

METHOD FOR FIELD SCALE PRODUCTION OPTIMIZATION

TECHNICAL FIELDS

The present invention relates generally to methods for controlling hydrocarbon production from a field of wells, and more particularly, to methods for optimizing production by enhancing fluid flow rate allocations among the wells.

BACKGROUND OF THE INVENTION

Field scale optimization is known which attempts to optimize or enhance the production of production fluids, including hydrocarbons, from a field containing one or more subterranean reservoirs. Wells or well bores connect the reservoirs with surface facilities which collect and process the captured production fluids. Typically, these production fluids include the components of oil, gas and water. Chokes or flow control devices are used to adjust the allocation of flow rates among the well bores in a field. The relative quantities and ratios of production of the different components of oil, gas and water for an individual well bore can be controlled by adjusting a choke to change the pressure in a well bore.

Surface facilities are needed to produce and process the production fluids. These facilities may include apparatus such as separators, pumps, storage tanks, compressors, etc. Ideally, the capital expenditures on these facilities are minimized by employing the smallest and least expensive surface facilities possible. However, fluid handling capacity should be sufficiently large so as not to unduly limit the production rate of the economically desirable oil and/or gas. Hence, the allocation of fluid flow in the well bores is ideally optimized to maximize monetary return while meeting production constraints such as those imposed by the fluid handling capacities of the surface facilities.

1 Optimization techniques are used predict the optimal allocation of fluid flows
2 in well bores for a given set of production constraints. First, a reservoir
3 simulator is used to mathematically model the flow of fluids throughout a field
4 including the reservoirs and well bores. The simulated flow is used to
5 establish component flow rate curves or rate equations for each well bore
6 which describe how the flow rate of one component, such as water, relates to
7 the flow rate of another component, i.e., oil. Typically, an objective function is
8 created which seeks to optimize an objective such as maximizing oil
9 production or minimizing water production. The objective function
10 incorporates the flow rates from the well bores which are predicted by the
11 reservoir simulation. A set of production constraints, such as oil production
12 targets or gas or water production limitations for the field, are specified.
13 Constraint equations are generated to meet these production constraints.
14 The fluid flow among the well bores must adhere to these production
15 constraints. The objective function is then optimized by a subroutine, referred
16 to as an optimizer, to determine the optimal allocation of flow rates among the
17 well bores. The optimizer utilizes the well bore component flow rate equations
18 and constraint equations in the optimization process.

19
20 A first shortcoming of typical field scale optimization schemes is that feasible
21 solutions to an optimization may not be possible for specified production
22 constraints. For example, a certain level of oil production may be desired
23 while not producing more than a specified quantity of water. A feasible
24 solution to the objective function with this set of constraints may not be
25 possible. In this event, one or more of the constraints must be adjusted and
26 the reservoir simulator and optimizer run again to determine when a feasible
27 solution is possible. Such iterative runs in solving numerous optimizations of
28 the objective function are computationally intensive and undesirable.

29
30 A second problem in some optimization schemes is that while a feasible
31 solution to the optimization of the objective function may be achieved, the
32 results may not be practical. For example, in a first run or time step, the
33 optimizer may determine that a first well bore should produce at a high level

1 while a second well bore is substantially closed down. In the next time step,
2 the optimizer may suggest that the second well bore produce at a high level
3 while the first well bore is substantially shut down. Therefore, production from
4 the well bores may oscillate if the suggested allocations from the optimizer are
5 followed. Generally, it is more practical if the production from well bores
6 having similar fluid flow characteristics are at a consistent level. This would
7 minimize the oscillations in production from the related well bores over time
8 steps.

9
10 A third shortcoming is that creating component flow rate curves or equations
11 for the production of fluids from a well bore can be computationally intensive.
12 One method of calculating these rate curves is to create a sub model of the
13 well bores and surrounding reservoirs and iteratively solve for the production
14 rates of the components, i.e., oil, gas and water, as the chokes are opened
15 and the pressure draw downs between the reservoirs and the well bores are
16 increased. Typically, several Newton iterations must be performed to produce
17 each data point relating the production of one component relative to another
18 component for a given pressure draw down in a well bore. Again, the
19 pressure draw down in a well bore is related to how open is a choke
20 controlling the well bore. This process is repeated many times until enough
21 data points, perhaps as many as 30-50 data points, have been calculated
22 such that an overall flow rate curve or equation can be developed. The
23 optimizer then uses the rates curves or equations during the optimization of
24 the objective function. Generating data points using these many Newton
25 iterations to create rate curves or equations is computationally costly.

26
27 The present invention provides solutions to the above described shortcomings
28 of conventional field scale optimization schemes. First, an objective function
29 and associated constraint equations are generated which can be solved in a
30 single run of an optimizer to produce a feasible solution. Second, constraint
31 equations may be created which requires the rates of production from similar
32 well bores to be related to prevent significant oscillation of well rates between
33 time steps of a reservoir simulation. Finally, an efficient method of generating

1 well bore component flow rate curves or equations relating production rates
2 between fluid components of a well bore is described.

3

4 SUMMARY OF THE INVENTION

5

6 The present invention includes a method for enhancing the allocation of fluid
7 flow rates among a plurality of well bores in fluid communication with at least
8 one subterranean reservoir. Fluid flow is simulated, using a numerical
9 reservoir simulator, in at least one subterranean reservoir and in a number of
10 well bores in fluid communication with the subterranean reservoir.

11 Component flow rate equations are generated from the simulated flow in the
12 well bores. Production constraints are selected with at least one of the
13 production constraints ideally being a soft constraint which may be violated if
14 necessary during an optimization process to provide a feasible solution.
15 Constraint equations corresponding to the production constraints are also
16 generated.

17

18 An objective function is generated which corresponds to the fluid flow in the
19 well bores. The objective function may also include constraint violation
20 penalties which correspond to the soft constraints and soft constraint
21 equations. The objective function is then optimized utilizing the component
22 flow rate equations and the constraint equations to determine an enhanced
23 allocation of fluid flow rates among the well bores. If necessary, soft
24 constraints may be violated to achieve a feasible solution to the optimizing of
25 the objective function. The presence of the constraint violation penalties
26 allows the soft constraints to be violated while still satisfying a corresponding
27 constraint equation. The fluid flow rates are then allocated among the well
28 bores as determined by the optimizing of the objective function.

29

30 The soft constraints may be prioritized as to which of the soft constraints
31 should be most difficult to violate if necessary to achieve a feasible solution to
32 the optimization of the objective function. Weighting scale factors may be
33 associated with the constraint violation penalties in the objective function.

1 The weighting scale factors may be weighted in accordance with the
2 prioritization of the soft constraints to make higher priority soft constraints
3 more difficult to violate than lower priority soft constraints.

