

Method of assessing risk

The invention concerns a computer implemented method for assessing risk attached to an asset. More precisely, the invention aims at assessing risk based on time series related to
5 option assets and their underlying.

In conventional art, risk is generally assessed from a statistical backward-looking point of view. This means that risk is usually computed as an historical measure uncertainty in the market. This view is classically accepted as less biased, but does not capture the full
10 depth and breadth of information that is actually available in the market. More precisely, it does not capture the different views of the investors, for which the intensity of the appetite for risk is not quantified if ever taken into account.

The invention aims at improving the situation. To this end, the Applicant proposes a
15 computer-implemented method for determining a risk indicator comprising:

- a) receiving time series for an option asset, each element of said time series comprising an option asset price, an option asset strike price, an option asset maturity chosen among a finite number of maturity values, as well as time series for the underlying asset values, and determining for each time instant of both time series a moneyness value based on the
20 ratio between an option asset strike price and an underlying asset value both associated with said time instant,
- b) calculating a risk neutral density function of the returns for said underlying asset by
 - b1) for each maturity value, using splines to interpolate a maturity based implied volatility surface based on the option asset strike price of the elements of the time series which
25 option asset maturity value corresponds to said each maturity value,
 - b2) for each maturity value, using splines to interpolate a moneyness based implied volatility surface based on the option asset strike price of the elements of the time series which option asset maturity value corresponds to said each moneyness value, and
 - b3) deriving adjusted option asset prices and option asset strike prices from the
30 moneyness and maturity based on the implied volatility surfaces of steps b2) and b3), such that the adjusted option asset prices and option asset strike prices define a substantially continuous function, and calculating the risk neutral density function of the

- returns for said underlying asset as a function of the second order derivative of the option asset price with respect to the option asset strike price,
- c) determining, for said asset, a historical density of returns function, based on historical asset values,
- 5 d) calculating, for at least some of the moneyness values in the set of moneyness values, and for at least some of the maturity values, a time series of relative risk aversion values as a function of difference between the ratio of the first order derivative of the risk neutral density function and the risk neutral density function and the ratio of the first order derivative of the historical density of returns function and the historical density of returns
- 10 function,
- e) performing a wavelet transform on the times series of relative risk aversion values regrouped by moneyness value and maturity value to derive corresponding risk aversion frequencies, and lowpass filtering the times series of relative risk aversion values based on the relative risk aversion frequencies, so as to obtain filtered time series of relative risk
- 15 aversion values regrouped by option asset strike price and maturity value,
- f) for each of at least some of the maturity values, calculating daily aggregates of relative risk aversion values based on relative risk aversion values relating to a same day and chosen within the filtered time series of relative risk aversion values associated with said each of at least some of the maturity values and a moneyness value chosen within the set
- 20 of moneyness values,
- g) performing steps b) to f) over a chosen period of the past, and training a classifier with the resulting daily aggregates, said classifier being arranged to associate a state with each aggregate depending on a return value of the asset value at a fixed date following the date associated with each daily aggregate,
- 25 h) performing steps b) to f) for the current day, and feeding the resulting daily aggregates to the classifier of step g) to derive a daily risk indicator.

This method is advantageous because it allows to capture all of the data available to robustly quantify the investor's appetite for risk. This is achieved by modelizing and

30 denoising the available data.

In various embodiments, the method may present one or more of the following features:

- operation b3) includes applying a polynomial fit to the time series of relative risk aversion values;
- operation c) includes applying a GARCH-like model or a Gaussian Mixture Model to calculate historical density of returns function;
- 5 - operation e) comprises applying an inverse wavelet transform to the lowpass filtered wavelet transformed times series of relative risk aversion values regrouped by moneyiness value and maturity value,
- for a given date, the daily aggregates constitute a vector comprising at least 3 rows, and
- 10 - operation g) comprises training one or more models based on Gaussian Naive Bayes Classifier.

The invention also concerns a computer program comprising instructions for performing the above method of any of the preceding claims, a data storage medium having recorded thereon this computer program, and a computer system comprising a processor coupled to a memory, the memory having recorded thereon this computer program.

Other features and advantages of the invention will readily appear in the following description of the drawings, which show exemplary embodiments of the invention and on which:

- [Fig 1] represents a general diagram of a system executing the method according to the invention,
- [Fig 2] represents an exemplary result of a function which determines an implied volatility as a function of maturity for a given asset,
- 25 - [Fig 3] represents an exemplary result of a function which determines an implied volatility as a function of moneyiness for a given asset,
- [Fig 4] represents an exemplary result of a function which combines Figure 2 and Figure 3 to provide an implied volatility surface,
- [Fig 5] represents an exemplary result of a function which determines a risk neutral
- 30 density function of the returns as a function of strike price, based on the implied volatility surface of Figure 4,

- [Fig 6] represents the first moments of the two Gaussians used in a preferred embodiment,
- [Fig 7] represents the second moments of the two Gaussians used in the same embodiment as Figure 6, and
- 5 - [Fig 8] represents on the same graph a relative risk aversion determined based on the risk neutral density of Figure 5, and the same after it has been wavelet transformed, low pass filtered and detransformed.

