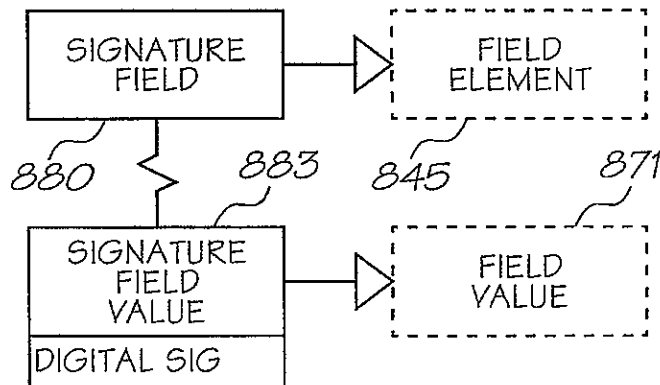


FIG. 37

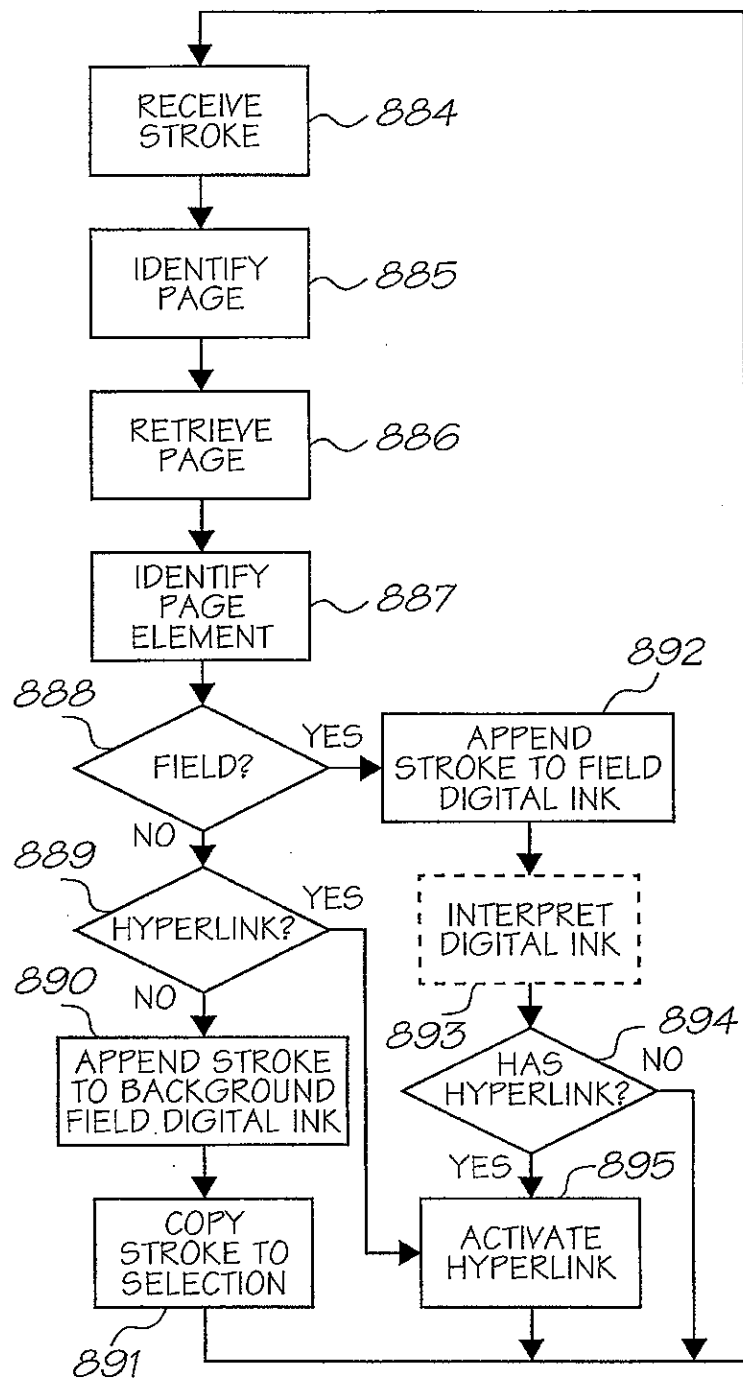


FIG. 38

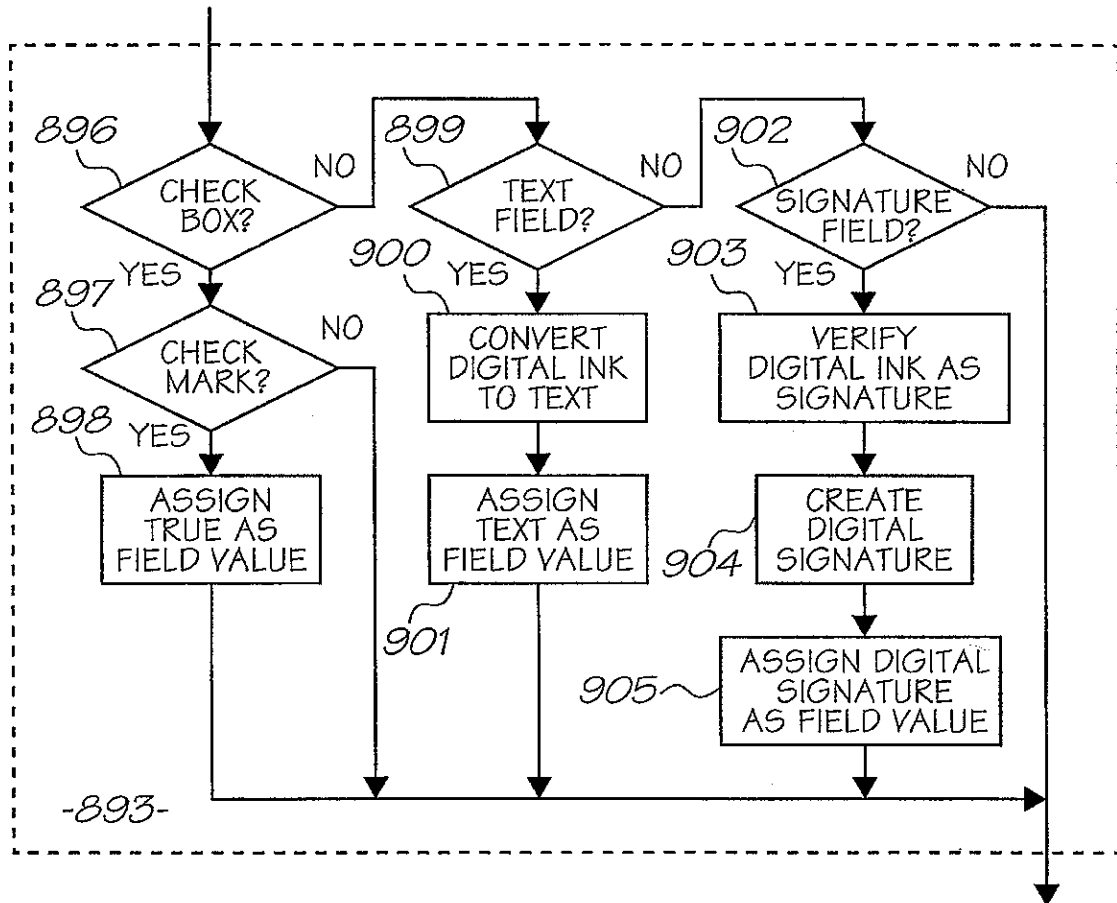


FIG. 38a

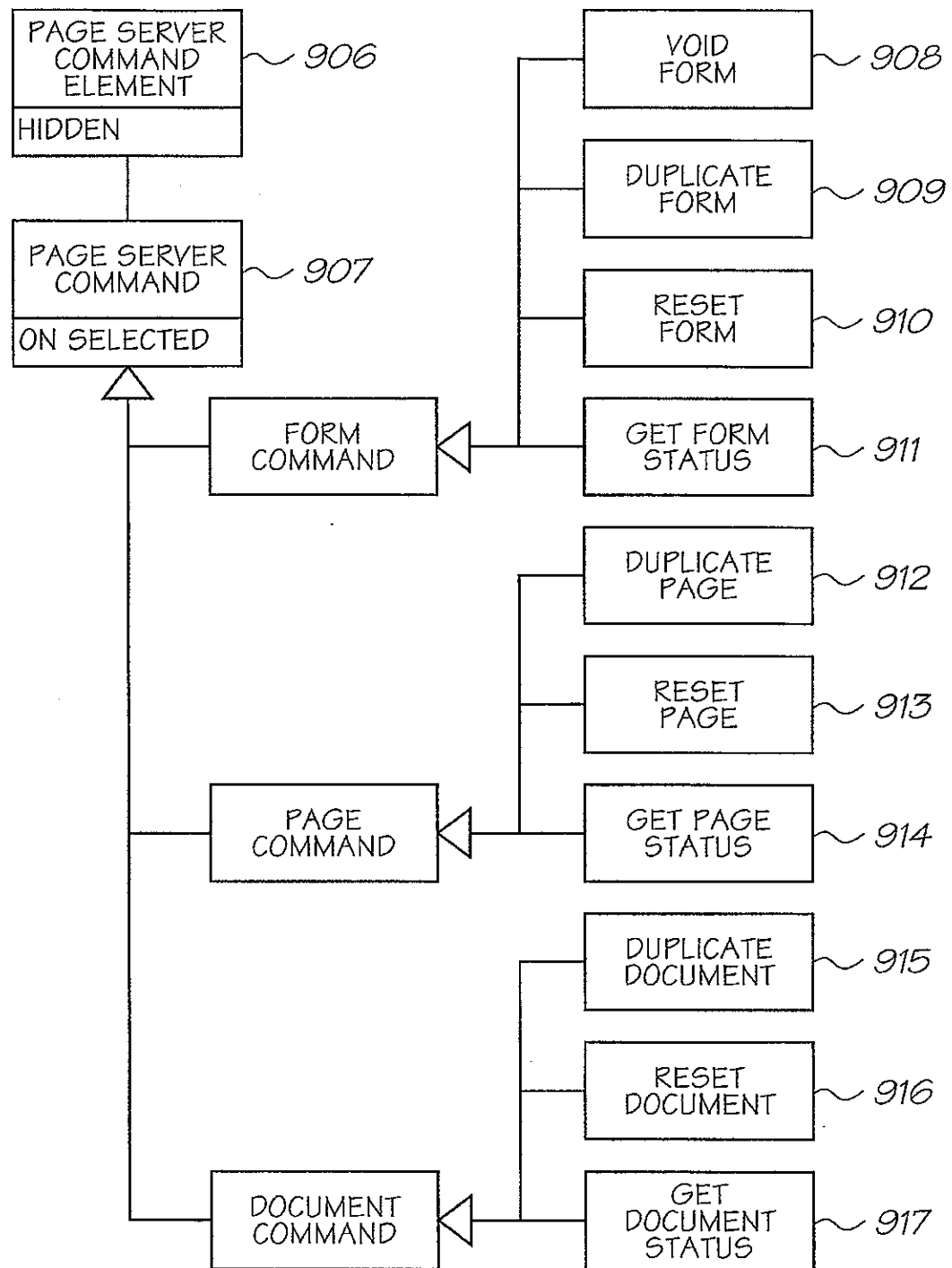


FIG. 39

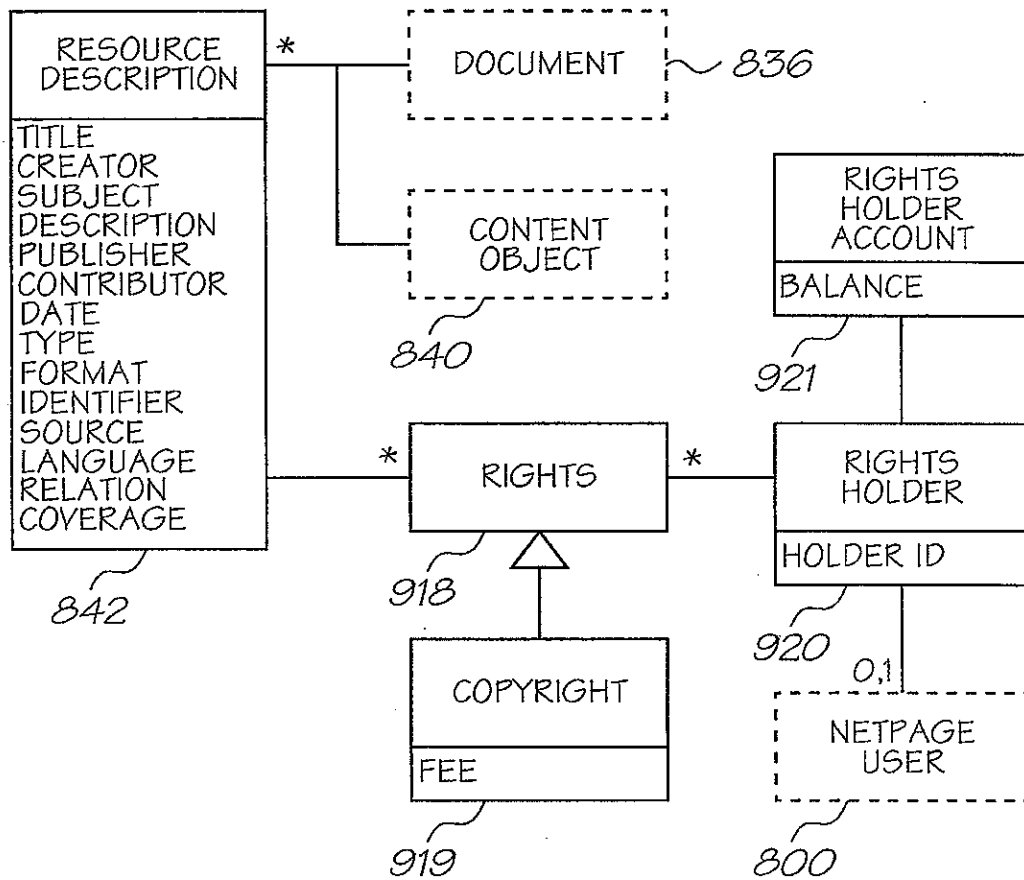