4

5 Flow rates between select well bores may have their flow rates related. In
6 particular, well bores exhibiting similar flow characteristics, such as gas-to-oil
7 ratio (*GOR*) or water-to-oil ratio (*WOR*), may have their well rates related to
8 one another. Again, constraint equations can be generated for these related
9 well bore flow rates. The enhanced allocation of flow rates among the related
10 well bores will then be related or tied to one another.

11

12 In another aspect of this invention, the simulated well bores include a plurality
13 of completion elements and the reservoir or reservoirs include a plurality of
14 reservoir elements. The reservoir simulator is run to determine pressures in
15 the reservoir elements and in the completion elements and to determine fluid
16 flows in the completion elements of at least two components, i.e., oil and
17 water, due to the pressure draw down between the reservoir elements and the
18 completion elements. Fluid flow component rate data points are then
19 generated over a range of fluid flows for each well bore. The data points are
20 ideally generated by scaling and summing the fluid flows in the completion
21 elements based upon the component flow rates determined by an initial
22 simulator run and in relation to an incremented range of pressure draw downs
23 between the reservoir and completion elements.

24

25 It is an object of the present invention to provide a method wherein an
26 objective function is created which includes at least one constraint violation
27 penalty corresponding to a soft constraint which allows the objective function
28 to be optimized wherein the soft constraint may be violated if necessary to
29 arrive at a feasible solution for the optimization.

30

31 It is another object to generate an objective function which incorporates
32 weighted constraint violation penalties which may be appropriately weighted
33 so that soft constraints may be violated in a prioritized order.

1 It is yet another object to relate production rates of well bores in an
2 optimization so that the flow rates among those well bores will have related
3 flow rates after an optimization has been performed resulting in limited flow
4 rate oscillations of those well bores between time steps in a reservoir
5 simulation.

6
7 It is still another object to generate component flow rate equations which are
8 generated by scaling component flow rates in individual completions elements
9 based upon flow rates originally determined in a reservoir simulation run and
10 a range of changing pressure profiles within the well bores.

11

12 BRIEF DESCRIPTION OF THE DRAWINGS

13

14 These and other objects, features and advantages of the present invention
15 will become better understood with regard to the following description,
16 pending claims and accompanying drawings where:

17

18 FIG. 1 is a schematic drawing of an exemplary hydrocarbon producing field
19 containing subterranean reservoirs which are fluidly connected by well bores
20 to the surface of the field with chokes being used to control well bore
21 pressures and flow rates so that production from the field may be optimized;

22

23 FIG. 2 is a flowchart of an exemplary method for field scale optimization made
24 in accordance with this invention;

25

26 FIGS. 3A and 3B illustrate component flow rates curves generated using a
27 "quick rates" method made in accordance with the present invention and
28 component flow rate curves generated using a computationally intensive
29 iterative Newton method;

30

31 FIGS. 4A and 4B are graphs showing how well rates are related between a
32 pair of well bores having similar fluid characteristics;

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1 provide fluid communication between reservoirs 22 and 24 and well bore 30,
2 32 and 34. Well bore 34 only connects with upper reservoir 22.

3

4 Chokes or well control devices 54, 56, and 60 are used to control the flow of
5 fluid into and out respective well bores 30, 32 and 34. As will be described
6 more fully below, chokes 54, 56 and 60 also control the pressure profiles in
7 respective well bores 30, 32 and 34. Although not shown, well bores 30, 32
8 and 34 will fluidly connect with surface facilities such as oil/gas/water
9 separators, compressors, storage tanks, pumps, pipelines, etc. The rate of
10 flow of fluids through well bores 30, 32 and 34 may be limited by the fluid
11 handling capacities of these surface facilities.

12

13 FIG. 2 shows a flowchart illustrating the general steps used in accordance
14 with the field scale optimization method of the present invention. Persons
15 skilled in the art of reservoir simulation could easily develop computer
16 software for performing the method outlined in FIG. 2 based on the teachings
17 contained in this description of the invention.

18

19 A reservoir simulator is used to model the fluid flow in field 50 which includes
20 the reservoirs and well bores (step 110). Generally, such a reservoir model
21 will include thousands or even millions of discrete elements to carry out a
22 numerical simulation. These discrete elements comprise reservoir elements
23 and well bore elements. The well bore elements include specific completion
24 elements which transfer fluid back and forth between adjacent reservoir
25 elements and other well bore elements which are in fluid communication with
26 the choke and the surface facilities (not shown).

27

28 Initial and boundary conditions are specified on the field model. These initial
29 and boundary conditions include, by way of example and not limitation, the
30 initial pressures and flow rates in the reservoir elements and well bore
31 elements, fluid compositions, viscosities, etc.

1 Next, a simulation run (step 120) is performed on the field model to calculate
2 reservoir and fluid flow characteristics for a time step. In particular, fluid flow
3 rates between the reservoirs and the well bores are determined as are the
4 pressures in the reservoir and well bore elements. Producing well bores will
5 receive producing fluids from the reservoirs, including oil, water and gas,
6 which are delivered to the surface facilities of the field. Injection wells may be
7 used to pressurize one or more of the reservoirs and/or to dispose of water.
8 Also, gas may be injected into the well bores to provide gas assisted fluid
9 production. Those skilled in the art will appreciate that many other operations
10 affecting production may be modeled with a reservoir simulator and these
11 operations are included within the scope of this invention.

12

13 Component fluid flow rates may be determined in terms of oil, gas and water
14 flow. Alternatively, the fluid components for which flow is to be optimized
15 could be compositional components such as light (C_3 - C_4), medium (C_5 - C_8)
16 and heavy ($>C_9$) hydrocarbons. By way of example, and not limitation, other
17 possible component combinations might include non-hydrocarbon
18 components such as H_2S and CO_2 .

19

20 Component flow rate equations for each of the well bores are next calculated
21 (step 130.) These component flow rate equations describe the estimated flow
22 of one fluid component relative to that of another fluid component over the
23 anticipated range of flow rates for a well bore. Physically, the chokes on the
24 well bores may be opened or closed to increase or decrease the overall fluid
25 output or input relative to a well bore. Because of changing pressure profiles
26 in the well bores, the relative ratios of oil, gas and water produced from a well
27 bore may change with the opening and closing of a choke.

28

29 Examples of component flow rate curves for a well are shown in FIGS. 3A and
30 3B. In FIG. 3A, the rate of production of gas in MSCF/D (million square cubic
31 feet per day) is plotted against the rate of production of oil in STB/D (stock
32 tank barrels/day). In FIG. 3B, the rate of production of water (STB/D) is
33 plotted against the rate of production of oil in STB/D. The rate of production

1 of gas versus oil is relatively linearly over a wide range of possible oil
2 production rates. However, the rate of water production is non-linear relative
3 to the production rate of oil. Much more water is produced at higher outputs
4 of oil production than at lower rates of oil production. High production outputs
5 correspond to a wide open choke position.