FIG. 40

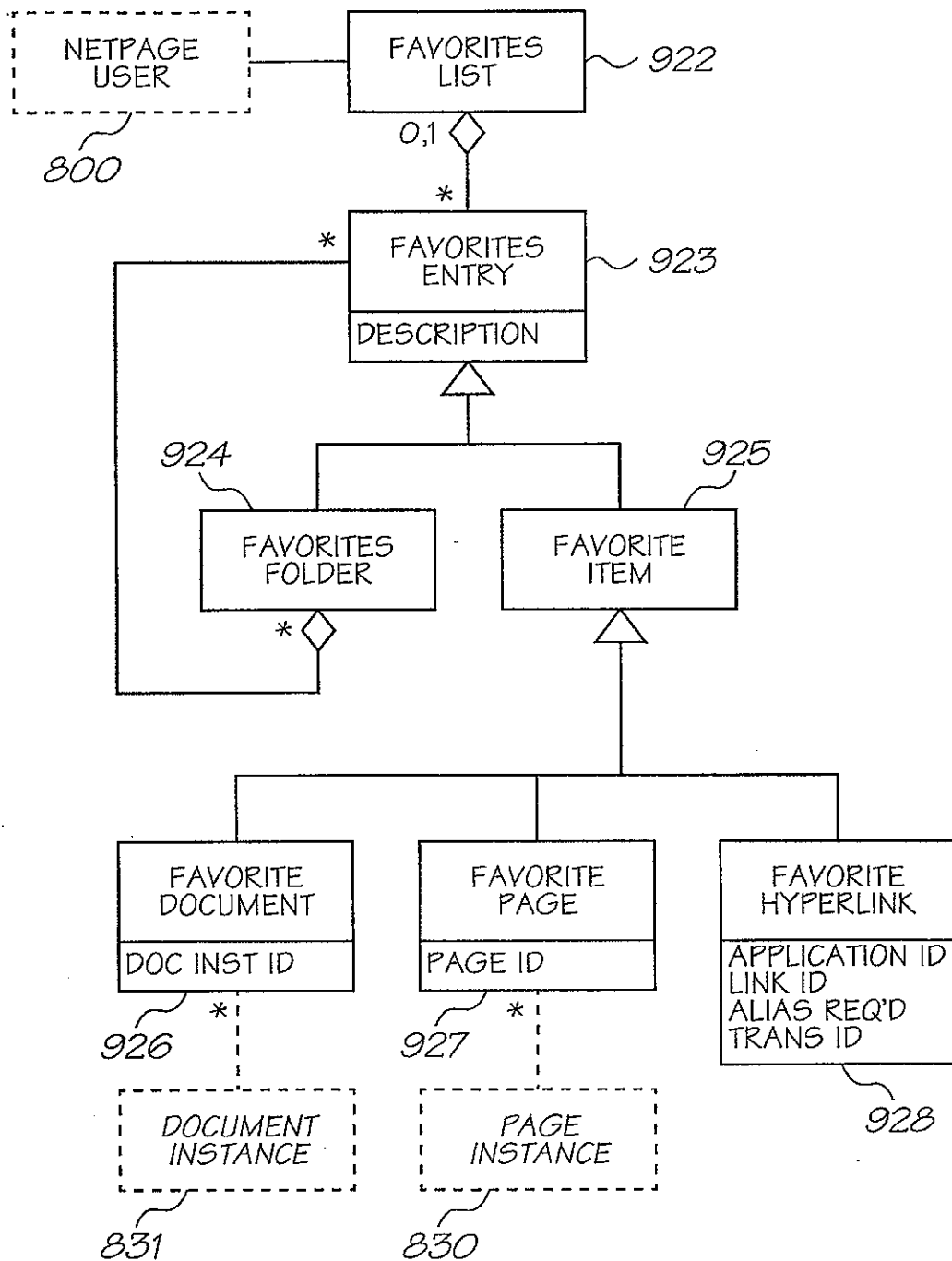


FIG. 41

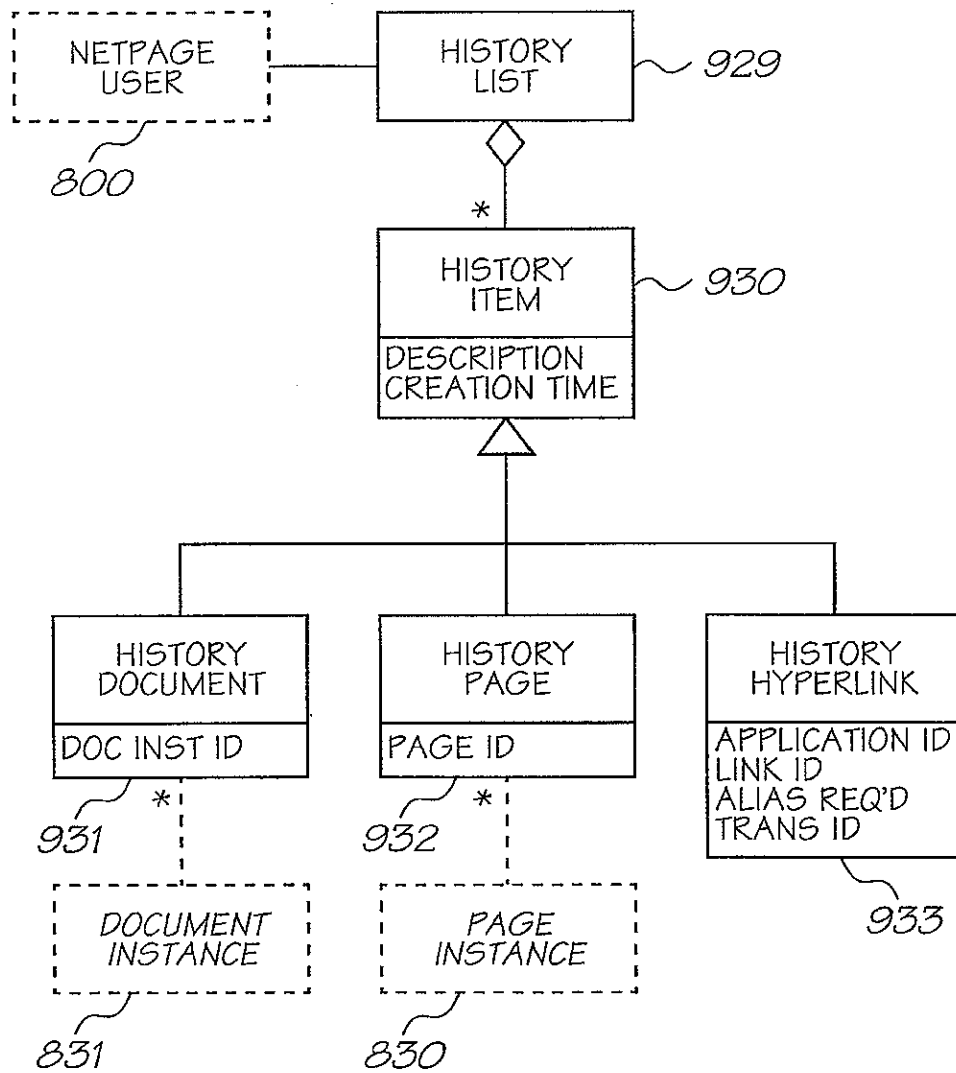


FIG. 42

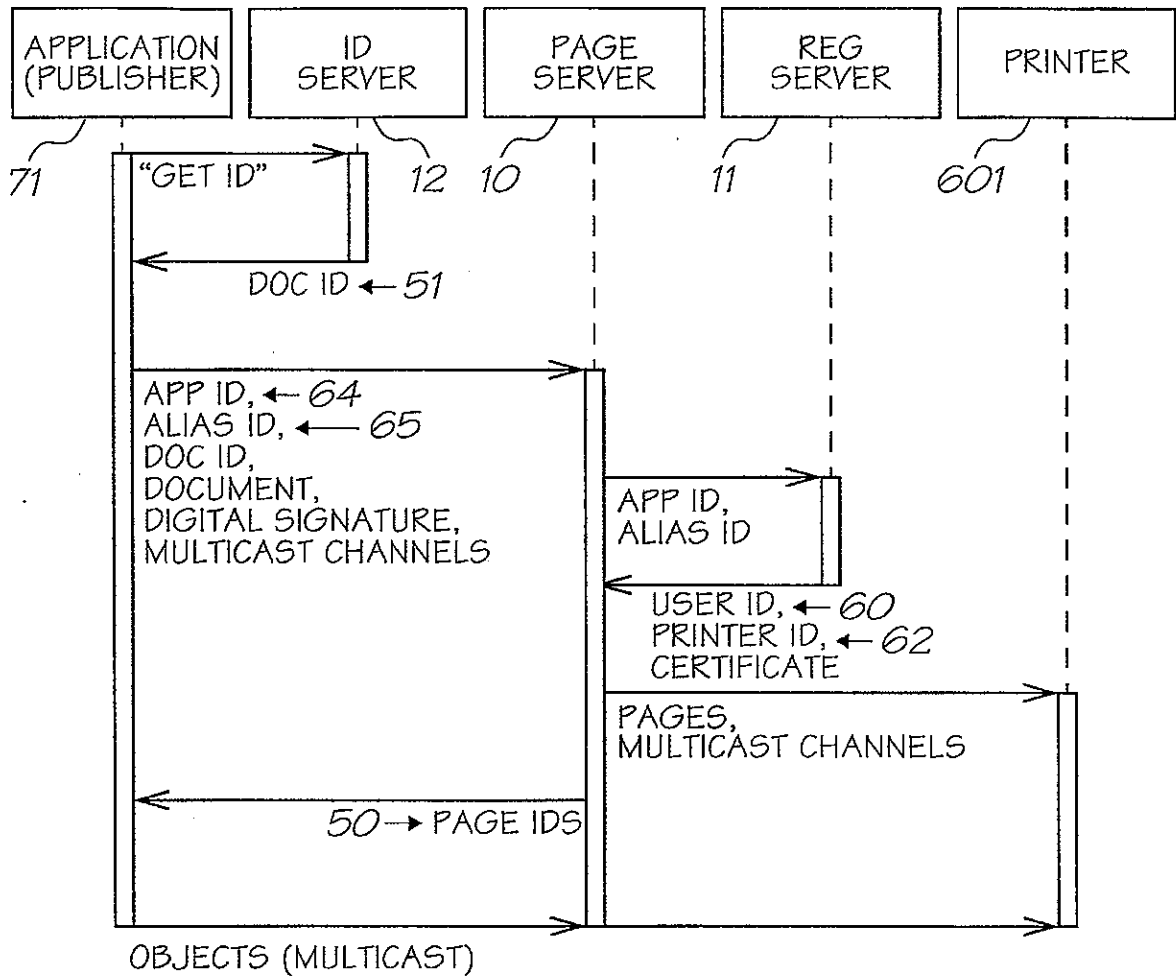


FIG. 43

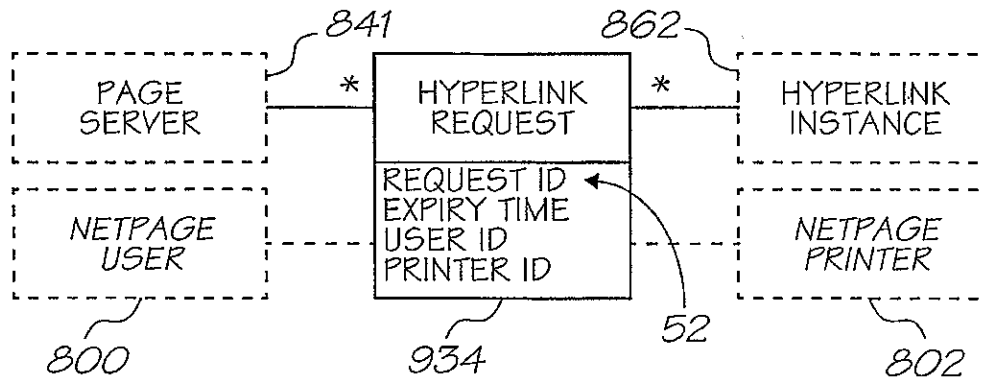


FIG. 44