6

7 In the preferred embodiment of this invention, a "quick-rates" method is used
8 to generate individual component rate data points which can then be used to
9 quickly construct graphs or generate component flow rate equations. More
10 details on the "quick-rates" method will be described below. Those skilled in
11 the art will appreciate that other methods may be used in generating
12 estimates of how the production of one component versus the rate of
13 production of another component may vary over the overall output range of a
14 well bore.

15

16 A user will specify production constraints (step 140) to be used in conjunction
17 with the field model. By way of example and not limitation, examples of
18 production constraints include (1) producing oil at a target level; (2) producing
19 gas at a target level; (3) limiting gas production below a predetermined limit;
20 (4) limiting water production below a predetermined limit; (5) limiting water
21 injection to an amount related to the water produced from the well bores; and
22 (6) limiting gas injection above a predetermined limit to provide gas assisted
23 lift. Further, these targets and limitations may be combined or scaled relative
24 to one another as well.

25

26 The production constraints may include hard or soft constraints. Hard
27 constraints are constraints which will not be allowed to be violated. Soft
28 constraints are constraints which may be violated if necessary to produce a
29 feasible solution to an optimization problem. Optionally, the order in which the
30 soft constraints are preferably allowed to be violated, if necessary to achieve
31 a feasible solution, may also be specified.

1 Another aspect of the present invention includes optionally specifying
2 (step 150) whether the well bore flow rates of certain well bores are to be
3 related. For example, well bores having similar fluid characteristics such as
4 gas-to-oil ratio (*GOR*) or water-to-oil ratio (*WOR*), may be related to one
5 another. The relating of production rates between well bores will insure that
6 rates of production (or injection) between these well bores will not arbitrarily
7 oscillate between time steps.

8
9 Constraint equations are then generated (step 160) from the production
10 constraints and the related well bore rates. Hard constraint equations are
11 created for those constraints which are not allowed to be violated. Soft
12 constraint equations corresponding to the soft constraints are generated
13 which include constraint violation penalties. The constraint violation penalties
14 allow the soft constraint equations to be satisfied even when the soft
15 constraints must be violated so that an optimization may produce a feasible
16 solution. The generation of this set of constraint equations will be described
17 in further detail below.

18
19 An objective function is created in step 170 which seeks to optimize an
20 objective, such as oil production from field 50. The objective function ideally
21 includes the component flow rates of the well bores and also the constraint
22 violation penalties associated with the soft constraint equations. Weighting
23 scale factors may be associated with the soft constraint penalties in the
24 objective function. By appropriately weighting these weighting scale factors,
25 the order in which related soft constraints may be violated, may be prioritized.
26 The objective function is then optimized (step 180) by an optimizing
27 subroutine (optimizer) to produce an optimized allocation of fluid flow rates
28 among the well bores. The optimizer uses the component flow rate equations
29 calculated in step 130 and the constraint equations set up in step 160 to
30 optimize the objective function.

31
32 The optimized fluid flow rates, and other fluid flow characteristics determined
33 from the optimizer such as constraint violation penalties, may then be

1 allocated among the well bores and reservoir (step 190). These optimized
2 flow rates and characteristics may then be imposed (step 200) as
3 initial/boundary conditions in the next iterative time step in the reservoir
4 simulation. Steps 120-200 are then repeated to provide enhanced field scale
5 production over many time steps until a satisfactory period of time has
6 elapsed and the simulation is then ended. More details on the above
7 aforementioned steps will now be described.

8

9 B. Creation of the Objective Function and Constraint Equations

10

11 1. System of Constraint Equations

12

13 A linear programming (LP) system is a set of linear equations and linear
14 constraints. A mixed integer programming (MIP) system is a set of linear or
15 non-linear equations and constraints. In the present invention, preferably a
16 MIP system augments a LP system when a set of non-linear equations or
17 constraints, represented by piecewise linear functions, needs to be solved to
18 achieve an optimized objective. An open source software package, which
19 uses LP and MIP techniques, is used in this exemplary embodiment to
20 optimize the objective function. In particular, the present invention uses a
21 package entitled LP-Solve, which is available from
22 <http://packages.debian.org/stable/math/lp-solve>. An alternative commercial
23 solver is also utilized entitled XA which is available from Sunset Software
24 Technology Corporation, of San Marino, California. Those skilled in the art
25 will appreciate that other commercial LP/MIP optimizer packages may be
26 used to optimize the objective function using fluid flow rates and constraint
27 conditions.

28

29 The constraint equations, component flow rate equations, and the objective
30 function are input into the optimizer. The optimizer then outputs a feasible
31 solution to the optimization problem including enhanced allocation of well bore
32 flow rates. Values for the violation of any soft constraints necessary to
33 achieve a feasible solution to the optimization are also ideally output. A user

1 may then make appropriate changes to production constraints or to the
 2 capacity of surface facilities to reflect the value of the violation of the soft
 3 constraints.

4
 5 An extrema of an objective function is sought. A simple LP system may have
 6 the following form:

$$7 \quad OBJ = \max \left\{ \sum_i c_i x_i \right\} \text{ subject to constraints in the form of: } (1)$$

$$9 \quad \sum_i [a_i x_i - b_i] \{ \leq, \geq, = \} 0$$

10
 11 where

12
 13 i = index
 14 c_i = weighing scale factor
 15 x_i = parameters being optimized
 16 a_i = multiplicative constant and;
 17 b_i = additive constant

18
 19 In one embodiment of this invention, the main variables are well bore rates.
 20 That is, the rates at which components of fluid production, i.e., oil, water and
 21 gas, are produced from a well bore. Component flow rate equations are
 22 preferably generated using a "quick rates" method which will be described
 23 below. The component rate equations describe how much of one component
 24 is transported through a well bore as compared to another fluid component.
 25 The rates of production of the components may remain linear with respect to
 26 one another or may be non-linear over the potential range of well bore
 27 production outputs. The present invention ideally handles nonlinear scaling
 28 between component or phase rates through piecewise linear functions by
 29 formulating the system as a MIP problem. Production constraints are set up
 30 as hard constraints, which are not allowed to be violated, and/or as soft
 31 constraints, which are allowed to be violated when necessary to achieve a

1 solution. The constraints may include target objectives and production
 2 limitations. The objective function is setup from information provided by a
 3 user.