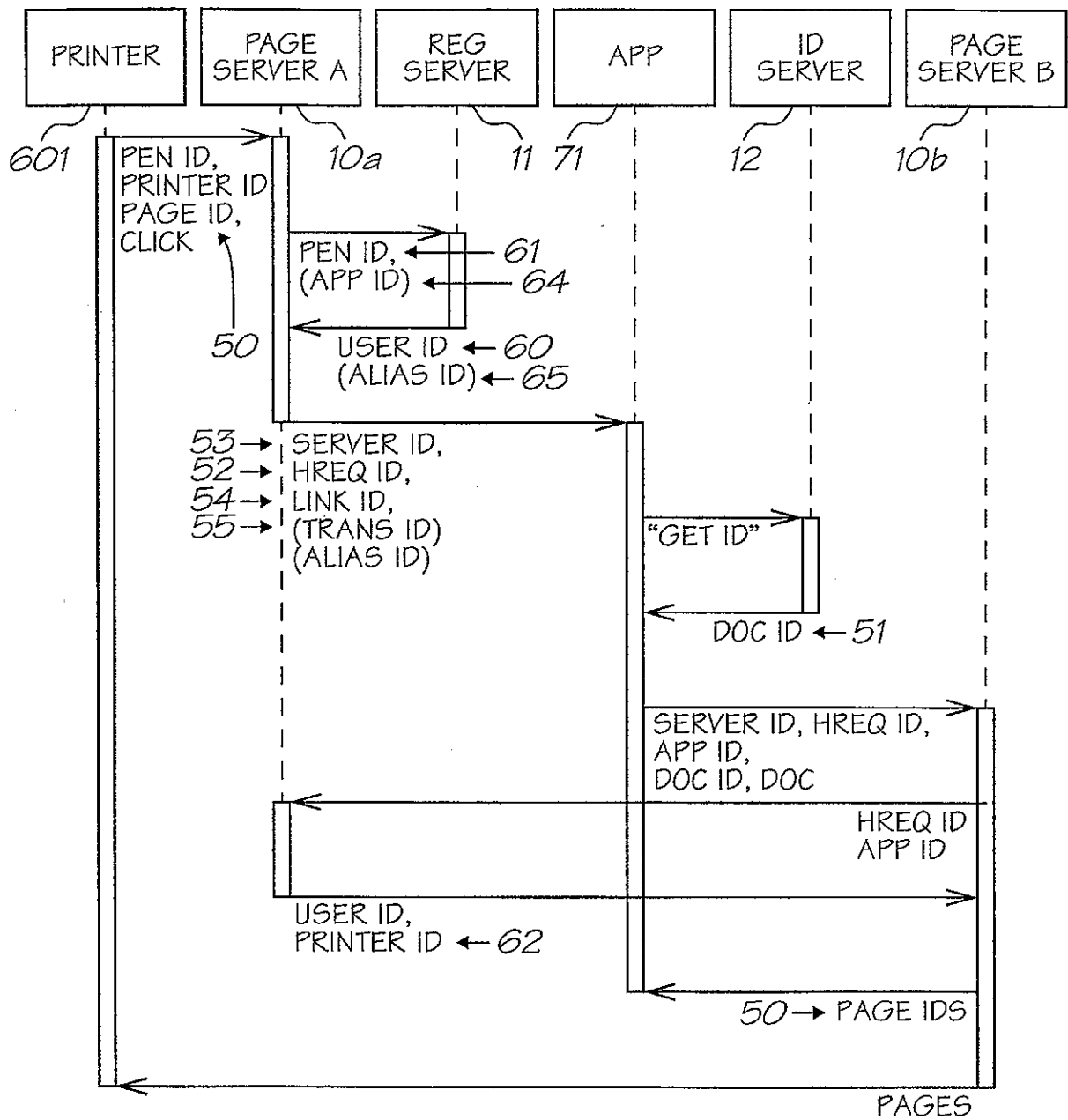


FIG. 45

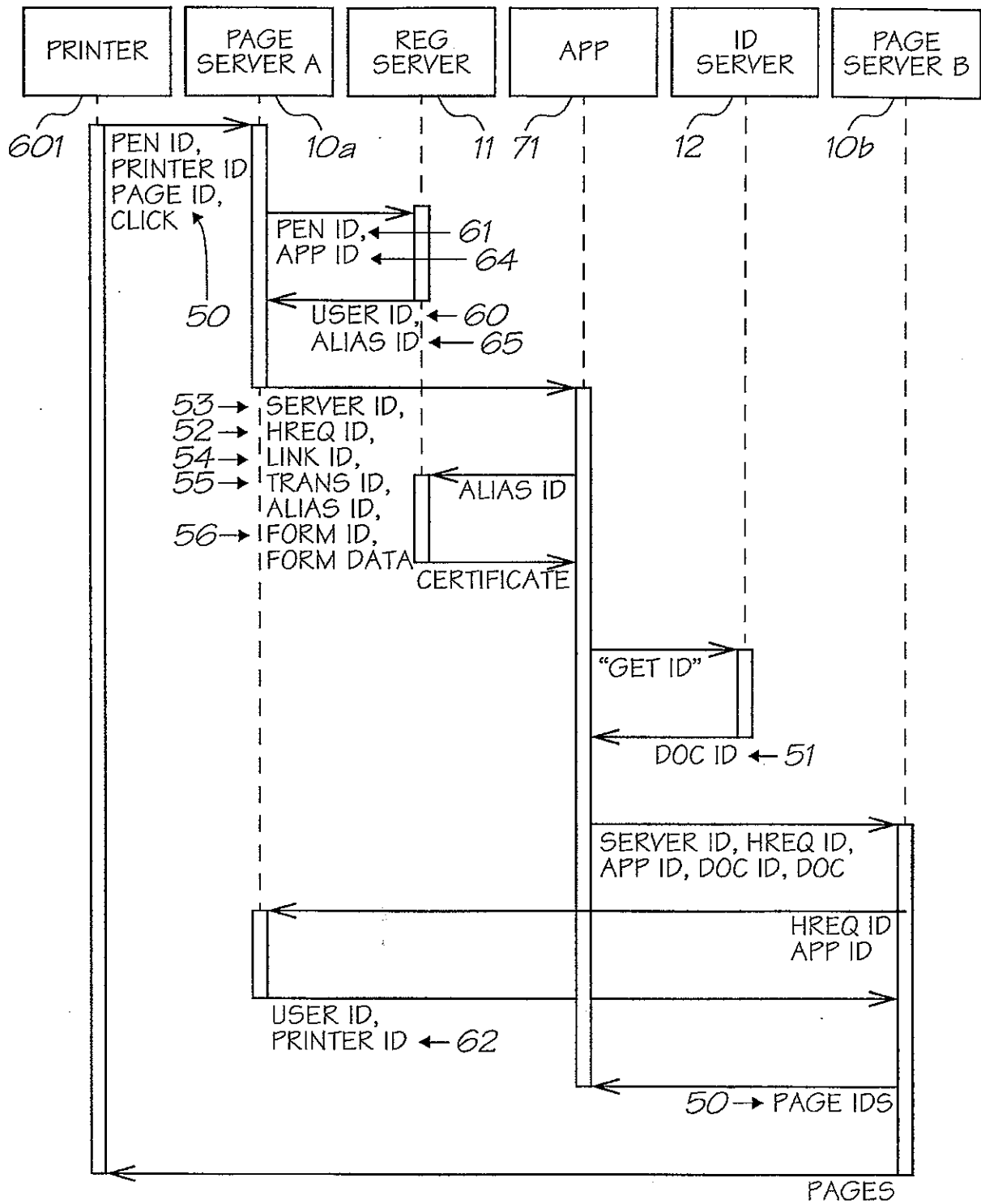


FIG. 46

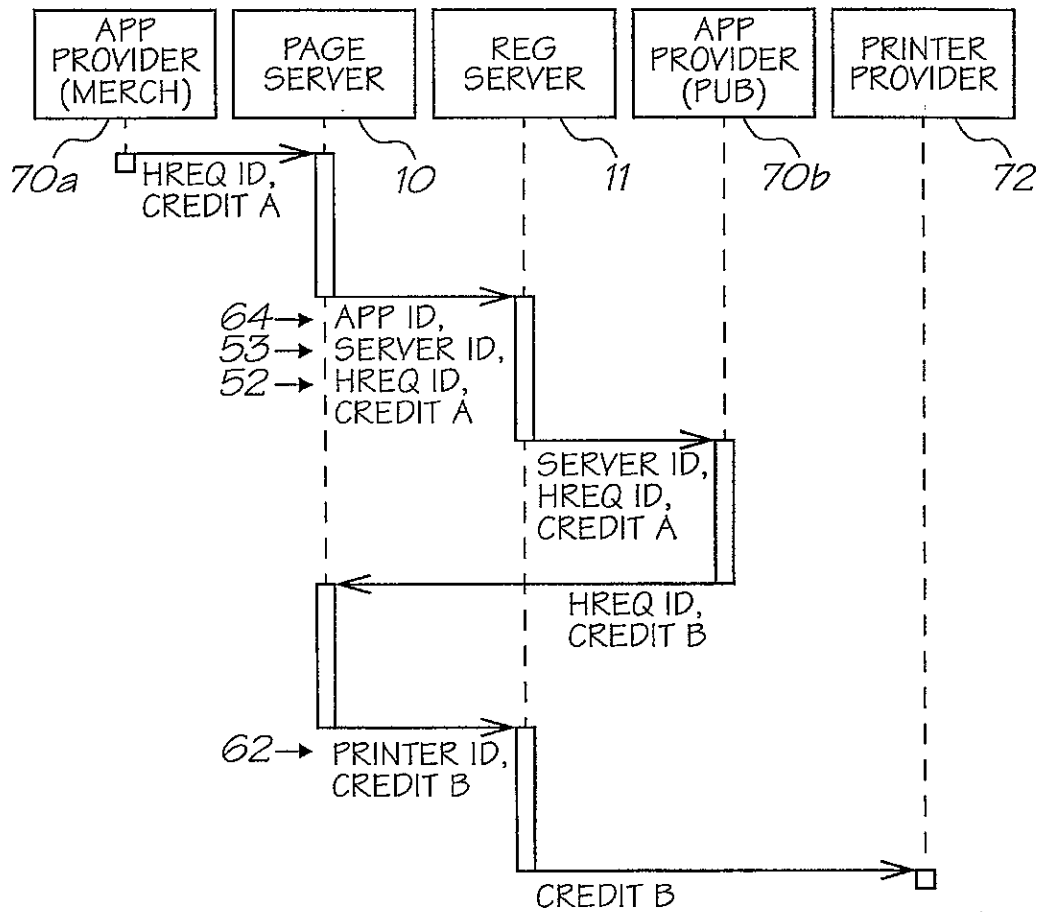


FIG. 47

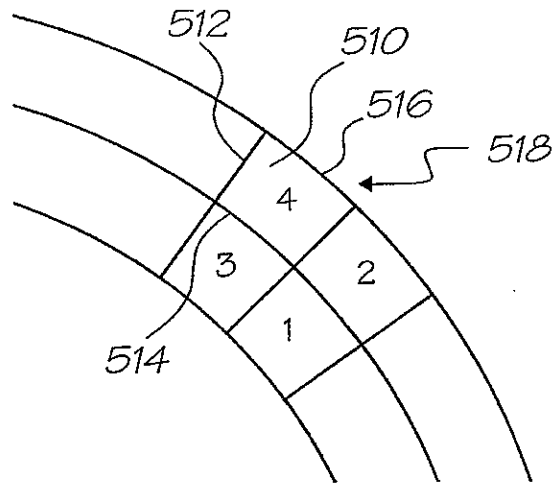


FIG. 48

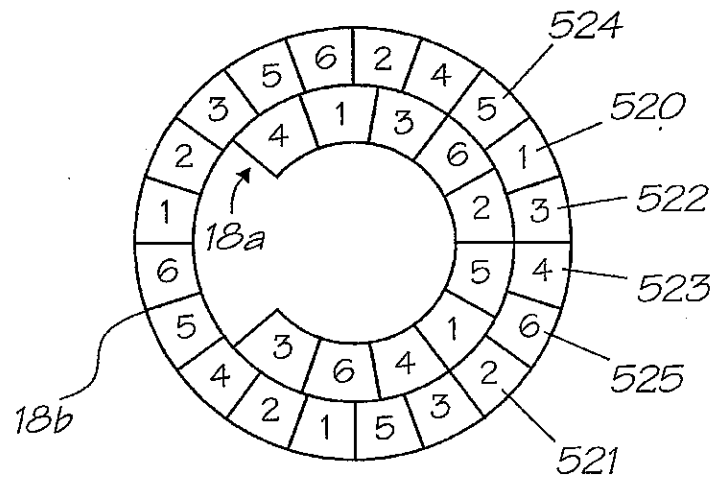


FIG. 49

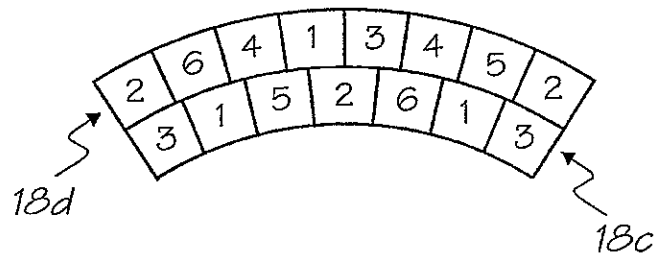


FIG. 50

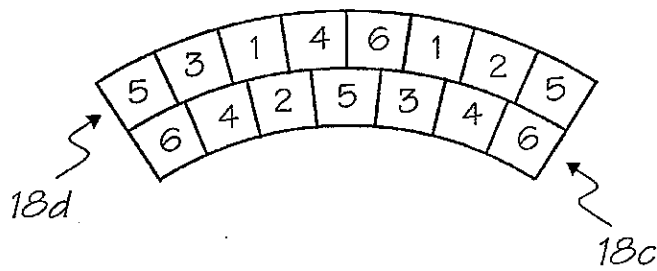


FIG. 51

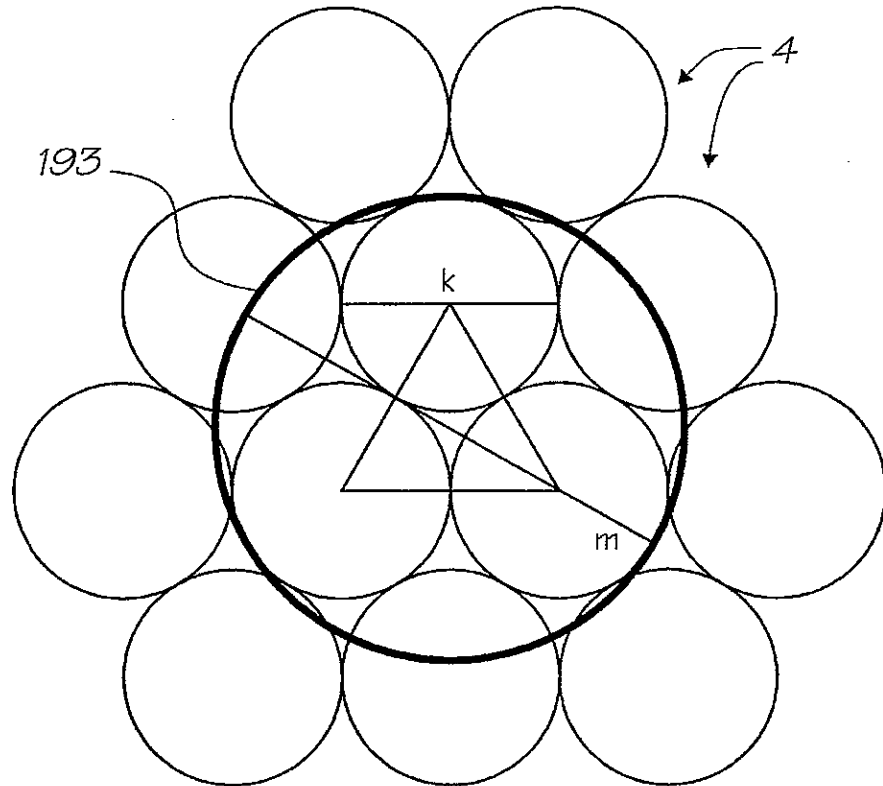


FIG. 52

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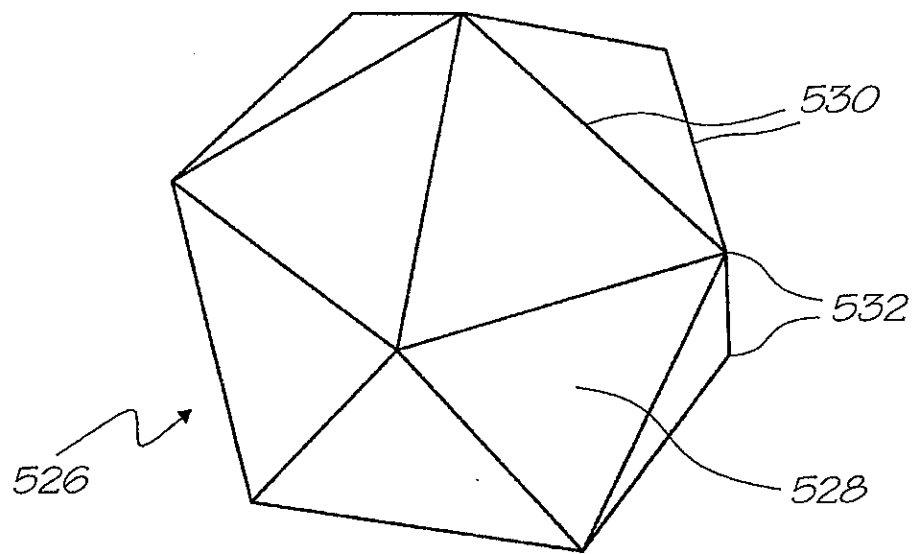