4

5 2. Setting up the Objective Function

6

7 In general, the objective function comports with the mathematical
 8 expression:

9

$$10 \quad OBJ = \sum_i w_i \sum_j q_{ij} - \sum_k w_k CVP_k \quad (2)$$

11 where

12

13 OBJ = objective to be optimized;

14 i = number of fluid components in a well bore fluid;

15 w_i = weighting scale factor for production of the i^{th} fluid
 16 component in a well bore;

17 j = the number of well bores;

18 q_{ij} = quantity of the i^{th} component produced by the j^{th} well;

19 k = number of constraint violation penalties associated with
 20 the production constraints;

21 w_k = weighting scale factor for the k^{th} constraint
 22 violation penalty; and

23 CVP_k = k^{th} constraint violation penalty.

24

25 A more specific exemplary objective function for the LP/MIP system might
 26 consist of the weighted sum of total production rates of oil, water and gas for a
 27 selected set of well bores. In the present invention, the objective function may
 28 also include constraint violation penalty variables (CVP_k) to accommodate the
 29 use of soft constraints. A typical objective function may be expressed in the
 30 following mathematical form:

$$OBJ = w_o \sum_i q_{oi} + w_g \sum_i q_{gi} + w_w \sum_i q_{wi} - \sum_k w_k CVP_k \quad (3)$$

2

3 where

4

5 OBJ = objective to be optimized;6 w_o = weighting scale factor for oil production;7 q_{oi} = quantity of oil produced by the i^{th} well;8 w_g = weighting scale factor for gas production;9 q_{gi} = quantity of gas produced by the i^{th} well bore;10 w_w = weighting scale factor for water production;11 q_{wi} = quantity of water produced by the i^{th} well bore;12 w_k = weighting scale factor for the k^{th} ; and13 CVP_k = k^{th} constraint violation penalty.

14

15 The weighting scale factors w_i or well rate parameters may be specified by a
 16 user. For example, a user might specify:

17

18 $w_o = 1.0$; $w_g = -0.1$; and $w_w = -0.2$.

19

20 These weighting scale factors correspond to the maximization of oil
 21 production rate while trying to minimize gas and water rates. In this case, the
 22 objective function is incremented by 1.0 for each stock tank barrel/day
 23 (STB/D) of oil produced ($w_{oil} = 1.0$) and penalized by 0.2 for every million
 24 standard cubic feet/day (MSCF/D) of gas and 0.1 for every STB/D of water
 25 produced. In this case, the units of the objective function are a combination of
 26 STB/D and MSCF/D units. Normalization of the objective function
 27 components is ideally carried out to render the objective function
 28 non-dimensional.

1 Another preferred way of handling this unit mismatch in the objective function
 2 is to make use of economical information, if available. For example, if oil
 3 revenues are 22\$/STB/D, gas revenues are 3\$/MSCF/D and every STB/D of
 4 water costs \$3.5 to handle, then:

5

$$6 \quad w_o = 22.0; \quad w_g = 3.0; \text{ and } \quad w_w = -3.5.$$

7

8 In this case, the units of the objective function are monetary (\$) and are
 9 consistent. It is preferred to scale the weighting scale factors so that w_o is
 10 1.0, hence the previous well rate parameter values would be normalized by
 11 22.0 to give:

12

$$13 \quad w_o = 1.0; \quad w_g = 0.136; \text{ and } \quad w_w = -0.159.$$

14

15 3. Production Constraints

16

17 Constraints may be based on physical limitations such as well
 18 production limits, injection rate limits or gas lift rate limits. Alternatively,
 19 constraints may be determined to meet engineering preferences such as
 20 production/injection targets for a group of wells. Other constraints by
 21 way of example and not limitation might include Gas to Oil Ratios
 22 (GOR), Water to Oil Ratios (WOR), and constraints on a subset of wells
 23 or completions.

24

25 The LP/MIP system constraints are classified as hard and soft constraints.
 26 For example, hard constraints may be imposed on a pair of wells such that
 27 the combined maximum oil production is 5,000 STB/D. These hard
 28 constraints are translated into the following LP/MIP constraints:

29

$$30 \quad q_{p=oil}^{w=PROD1} \leq 5,000 \quad (4)$$

$$q_{p=oil}^{w=PROD2} \leq 5,000$$

2

3 where

4

5 $q_{p=oil}^{w=PROD1}$ = the quantity of oil produced from a first well; and

6 $q_{p=oil}^{w=PROD2}$ = the quantity of oil produced from a second well.

7

8 4. Prioritization of Soft Constraints

9

10 Soft constraints are constraints that are allowed to be violated if-and-only-if
 11 there is no other way to honor the soft constraints while obtaining a feasible
 12 solution for the system. Ideally, this violation of constraints will be the
 13 minimum possible necessary for obtaining a solution. Constraint violations
 14 may occur when the system has conflicting limits/targets. Consider the
 15 following situation where the field has constraints including an oil production
 16 target and a water handling limit on a group of wells as follows:

17

18 Oil Production Target = 7,500 STB/D (5)

19 Water Production Limit > 5,000 STB/D

20

21 There might, and most probably, will be a point in the simulation where the
 22 group of wells will not be able to produce 7,500 STB/D of oil without producing
 23 more than 5,000 STB/D of water. Wells tend to produce more water as they
 24 age or mature. In such a case, the optimizer will not report a no-solution but
 25 instead will allow the violation of one of the soft constraints. Preferably, a flag
 26 will be raised indicating that the constraint has been violated. Which
 27 constraint is chosen to be violated first may be determined by the user as well
 28 in this preferred embodiment of this invention.

29

30 These oil target and water limit conditions are translated into the following
 31 three soft constraint equations:

Whether the LP/MIP system chooses to scale back production or meet the water limit depends on the coefficients or weighting scale factors w_k of the CVP_k variables. Suppose the water capacity limit is absolute and that the oil production is allowed to be scaled back to meet the water limit. In this case, suppose $w_1 = 1$, $w_2 = 1$ and $w_3 = 2$ which corresponds to constraint-3 (water production limit) having more priority than the other two constraints (oil production target). Note that the weighting scale factor w_3 is given greater weight than the other two weighting scale factors w_1 and w_2 associated with the oil production. When the well rates are scaled back to meet the water production limit, suppose the oil production drops to 7,400 STB/D when water production is exactly 5,000 STB/D. In this case, CVP_1 will have to be non-zero to satisfy constraint 1, to be exactly $CVP_1 = 100$. In this setting, the LP/MIP system will choose to scale back the rates rather than produce more water due to the specific values of CVP coefficients w_k . The objective function entries will appear as follows for these two cases.

If the oil production target is disregarded and oil production is allowed to be scaled back to meet the water limit, then:

$$q_{p=oil}^{w=PROD1} + q_{p=oil}^{w=PROD2} = 7,400 \quad (8)$$

$$q_{p=water}^{w=PROD1} + q_{p=water}^{w=PROD2} = 5,000$$

$$CVF_1 = 100 \quad CVF_2 = 0 \quad CVF_3 = 0$$

$$OBJ = \dots -1CVF_1 -1CVF_2 -2CVF_3 = \dots -100 \quad (9)$$

If the oil production target is enforced but the limit on the water production limit is allowed to be violated, then:

$$q_{p=oil}^{w=PROD1} + q_{p=oil}^{w=PROD2} = 7,500 \quad (10)$$

2

$$q_{p=water}^{w=PROD1} + q_{p=water}^{w=PROD2} = 5,100$$

4

$$CVF_1 = 0 \quad CVF_2 = 0 \quad CVF_3 = 100$$

6

$$OBJ = \dots - 1CVF_1 - 1CVF_2 - 2CVF_3 = \dots - 200 \quad (11)$$

8

9 Since, everything else being the same, scaling back rates results in a higher
 10 objective function value (+100), the LP/MIP optimizer will prefer to scale back
 11 the rates. The same approach may be used to handle n soft constraints and
 12 put them in a desired priority order of violation.

13

14 If the order in which the soft constraints are to be violated is not specified and
 15 remains unprioritized, then all of the weighting scale factors w_k are equal and
 16 no preference is given as to which constraint is allowed to be violated first. In
 17 this event, $w_1 = w_2 = w_3 = 1$. Alternatively, a first soft constraint may be given
 18 the lowest priority, a second soft constraint is given a slightly higher priority,
 19 and a third soft constraint is given the highest priority. In this exemplary
 20 embodiment of the invention, the weighting scale factors w_i are then given
 21 values corresponding to 10×10^p where p is order of priority in which the soft
 22 constraints may be violated. For example,

23

$$w_1 = 10 \times 10^1; \quad w_2 = 10 \times 10^2; \quad \text{and} \quad w_3 = 10 \times 10^3$$

25

26 The general equation for the objection function is:

27

$$OBJ = \dots - w_1 CVP_1 - w_2 CVP_2 - w_3 CVP_3 \quad (12)$$

29

30 The objective function with weighting scale factors then becomes:

$$OBJ = \dots - 10 \times 10^1 CVP_1 - 10 \times 10^2 CVP_2 - 10 \times 10^3 CVP_3 \quad (13)$$

2

3 Preferably, these coefficients are normalized to give values of between
4 0 and 1. The normalization is partially based upon the potential range of a
5 constraint violation penalty.

6

$$7 \quad \text{Constraint1} \quad 0 \leq CVP_{\text{normi}} \leq 1$$

8

$$9 \quad CVP_{\text{normi}} = (CVP - CVP_{\text{min}}) / (CVP_{\text{max}} - CVP_{\text{min}}) \quad (14)$$

10

11 Or, since CVP_{min} is always zero:

12

$$13 \quad w_i = 10 \times 10^p / (CVP_{\text{maxi}}) \quad (15)$$

14

15 CVP_k parameters are optimized along with the other parameters in the
16 optimization system (production/injection rates). Since any positive value of
17 CVP_k imposes a penalty through the objective function, the system tries to
18 keep CVP_k values as zero. CVP_k gains a positive value if and only if there is
19 no other way to achieve a feasible solution.

20

21 Note that if there are no conflicting objectives for optimization, all of the CVP
22 variables will be zero and soft constraints will be equivalent to hard
23 constraints.

24

25 The operators used with the soft constraints are translated into LP/MIP
26 equations as follows:

1

op	Soft constraint criteria		op	LP/MIP equation
>	WATPR > 5,000	becomes	$\leq$	$q_w \leq 5,000$
<	GASPR < 10,000	becomes	$\geq$	$q_g \geq 5,000$
=	OILPR = 7,500	becomes	$\leq$ and $\geq$	$q_o \leq 7,500$ and $q_o \geq 7,500$

2

3 Note that the (=) operator is the target operator and would satisfy a condition
4 (thus trigger an action) if the criteria left-hand-side is *not* equal to the criteria
5 right-hand-side.

6

7 5. Relating Well Rates

8

9 LP/MIP systems are strictly mathematical and thus have no notion of the
10 physics underlying the variables, equations and constraints. Therefore, in
11 some cases, the LP/MIP results, although mathematically sound, may make
12 little practical sense. Such a case may occur when the LP/MIP optimizer
13 decides to significantly choke back only one well bore in a group of well bores
14 that all have insignificant differences in their properties. This might result in
15 large rate oscillations for individual wells between time steps. To prevent
16 such an occurrence, the present invention provides the option that well rates
17 of well bores with close characteristics be related.

18

19 If it is determined that the well rates should be related, in addition to the
20 existing constraint equations, further constraints equations that relate certain
21 well bore flow rates are setup. For example, if well bores which have fluid
22 characteristics which are within a predetermined range of one another, such
23 as gas-to-oil ratios (*GOR*) and/or water-to-oil ratios (*WOR*), then the flow
24 rates of these well bores may be related. Similar to the soft constraint
25 equations described above, these rate relating equation may have weighting
26 scale factors which are close to one another and include constraint violation
27 penalties.

1 Referring now to FIG. 4A, for instance, given the flow rate of a well bore with
 2 the maximum GOR (q_1), the flow rate of the related well (q_2) is allowed to be
 3 in the shaded area. This is achieved by adding the following constraints to
 4 the system:

$$5 \quad q_2 - \frac{q_{2f}}{q_{1f}} \cdot q_1 - a - RVP \leq 0 \quad (16)$$

$$7 \quad q_2 - \frac{q_{2f}}{q_{1f}} \cdot q_1 - RVP \geq 0$$

8

9 where

10

11 q_1, q_2 = rates that are being related to one another

12 q_{1f}, q_{2f} = maximum possible value of rates

13 RVP = Rate Violation Penalty

14 a = value determining "strictness" of relation

15

16 all RVP s are added to the objective function with a negative weight:

17

$$18 \quad OBJ = \dots - \sum_i w_i RVP_i \quad (17)$$

19

20 where w_i is chosen to be -10 in this particular example.

21

22 a is given by:

23

$$24 \quad a = f \left(\frac{GOR_1 - GOR_2}{GOR_1} \right) q_{2f} \quad (18)$$

25

26 This means that when $q_1 = q_1^*$, q_2 needs to be in the range $[q_{2\min}^*, q_{2\max}^*]$. The

27 function f is a simple linear function as shown in FIG. 4B.

1 The present invention allows a user to change a threshold value t , however,
2 $t = 1.0$ should work for most cases. With this setting, given $t = 1.0$, a well with
3 $GOR_2 = 0.0$ will not be related, and will have an independent rate scaling
4 factor, whereas on the other extreme when $GOR_2 = GOR_1$, the shaded area in
5 FIG. 4A will collapse into a line as shown in FIG. 4B and the second well bore
6 will be forced to have the same scaling factor as well bore 1.

7

8 Another way to relate flow rates is through scaling flow rates in a group of well
9 bores by the same factor. For example, the injection rates of all the injectors
10 in a first injector well group and the production rates of all the producers in a
11 first production group of well bores may be related. This relation is not based
12 on GOR or WOR in this case; the relation simply implies that when the rate of
13 a well bore is scaled by a factor, the other wells in the related group will be
14 scaled with the same factor.