FIG. 53

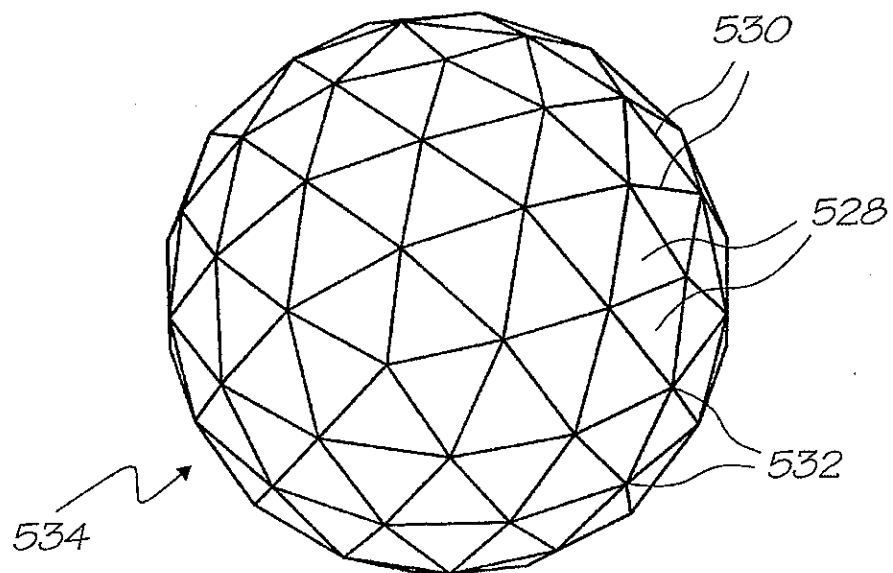


FIG. 54

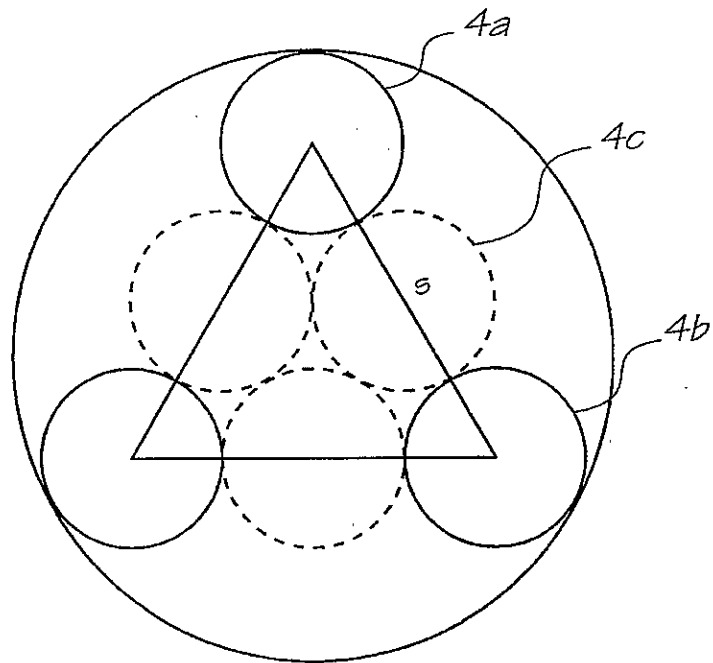


FIG. 55

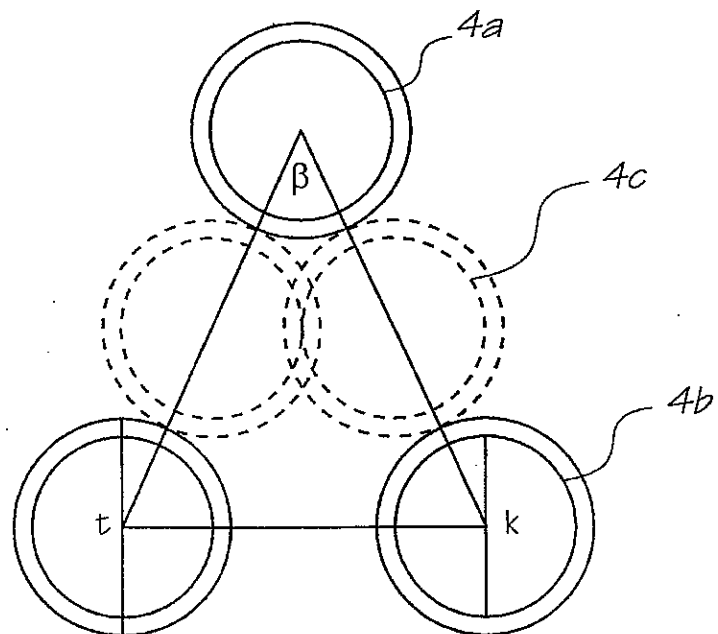


FIG. 56

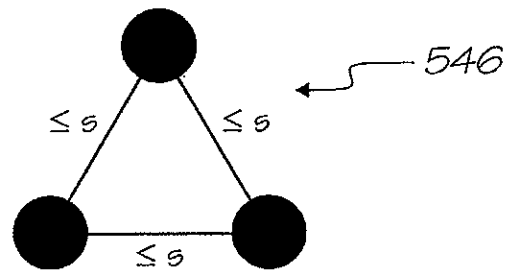


FIG. 57

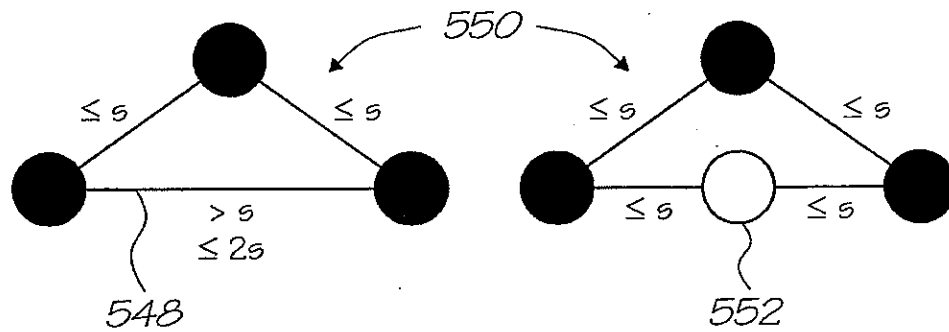


FIG. 58

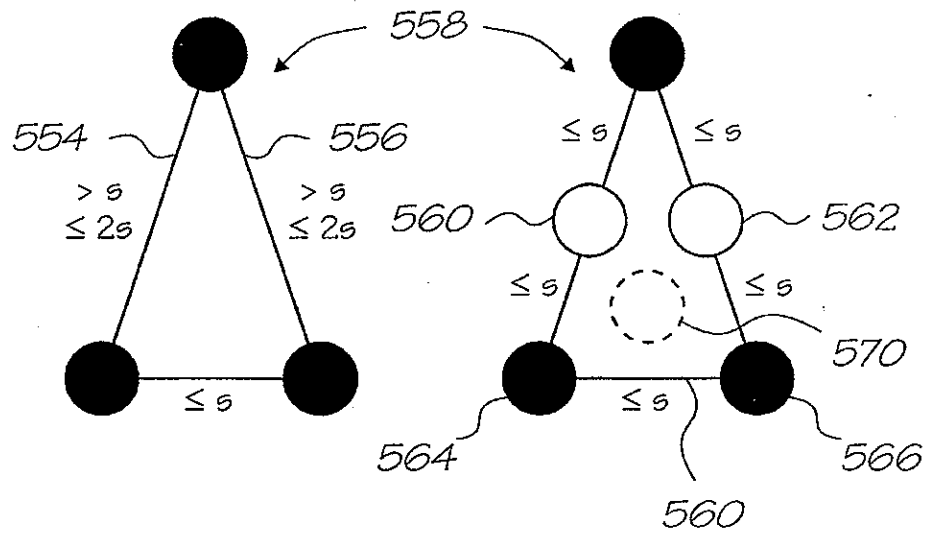


FIG. 59

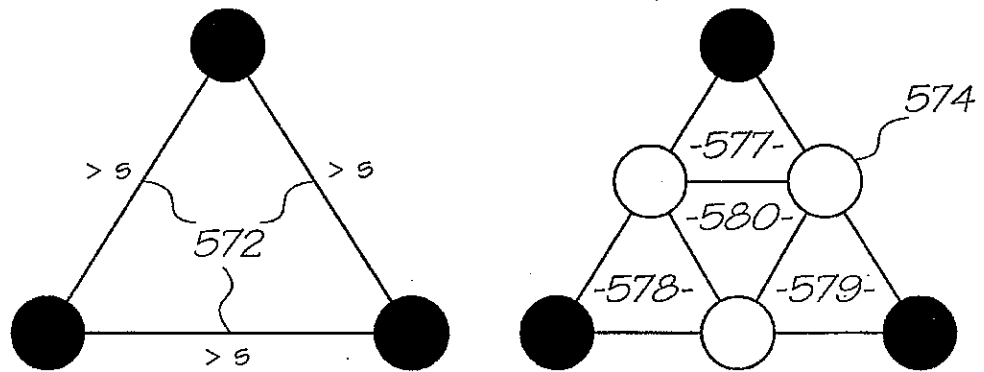


FIG. 60

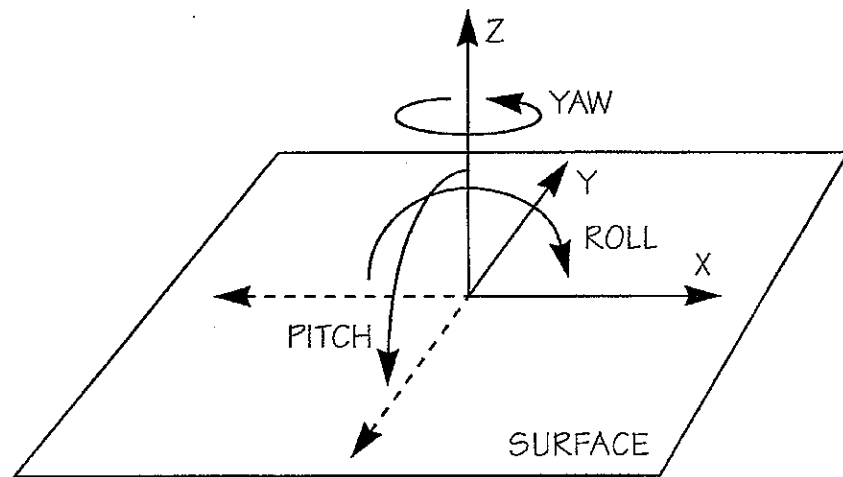


FIG. 61

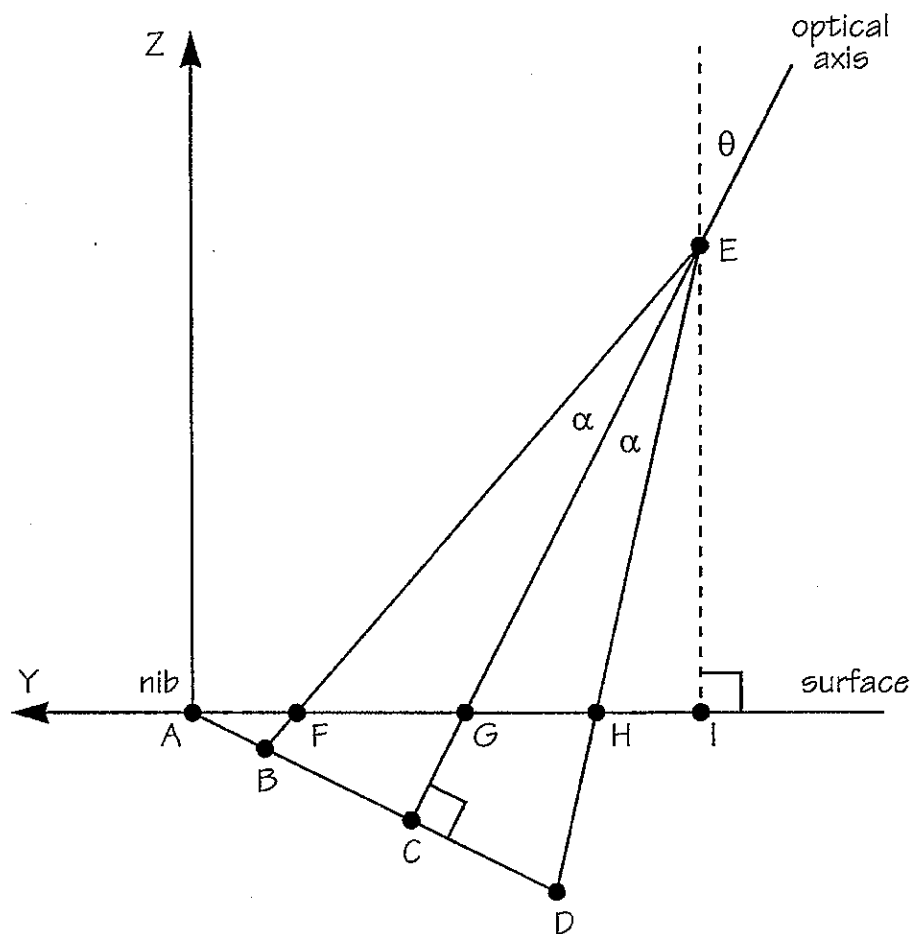


FIG. 62

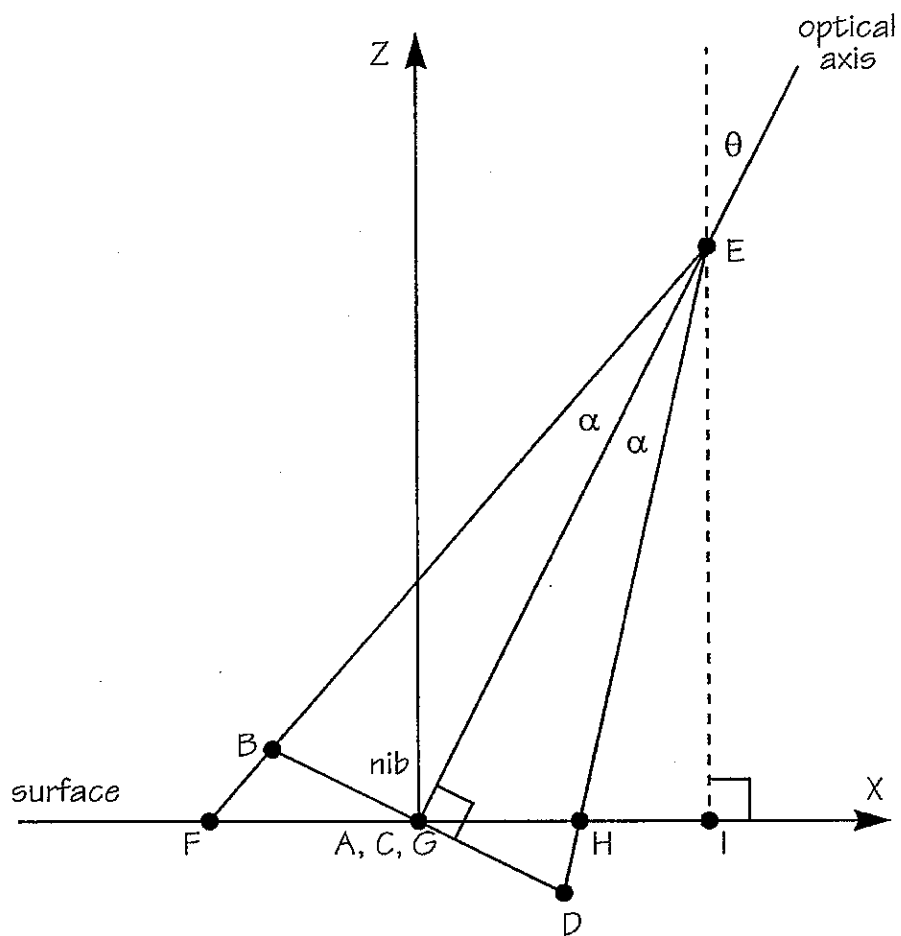


FIG. 63

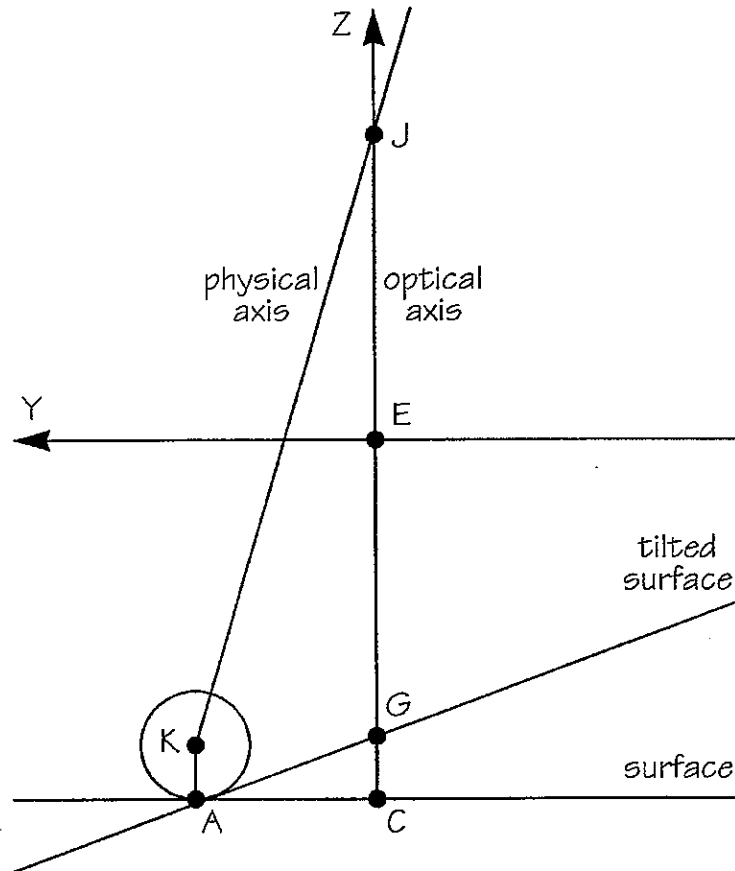


FIG. 64

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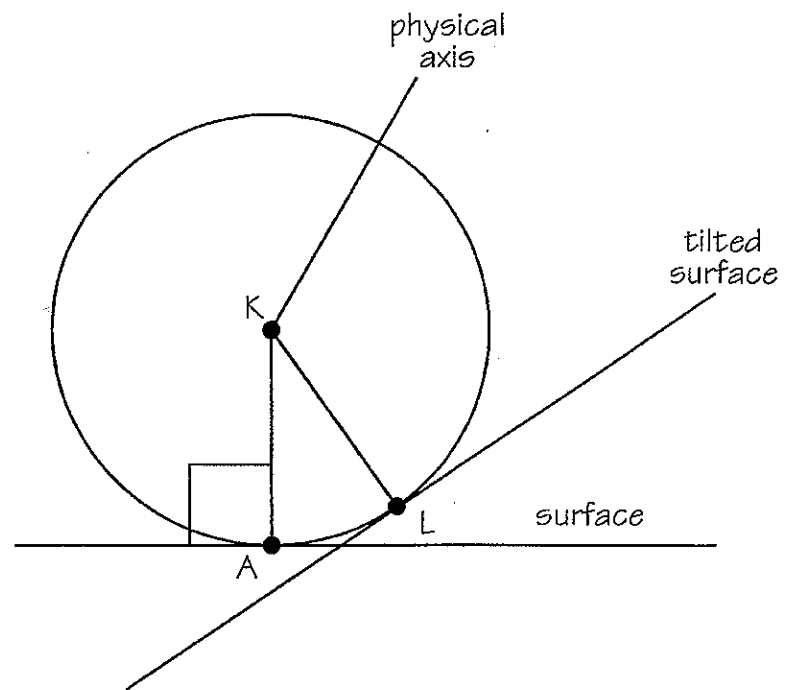


FIG. 65

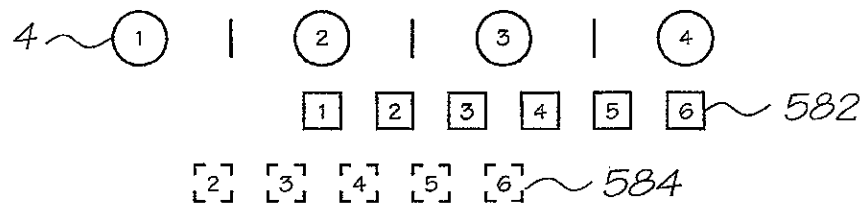


FIG. 66

EQ NUMBER	EQUATION
1	$m = k\left(\frac{2}{\sqrt{3}} + 1\right)$
2	$m = \frac{2s}{\sqrt{3}} + k$
3	$u = k\left(\frac{2}{\sqrt{3}} - 1\right)$
4	$m = \frac{7k}{3}$
5	$\theta = 2\arcsin\left(\frac{2}{\sqrt{10 + 2\sqrt{5}}}\right) \cong 63.4^\circ \cong 1.11 \text{ radians}$
6	$n = 10v^2 + 2 = 10\left[\frac{\theta r}{K}\right]^2 + 2$
7	$r \leq \frac{K}{\theta} \left[\sqrt{\frac{n-2}{10}} \right]$
8	$s \geq 2k$
9	$\beta = 2\arcsin\frac{k}{2t}$
10	$s \geq 2t$

FIG. 67

EQ NUMBER	EQUATION
11	$\sin \theta = \cos \phi \cos \psi$
12	$2D \tan \alpha$
13	$S = T + D \tan \alpha$
14	$d = D - S \tan \theta$
15	$d \cos \theta (\tan(\theta + \alpha) - \tan(\theta - \alpha))$
16	$d \cos \theta (\tan(\theta + \alpha) - \tan(\theta - \alpha)) \geq m$
17	$\frac{d \cos \theta}{\cos(\theta + \alpha)}$
18	$\cos(\theta + \alpha)$
19	$\omega = \frac{fd \cos \theta}{\cos^2(\theta + \alpha)}$
20	$\omega_0 = \frac{fD}{\cos^2 \alpha}$

FIG. 68

EQ NUMBER	EQUATION
21	$q = 2\alpha'n\omega\cos^2(\theta' + \alpha')$
22	$q = \frac{2\alpha'nfd\cos\theta\cos^2(\theta' + \alpha')}{\cos^2(\theta + \alpha)}$
23	$q = \frac{2\alpha nfd\cos\theta\cos^2\alpha}{\cos^2(\theta + \alpha)}$
24	$q = \frac{2\alpha nfd}{\cos\theta(1 - \tan \theta \tan\alpha)^2}$
25	$q_0 = 2\alpha nfd$
26	$q = 2\alpha nfd\cos\theta$

FIG. 69

EQ NUMBER	EQUATION
27	$P_{sensed} = \begin{pmatrix} 0 \\ 0 \\ -d \end{pmatrix}$
28	$P_{pivot} = \begin{pmatrix} 0 \\ S \\ R - D \end{pmatrix}$
29	$\vec{V}_{pivot-sensed} = P_{pivot} - P_{sensed} = \begin{bmatrix} 0 \\ S \\ R - D + d \end{bmatrix}$
30	$\vec{N} = M \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
31	$\vec{V}_{contact-pivot} = R \frac{\vec{N}}{ \vec{N} }$
32	$\vec{V}_{contact-sensed} = \vec{V}_{contact-pivot} - \vec{V}_{pivot-sensed}$
33	$\vec{V}_{contact-tag} = \vec{V}_{sensed-tag} + M^{-1} \vec{V}_{contact-sensed}$
34	$P_{contact} = P_{tag} + \vec{V}_{contact-tag}$

FIG. 70

EQ NUMBER	EQUATION
35	$\gamma_{pen} = -\gamma_{tag}$
36	$\phi_{pen} = -\phi_{tag} - \phi_{sensor}$
37	$\psi_{pen} = -\psi_{tag}$
38	$\lambda_S < \frac{\lambda_T}{2}$
39	$P_{i+1} - P_i = \lambda_S$
40	$ \lambda_T - (P_{i+1} - P_i) = \lambda_S$
41	$\lambda_S = \frac{\lambda_T}{2}$

FIG. 71

EQ NUM	EQUATION
42	$M_1 = T_{xy} = \begin{bmatrix} 1 & 0 & 0 & A \\ 0 & 1 & 0 & B \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
43	$R_z = \begin{bmatrix} C & -D & 0 & 0 \\ D & C & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
44	$M_2 = R_z M_1 = \begin{bmatrix} C & -D & 0 & AC-BD \\ D & C & 0 & AD+BC \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