15

16 For instance, if a well bore in a first production group needs to cut production
17 by half (to satisfy another constraint perhaps), then all the well bores in a first
18 production group will cut production by half. Ideally, the default for this
19 relation is to have less weight in the LP/MIP system than the specified
20 constraints. This means that rate-relations may be broken for the sake of
21 satisfying the constraints. Parameters can be used to determine the relative
22 weights of the constraints and rate-relations in the LP/MIP system. The
23 smaller (more negative) these coefficients, the more influence these
24 coefficients will have on the system.

25

26 C. Generation of Rate Curves and Component Flow Rate Equations

27

28 1. Quick Rates Method

29

30 The following "quick rates" method is preferably used in generating fluid flow
31 component flow rate curves and equations. A rate curve relates how the
32 production of one component compares with the production of another. For
33 example, as a choke or valve is opened on a well, oil, water and gas

1 production will generally increase. The increase between any two of the
2 components may be linear or non-linear over the range of overall fluid
3 production. Referring again to FIGS. 3A and 3B, gas and oil production are
4 shown to be generally linear while water and oil production are generally non-
5 linear. The rate curves are generated from a series of data points. Data
6 points generated using an iterative Newton-Raphson procedure in conjunction
7 with a sub-portion of the reservoir model are indicated by "x" marks. Data
8 points indicated by "diamond" indicia were created using a "quick rates"
9 method. Note that both methods provide similar results. However, the "quick
10 rates" method is much more computationally efficient.

11

12 The quick-rates method utilizes the fact that at a fixed point in time, production
13 from individual completion elements is generally linearly proportional to
14 pressure draw down. Pressure draw down is the pressure differential
15 between the pressure in a well bore completion element and adjacent
16 reservoir elements. It is this pressure differential which drives fluids into and
17 out of the completion elements during respective production and injection
18 operations. Using a number of different pressure draw down profiles for each
19 well bore, a set of data points is generated. Then, a piecewise linear function
20 that best fits these points is ideally constructed. A component flow rate
21 equation is then generated from this piecewise linear function which is to be
22 used by the optimizer.

23

24 The oil-water total component flow rate curve is piecewise linear, which is not
25 a linear function. FIGS. 5A-D show the flow rates of individual completion
26 elements for four different overall production outputs for a well bore. Also
27 shown are the pressure profiles for the reservoir and well bore elements for
28 these different production rates. FIGS. 5A-D illustrate cases where oil
29 production is being sequentially reduced, such as occurs when a well head
30 choke valve of a well bore is being closed. Note as oil production is reduced,
31 water production is reduced until almost no water is produced.

1 While the rate of production is decreased, the well bore pressure profile of the
 2 well bore will increase. The pressure profile of the reservoir is assumed to
 3 remain constant at a given time step. This will result in the pressure draw
 4 down in the well decreasing as the well bore pressure profile increases toward
 5 the reservoir pressure profile. Note that the pressure at deeper completions
 6 will be greater than at shallower depth completions due to pressure
 7 head/gravity effects. Consequently, pressure draw down will be lower at
 8 greater depths where denser water underlies less dense layers of oil and gas.

9
 10 The present invention exploits the linear rate scaling for individual well bore
 11 completions. The total production rate of component p , i.e., oil, water or gas,
 12 from a well is the sum of rates from its flowing completions:

$$14 \quad q_{pT} = \sum_{i=1}^{n_{comp}} q_{pi} \quad (19)$$

15
 16 where

17
 18 q_{pT} = total quantity of flow from a well;

19 n_{comp} = number of completion elements s in a well; and

20 q_{pi} = quantity of flow of a component from the i^{th} well bore.

21
 22 The baseline flow rate of each component at each individual completion is
 23 extracted from the reservoir simulation run at a particular time step and well
 24 production level. It is assumed that at a fixed point in time the completion rate
 25 for each individual well completion element is linearly proportional to the
 26 pressure draw down. Thus, if the pressure draw down in a well is reduced by
 27 an amount, c , individual completion rates will be scaled back accordingly and
 28 the new total rate will be given by:

$$q_{pT}^* = \sum_{i=1}^{n_{comp}} \frac{\Delta P_i - c}{\Delta P_i} q_{pi} \quad (20)$$

2

3 where

4

5 q_{pT}^* = new total quantity of flow from a well;6 c = reduction in pressure draw down;7 n_{comp} = number of completion elements in a well;8 ΔP_i = original pressure draw down in the i^{th} completion element;

9 and

10 q_{pi} = quantity of flow of a phase from the i^{th} well bore;

11

12 Thus, the amount of pressure shift, c , required to reduce the well oil rate from
 13 q to q^* is given by:

14

$$c = \frac{q_{oT} - q_{oT}^*}{\sum_{i=1}^{n_{comp}} \frac{q_{oi}}{\Delta P_i}} \quad (21)$$

16

17 This pressure shift dictates a parallel shift in the well bore pressure profile, as
 18 illustrated in FIG 6. Having calculated c , equation 20 can be used to
 19 calculate the well rates of other components flowing in the well bore. The
 20 same procedure can be used for injection rates as well. Repeating this
 21 process, a number of component flow data points may be generated and a
 22 curve may be generated as has been considered previously with respect to
 23 FIGS. 3A and 3B.

24

25 2. Piecewise Linear Function Construction

26

27 Piecewise linear functions are generated which best represent these data
 28 point sets generated by the "quick-rates" method for each of the well bores.

1 The piecewise linear functions include a number of line segments and
 2 breakpoints. The number and location of the breakpoints are ideally selected
 3 using a least squares fit to the data set generated by the "quick-rates"
 4 method. In this exemplary embodiment, a Levenberg-Marquardt least
 5 squares fit method is preferably used to locate breakpoints. Those skilled in
 6 the art will appreciate other curve or equation generating techniques may be
 7 used to represent the generated data points which is to be used by the
 8 optimizer.

9
 10 Referring now to FIGS. 7A and 7B, given a segment k , the coordinates of the
 11 end points of the segment is given by:

$$12 \quad (a_{2k-1}, a_{2k}) \text{ and } (a_{2k+1}, a_{2k+2}) \quad (22)$$

14
 15 Least square methods, such as the Levenberg-Marquardt, require the
 16 derivatives of this function, y , be determined with respect to the parameters,
 17 a . These derivatives are:

$$18 \quad \frac{\partial y}{\partial a_{2k-1}} = (a_{2k+2} - a_{2k}) \frac{-a_{2k+1} + x}{(a_{2k+1} - a_{2k-1})^2} \quad (23)$$

$$19 \quad \frac{\partial y}{\partial a_{2k}} = 1 - \frac{x - a_{2k-1}}{a_{2k+1} - a_{2k-1}}$$

$$20 \quad \frac{\partial y}{\partial a_{2k+1}} = - \frac{(x - a_{2k-1})(a_{2k+2} - a_{2k})}{(a_{2k+1} - a_{2k-1})^2}$$

$$21 \quad \frac{\partial y}{\partial a_{2k+2}} = \frac{x - a_{2k-1}}{a_{2k+1} - a_{2k-1}}$$

22
 23
 24
 25
 26 In the preferred embodiment, ideally an appropriate number of breakpoints as
 27 well as their optimum locations are determined. The algorithm shown in
 28 FIG. 8 is used for the selection of the number of breakpoints.