FIG. 72

EQ NUM	EQUATION			
45	$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & E & -F & 0 \\ 0 & F & E & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$			
46	$M_3 = R_x M_2 = \begin{bmatrix} C & -D & 0 & AC - BD \\ DE & CE & -F & ADE + BCE \\ DF & CF & E & ADF + BCF \\ 0 & 0 & 0 & 1 \end{bmatrix}$			
47	$R_y = \begin{bmatrix} G & 0 & H & 0 \\ 0 & 1 & 0 & 0 \\ -H & 0 & G & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$			

FIG. 73

EQ NUM	EQUATION
48	$M_4 = R_y M_3 = \begin{bmatrix} CG + DFH & CFH - DG & EH & GK + HL \\ DE & CE & -F & ADE + BCE \\ DFG - CH & DH + CFG & EG & GL - HK \\ 0 & 0 & 0 & 1 \end{bmatrix}$
49	$K = AC - BD$
50	$L = ADF + BCF$
51	$T_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

FIG. 74

EQ NUM	EQUATION				
52	$M_5 = T_z M_4 = \begin{bmatrix} CG + DFH & CFH - DG & EH & GK + HL \\ DE & CE & -F & ADE + BCE \\ DFG - CH & DH + CFG & EG & GL - HK + I \\ 0 & 0 & 0 & 1 \end{bmatrix}$				
53	$M_p = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & J & 1 \end{bmatrix}$				
54	$M_6 = M_p M_5 = \begin{bmatrix} CG + DFH & CFH - DG & EH & GK + HL \\ DE & CE & -F & ADE + BCE \\ 0 & 0 & 0 & 0 \\ J(DFG - CH) & J(DH + CFG) & EGJ & J(GL - HK + I) + 1 \end{bmatrix}$				

FIG. 75

EQ NUM	EQUATION
55	$M_s = \begin{bmatrix} S & 0 & 0 & 0 \\ 0 & S & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
56	$M_7 = M_s M_6 = \begin{bmatrix} S(CG + DFH) & S(CFH - DG) & SEH & S(GK + HL) \\ SDE & SCE & -SF & S(ADE + BCE) \\ 0 & 0 & 0 & 0 \\ J(DFG - CH) & J(DH + CFG) & EGJ & J(GL - HK + I) + 1 \end{bmatrix}$
57	$P_1 = M_7 \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$