1 The first step is to start with a linear function (i.e. a single segment, two end
 2 points, hence $i=2$. The χ_i^2 for this linear function is calculated (χ_2^2). Then a
 3 break point is added to the linear function making it a piecewise linear function
 4 with two segments and three end points ($i=i+1$, i.e., $i=3$). The breakpoint
 5 coordinates is optimized for minimum χ_i^2 . If the fit is improved by more than a
 6 factor of f from the initial fit, then a new breakpoint is added and the process is
 7 repeated until the improvement is not significant. This algorithm keeps adding
 8 more breakpoints only if this improves the fit by the fraction f .

9
 10 A better fit can be made by decreasing the value of f at the expense of having
 11 a larger number of segments. This approach is generally robust. A check
 12 may be made to make sure that the breakpoints are always in the feasible
 13 region (first quadrant). This is ensured by penalizing (P) the solutions that fall
 14 into infeasible areas, as shown in FIG. 9.

15

16 3. Incorporation of Piecewise Linear Functions Into the Linear 17 Programming

18

19 Incorporation of piecewise linear curve to the LP setup requires the
 20 introduction of binaries, additional continuous parameters and some
 21 constraints. Following is the set of equations and variables that need to be
 22 added:

23

24 Break points:

25

$$26 \quad (x_{bi}, y_{bi}) \quad i = 1, 2, \dots, n \quad (24)$$

27

28 Replace rate term with:

29

$$30 \quad q = z_1 y_{b1} + z_2 y_{b2} + \dots + z_n y_{bn}$$

31

32 Add constraints:

$$\begin{aligned}
1 \quad & z_1 \leq y_1 \quad z_2 \leq y_1 + y_2 \quad z_3 \leq y_2 + y_3 \quad \dots \quad z_{n-1} \leq y_{n-2} + y_{n-1} \quad z_n \leq y_{n-1} \\
2 \quad & y_1 + y_2 + \dots + y_{n-1} = 1 \quad (25) \\
3 \quad & z_1 + z_2 + \dots + z_n = 1 \\
4 \quad & q_1 = z_1 x_{b1} + z_2 x_{b2} + \dots + z_n x_{bn} \\
5 \quad & y_i \in \{0,1\} \quad i = 1,2,\dots,n-1 \\
6 \quad & z_i \geq 0 \quad i = 1,2,\dots,n
\end{aligned}$$

7

8 Here, q is the dependent rate and q_1 is the controlling rate. Now it will be
9 demonstrated why such a setup results in the correct behavior with a simple
10 piecewise linear function with the two segments. Suppose the function
11 appears as in FIG. 10. The value for the function at $x=15$ is to be
12 determined. The formulation corresponding to this problem would be:

13

$$14 \quad f(x) = z_1 0 + z_2 3 + z_3 9 \quad (26)$$

$$15 \quad x = z_1 0 + z_2 20 + z_3 30$$

$$16 \quad z_1 \leq y_1 \quad z_2 \leq y_1 + y_2 \quad z_3 \leq y_2$$

$$17 \quad y_1 + y_2 = 1$$

$$18 \quad z_1 + z_2 + z_3 = 1$$

$$19 \quad y_i \in \{0,1\} \quad i = 1,2$$

$$20 \quad z_i \geq 0 \quad i = 1,2,3$$

21

22 The binary y indicates the segment that x belongs to. In this case, y_1 should
23 be one and y_2 should be zero. First, check to see if y_2 can ever be one. If
24 y_2 was one, then y_1 has to be zero, which means z_1 is zero and z_2 and z_3
25 are non-zero. However, if z_2 and z_3 are non-zero, $x=15$ for the second
26 equation can never be satisfied, thus y_2 cannot be 1. Thus if y_1 is one, then
27 solving for z obtains:

1 z_1 = 0.25

2 $f(x)$ = 2.25.

3 z_2 = 0.75

4

5 Incorporation of the equations and variables in equation 24 force the LP/MIP
6 optimizer to honor the component flow rate curves.

7

8 While in the foregoing specification this invention has been described in
9 relation to certain preferred embodiments thereof, and many details have
10 been set forth for purpose of illustration, it will be apparent to those skilled in
11 the art that the invention is susceptible to alteration and that certain other
12 details described herein can vary considerably without departing from the
13 basic principles of the invention.

1 WHAT IS CLAIMED IS:

2

3 1. A method for enhancing the allocation of fluid flow rates among a
4 plurality of well bores in fluid communication with at least one
5 subterranean reservoir, the method comprising:

6

7 (a) simulating fluid flow of a fluid containing multiple components in
8 at least one subterranean reservoir and in a plurality of well
9 bores which are in fluid communication with the at least one
10 subterranean reservoir;

11

12 (b) selecting production constraints including at least one soft
13 constraint wherein the at least one soft constraint may be
14 violated;

15

16 (c) generating system equations including component flow rate
17 equations corresponding to the simulated fluid flow in the well
18 bores and constraint equations including at least one soft
19 constraint equation associated with the at least one soft
20 constraint, the at least one soft constraint equation including a
21 constraint violation penalty (*CVP*) which allows the at least one
22 soft constraint equation to satisfy the soft constraint;

23

24 (d) generating an objective function corresponding to the fluid flow
25 in the well bores and to the constraint violation penalty;

26

27 (e) optimizing the objective function utilizing an optimizer and the
28 system equations to determine an enhanced allocation of fluid
29 flow rates among the plurality of well bores wherein the at least
30 one soft constraint may be violated if necessary to achieve a
31 feasible solution to the optimization; and

1 (f) allocating the fluid flow rates among the plurality of well bores as
2 determined in step (e).

3

4 2. The method of claim 1 wherein:

5

6 the production constraints include a plurality of soft constraints which
7 may be violated;

8

9 the system equations include a plurality of soft constraint equations
10 corresponding to the soft constraints, each of the soft constraint
11 equations including a respective constraint violation penalty (*CVP*)
12 which allows that soft constraint equation to satisfy the respective soft
13 constraint; and

14

15 the objective function corresponds to the fluid flow in the well bores and
16 to the constraint violation penalties;

17

18 wherein the soft constraints may be violated if necessary to achieve a
19 feasible solution to the optimization.

20

21 3. The method of claim 2 wherein:

22

23 the soft constraints are prioritized as to the difficulty to which the soft
24 constraints are to be violated.