FIG. 76

EQ NUM	EQUATION
58	$P_1 = \frac{S(CG + DFH)x + S(CFH - DG)y + S(GK + HL)}{SDEx + SCEy + S(ADE + BCE)}$ $J(DFG - CH)x + J(DH + CFG)y + J(GL - HK + I) + 1$
59	$P_1 = \frac{S(CG + DFH)x + S(CFH - DG)y + S(ACG - BDG + ADFH + BCFH)}{SDEx + SCEy + S(ADE + BCE)}$ $J(DFG - CH)x + J(DH + CFG)y + J(BDH - ACH + ADFG + BCFG + I) + 1$

FIG. 77

EQ NUMBER	EQUATION
60	$M_{p2Dinf} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & 1 \end{bmatrix}$
61	$M_{p2D} = \begin{bmatrix} ai & bi & ci \\ di & ei & fi \\ gi & hi & i \end{bmatrix}$
62	$P_2 = M_{per2D} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
63	$P_2 = \begin{bmatrix} aix + biy + ci \\ dix + eiy + fi \\ gix + hiy + i \end{bmatrix}$

FIG. 78

EQ NUMBER	EQUATION
64	$\frac{ai}{S} = CG + DFH$
65	$\frac{bi}{S} = CFH - DG$
66	$\frac{ci}{S} = ACG - BDG + ADFH + BCFH$
67	$\frac{di}{S} = DE$
68	$\frac{ei}{S} = CE$
69	$\frac{fi}{S} = ADE + BCE$
70	$gi = J(DFG - CH)$
71	$hi = J(DH + CFG)$
72	$i = J(BDH - ACH + ADFG + BCFG + I) + 1$
73	$\sin^2 \theta + \cos^2 \theta = 1$
74	$\theta = \arctan(\sin \theta, \cos \theta)$

FIG. 79

EQ NUMBER	EQUATION
75	$\frac{ci}{S} = A(CG + DFH) + B(CFH - DG)$
76	$c = Aa + Bb$
77	$\frac{fi}{S} = A(DE) + B(CE)$
78	$f = Ad + Be$
79	$i = AJ(DFG - CH) + BJ(DH + CFG) + IJ + 1$
80	$i = Agi + Bhi + IJ + 1$
81	$A = \frac{c-Bb}{a}$
82	$A = \frac{f-Be}{d}$
83	$\frac{c-Bb}{a} = \frac{f-Be}{d}$
84	$cd - Bbd = af - Bae$
85	$B(ae - bd) = af - cd$
86	$B = \frac{af - cd}{ae - bd}$

FIG. 80

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EQ NUMBER	EQUATION
87	$A = \frac{f - \left(\frac{af - cd}{ae - bd} \right) e}{d}$
88	$A = \frac{f(ae - bd) - e(af - cd)}{d(ae - bd)}$
89	$A = \frac{aef - bdf - aef + cde}{d(ae - bd)}$
90	$A = \frac{-bdf + cde}{d(ae - bd)}$
91	$A = \frac{-bf + ce}{ae - bd}$
92	$C = \frac{ei}{ES}$
93	$D = \frac{di}{ES}$
94	$a = \frac{e}{E}G + \frac{d}{E}FH$
95	$b = \frac{-d}{E}G + \frac{e}{E}FH$
96	$g = \frac{-e}{ES}H + \frac{d}{ES}FG$
97	$h = \frac{d}{ES}H + \frac{e}{ES}FG$
98	$E = \frac{eG + dFH}{a}$
99	$E = \frac{-dG + eFH}{b}$

FIG. 81

EQ NUMBER	EQUATION
100	$ES = \frac{-eH + dFG}{g}$
101	$ES = \frac{dH + eFG}{h}$
102	$\frac{eG + dFH}{a} = \frac{-dG + eFH}{b}$
103	$\frac{FH}{G} = \frac{ad + be}{ae - bd}$
104	$\frac{-eH + dFG}{g} = \frac{dH + eFG}{h}$
105	$\frac{FG}{H} = \frac{dg + eh}{dh - eg}$
106	$\frac{FH}{G} \times \frac{FG}{H} = \frac{(ad + be)(dg + eh)}{(ae - bd)(dh - eg)} = F^2$
107	$F = \pm \sqrt{\frac{(ad + be)(dg + eh)}{(ae - bd)(dh - eg)}}$
108	$E = \pm \sqrt{1 - F^2}$
109	$\phi = \arctan(E, F)$
110	$\frac{1}{F} \times \frac{FH}{G} = \frac{H}{G} = \left(\pm \sqrt{\frac{(ae - bd)(dh - eg)}{(ad + be)(dg + eh)}} \right) \times \frac{(ad + be)}{(ae - bd)}$ $= \pm \sqrt{\frac{(ad + be)(dh - eg)}{(ae - bd)(dg + eh)}}$
111	$\frac{H}{G} = \frac{H}{\pm \sqrt{1 - H^2}} = \frac{1}{\pm \sqrt{\frac{1}{H^2} - 1}}$

FIG. 82

EQ NUMBER	EQUATION
112	$H = \frac{1}{\pm \sqrt{\frac{1}{\left(\frac{H}{G}\right)^2} + 1}}$
113	$G = \pm \sqrt{1 - H^2}$
114	$\psi = \arctan(G, H)$
115	$\left(\frac{ei}{ES}\right)^2 + \left(\frac{di}{ES}\right)^2 = 1$
116	$\frac{S}{i} = \frac{\pm \sqrt{d^2 + e^2}}{E}$
117	$C = \frac{e}{E} \times \frac{E}{\pm \sqrt{d^2 + e^2}}$
118	$C = \frac{e}{\pm \sqrt{d^2 + e^2}}$
119	$D = \frac{d}{E} \times \frac{E}{\pm \sqrt{d^2 + e^2}}$
120	$D = \frac{d}{\pm \sqrt{d^2 + e^2}}$

FIG. 83

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EQ NUMBER	EQUATION
121	$i(1 - Ag - Bh) = IJ + 1$
122	$\text{sign}(i) = -\text{sign}(1 - Ag - Bh)$
123	$\gamma = \arctan(D, C)$
124	$S = \frac{di}{DE}$
125	$S = \frac{ei}{CE}$
126	$\text{sign}(FH) = \text{sign}\left(\frac{ad + be}{ae - bd}\right)$
127	$J = \left \frac{gi}{-CH + \text{sign}(FH)DFG} \right $
128	$J = \left \frac{hi}{DH + \text{sign}(FH)CFG} \right $
129	$I = \frac{(i - Agi - Bhi - 1)}{J}$

FIG. 84

case	C,D	E,F	G,H	ai	bi	ci	di	ei	fi	gi	hi
1a	$\pm 1,0$	1,0	1,0	$\pm S$	0	$\pm AS$	0	$\pm S$	$\pm BS$	0	0
1b	$0, \pm 1$	1,0	1,0	0	$\pm(-S)$	$\pm(-BS)$	$\pm S$	0	$\pm AS$	0	0
1c	C,D	1,0	1,0	CS	-DS	Aai+Bbi	DS	CS	BS	0	0
2a	$\pm 1,0$	E,F	1,0	$\pm S$	0	$\pm AS$	0	$\pm ES$	Bei	0	$\pm FJ$
2b	$0, \pm 1$	E,F	1,0	0	$\pm(-S)$	$\pm(-BS)$	$\pm ES$	0	Adi	$\pm FJ$	0
2c	C,D	E,F	1,0	CS	-DS	Aai+Bbi	DES	CES	Adi+Bei	DFJ	CFJ
3a	$\pm 1,0$	1,0	G,H	$\pm GS$	0	Aai	0	$\pm S$	$\pm BS$	$\pm(-HJ)$	0
3b	$0, \pm 1$	1,0	G,H	0	$\pm(-GS)$	Bbi	$\pm S$	0	$\pm AS$	0	$\pm HJ$
3c	C,D	1,0	G,H	CGS	-DGS	Aai+Bbi	DS	CS	Adi+Bei	-CHJ	DHJ
4a	$\pm 1,0$	E,F	G,H	$\pm GS$	$\pm FHS$	Aai+Bbi	0	$\pm ES$	Bei	$\pm(-HJ)$	$\pm FGJ$
4b	$0, \pm 1$	E,F	G,H	$\pm FHS$	$\pm(-GS)$	Aai+Bbi	$\pm ES$	0	Adi	$\pm FGJ$	$\pm HJ$
4c	C,D	E,F	G,H	CGS+ DFHS	-DGS+ CFHS	Aai+Bbi	DES	CES	Adi+Bei	-CHJ+ DFGJ	DHJ+ CFGJ

FIG. 85

description	case	condition	handling
zero pitch & zero roll	1	$g = h = 0$	$E \leftarrow 1$ $F \leftarrow 0$ $G \leftarrow 1$ $H \leftarrow 0$
zero roll	2a	$b = d = g = 0$	$E \leftarrow \frac{e}{a}$ $\frac{FJ}{S} = \frac{h}{a}$
	2b	$a = e = h = 0$	$E \leftarrow \frac{-d}{b}$ $\frac{FJ}{S} = \frac{-g}{b}$
	2c	$\frac{a}{b} = \frac{-h}{g}$	handle via 2a or 2b
	2		$G \leftarrow 1$ $H \leftarrow 0$ $F \leftarrow \text{sign}\left(\frac{FJ}{S}\right) \sqrt{1 - E^2}$

FIG. 86

description	case	condition	handling
zero pitch	3a	$b = d = h = 0$	$G \leftarrow \frac{a}{e}$ $\frac{HJ}{S} = \frac{-g}{e}$
	3b	$a = e = g = 0$	$G \leftarrow \frac{-b}{d}$ $\frac{HJ}{S} = \frac{h}{d}$
	3c	$\frac{a}{b} = \frac{g}{h}$	handle via 3a or 3b
	3		$E \leftarrow 1$ $F \leftarrow 0$ $H = \text{sign}\left(\frac{HJ}{S}\right) \sqrt{1 - G^2}$
non-zero pitch & non-zero roll	4	$(g \neq 0) \wedge (h \neq 0)$	handle via general solution

FIG. 87