25

26 4. The method of claim 3 wherein:

27

28 weighting scale factors are associated in the objective function with
29 constraint violation penalties of respective soft constraint equations and
30 are weighted in accordance with the prioritization of the soft constraints
31 associated with the respective soft constraint equations to make the
32 higher priority soft constraints more difficult to violate.

1 5. The method of claim 1 wherein:

2

3 the objective function comports with the mathematical expression:

4

$$5 \quad OBJ = \sum_i w_i \sum_j q_{ij} - \sum_k w_k CVP_k$$

6

7 where

8

9 OBJ = objective to be optimized;

10 i = number of fluid components in the fluid;

11 w_i = weighting scale factor for production of the i^{th} fluid in a
12 well bore;

13 j = the number of well bores;

14 q_{ij} = quantity of the i^{th} component produced by the j^{th} well;

15 k = number of constraint violation penalties associated with
16 the soft constraints;

17 w_k = weighting scale factor for the k^{th} constraint violation
18 penalty; and

19 CVP_k = k^{th} constraint violation penalty.

20

21 6. The method of claim 2 wherein:

22

23 the objective function comports with the mathematical expression:

24

$$25 \quad OBJ = \sum_i w_i \sum_j q_{ij} - \sum_k w_k CVP_k$$

26

27 where

28

29 OBJ = objective to be optimized;

30 i = number of fluid components in the fluid;

1 9. The method of claim 1 wherein:

2

3 the well bores include a plurality of completion elements and the at
4 least one subterranean reservoir includes a plurality of reservoir
5 elements which are in fluid communication with the completion
6 elements;

7

8 the step of simulating fluid flow includes determining pressures in the
9 reservoir elements and in the completion elements and includes
10 determining the corresponding component fluid flow rates in the
11 completion elements due to the pressure draw down between the
12 reservoir elements and the completion elements; and

13

14 the component flow rate equations are generated from component rate
15 data points which are created by scaling and summing the component
16 fluid flows in the completion elements of each well bore based upon the
17 component fluid flow rates determined in the simulation of fluid flow
18 and in relation to the changing pressure draw down between the
19 reservoir and completion elements.

20

21 10. The method of claim 9 wherein:

22

23 the component rate data points are generated utilizing the following
24 mathematical expression:

25

$$26 \quad q_{pT}^* = \sum_{i=1}^{n_{comp}} \frac{\Delta P_i - c}{\Delta P_i} q_{pi}$$

27

28 where

29

30 q_{pT}^* = new total quantity of flow from a well bore;

31 n_{comp} = number of completion elements in a particular well bore;

- 1 ΔP_i = original pressure draw down of the i^{th} completion
2 element;
3 c = change in pressure draw down from the original
4 simulated pressure draw down for a completion element;
5 and
6 q_{pi} = original simulated quantity of component flow from the i^{th}
7 completion element.

- 8
9 11. A method for enhancing the allocation of fluid flow rates among a
10 plurality of well bores in fluid communication with at least one
11 subterranean reservoir, the method comprising:
12
13 (a) simulating fluid flow of a fluid containing multiple components in
14 a plurality of well bores and in at least one subterranean
15 reservoir, the well bores including a plurality of completion
16 elements and the at least one subterranean reservoir including a
17 plurality of reservoir elements which are in fluid communication
18 with the completion elements, and determining pressures in the
19 reservoir elements and in the completion elements and
20 determining the corresponding component flow rates in the
21 completion elements due to the pressure draw down between
22 the reservoir elements and the completion elements;
23
24 (b) generating component rate data points for the well bores over a
25 range of fluid flows by scaling and summing the component fluid
26 flows in the completion elements based upon component flow
27 rates determined in step (a) and changing pressure draw downs
28 between the reservoir and completion elements;
29
30 (c) generating component flow rate equations for the well bores
31 based upon the data points for the respective well bores;

- 1 (d) generating constraint equations corresponding to production
2 constraints;
3
- 4 (e) generating an objective function corresponding to the fluid flow
5 in the well bores;
6
- 7 (f) optimizing the objective function utilizing an optimizer and the
8 constraint and component flow rate equations to determine an
9 enhanced allocation of fluid flow rates among the plurality of well
10 bores; and
11
- 12 (g) allocating the fluid flow rates among the plurality of well bores as
13 determined in step (f).
14
- 15 12. The method of claim 11 further comprising:
16
17 generating piecewise linear functions from the data points for each of
18 the well bores; and
19
20 generating the component flow rate equations from the piecewise
21 linear functions.
22
- 23 13. The method of claim 12 wherein:
24
25 the component flow rate equations include binary variables to describe
26 the piecewise linear function; and
27
28 the optimizing step includes using mixed integer programming.
29
- 30 14. The method of claim 11 wherein:
31
32 the flow rates between at least two of the well bores is related to one
33 another.

1 15. The method of claim 11 wherein:

2

3 the optimizer utilizes at least one of linear programming and mixed
4 integer programming in optimizing the objective function.

5

6 16. The method of claim 15 wherein:

7

8 the optimizer utilizes mixed integer programming in optimizing the
9 objective function.

10

11 17. The method of claim 11 wherein:

12

13 the component rate data points are constructed utilizing the following
14 mathematical expression:

15

$$16 \quad q_{pT}^* = \sum_{i=1}^{n_{comp}} \frac{\Delta P_i - c}{\Delta P_i} q_{pi}$$

17

18 where

19

20 q_{pT}^* = new total quantity of flow from a well bore;

21 n_{comp} = number of completion elements in a particular well bore;

22 and

23 ΔP_i = original pressure draw down of the i^{th} completion
24 element;

25 c = change in pressure draw down from the original
26 simulated pressure draw down for a completion element;

27 and

28 q_{pi} = original simulated quantity of flow of a component from
29 the i^{th} completion element.

- 1 18. A method for enhancing the allocation of fluid flow rates among a
2 plurality of well bores in fluid communication with at least one
3 subterranean reservoir, the method comprising:
4
- 5 (a) simulating fluid flow of a fluid containing multiple components in
6 at least one subterranean reservoir and in a plurality of well
7 bores which are in fluid communication with the at least one
8 subterranean reservoir;
9
- 10 (b) generating rate relating equations between at least two of the
11 well bores;
12
- 13 (c) generating an objective function corresponding to the fluid flow
14 in the well bore;
15
- 16 (d) optimizing the objective function utilizing the rate relating
17 equations to determine an enhanced allocation of fluid flow rates
18 among the plurality of well bores with the at least two well bores
19 having related flow rates; and
20
- 21 (e) allocating the fluid flow rates among the plurality of well bores as
22 determined in step (d).
23
- 24 19. The method of claim 18 wherein:
25
- 26 fluid flow characteristics are compared between the at least two well
27 bores; and
28
- 29 the generation of the rate relating equations is triggered if the
30 compared fluid flow characteristics are within a predetermined range of
31 one another.