The drawings and the following description are comprised for the most part of positive
10 and well-defined features. As a result, they are not only useful in understanding the invention, but they can also be used to contribute to its definition, should the need arise.

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15 as it is limited to the necessary legal publications, however this should not be construed as a waiver of rights or any form of license.

The description further contains an annex referenced as Annex A, which provides the details of some mathematical formulas used in some of the embodiments of the invention.
20 This annex is included as a means of clarification and in order to simplify the references. It forms an integral part of the description and can thus be used to defined the invention should the need arise.

Figure 1 represents a general diagram of a system executing the method according to the
25 invention. System 2 comprises a memory 4, a risk neutral density of returns unit 6, an empirical density of returns unit 8, a relative risk aversion unit 10, a denoiser 12, an aggregator 14 and a modelizer 16.

Memory 4 stores option asset data, corresponding underlying asset data, as well as all the
30 data which is produced for carrying in out the invention. The option asset data represents a time series for a given option asset. In this time series, for a given option asset, triplets of value are associated to a given date. These triplets include an option asset price, an

option asset strike price and a chosen option asset maturity. The data may be organized differently, but it will appear that these elements can always be linked in the form of this triplet. The underlying asset data is organized similarly to the option asset data and it represents a times series for the underlying asset corresponding to the given option asset.

- 5 This time series contains the underlying asset value for time instant corresponding to at least some of the time instants of the time series for the given option asset. The fact that option asset values and underlying asset values are available for corresponding time instants helps defining moneyness values attached to each time series. Moneyness may be defined in different manners. In the following, for a given time instant, moneyness is
- 10 defined as the ratio between an option asset strike price at this time instant and the corresponding underlying asset value at the same time instant. As a result, moneyness is a value which is usually centered around 1 (it may generally oscillates between 0.9 and 1.1). It will be appear that the moneyness may be explicit (that is an actual value stored, and/or calculated) or implicit (that is calculated every time it is used).

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In the example described herein, the memory 4 may be realized in any way suitable, that is by means of a hard disk drive, a solid-state drive, a flash memory, a memory embedded in a processor, a distant storage accessible in the cloud, etc. The data described above may be stored together or across one of several locations.

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- The risk neutral density of returns unit 6, the historical density of returns unit 8, the relative risk aversion unit 10, the denoiser 12, the aggregator 14 and the modelizer 16 are computer programs or functions which are executed on one or more processors. Such processors include any means known for performing automated calculus, such as CPUs,
- 25 GPUs, CPUs and/or GPUs grids, remote calculus grids, specifically configured FPGAs, specifically configured ASICs, specialized chips such as SOC or NOCs, AI specialized chips, etc.

- Generically speaking the philosophy followed by the invention is to compute daily
- 30 aggregate of lowpass filtered relative risk aversion values for specific moneyness values and maturity values. These aggregates are calculated on a longstanding base of data (ideally 15 years of more), and are reused every day in order to train prediction models.

In turn, the prediction model can be used to predict a risk score value which the Applicant has validated mathematically and empirically as properly reflecting the actual risk appetite of the investors. In order to calculate the relative risk aversion values, the option asset data and the underlying asset data are used to compute on the one hand a risk neutral
5 density function of returns, and a historical density of returns.

As a result, the system 2 first feeds option asset data and underlying asset data to the risk neutral density of returns unit 6, which produces a risk neutral density function of returns 7 which will be described with respect to Figures 2 to 5. After that, before, or in
10 parallel, the system 2 feeds option asset data and underlying asset data to the historical density of returns unit 8, which produces a historical density of returns 9 which will be described with respect to Figures 6 and 7.

The risk neutral density function of returns 7 and the historical density of returns 9 are
15 then fed to the relative risk aversion unit 10, which computes relative risk aversion values 11. The relative risk aversion values 11 are fed to the denoiser 12, which produces filtered relative risk aversion values 13. This operation will be described with respect to Figure 8.

20 The filtered relative risk aversion values 13 are then fed to the aggregator 14, which produces moneyiness and maturity based daily aggregates 15 and will be described with respect to Figure Z.

Finally, the daily aggregates 15 are fed to a modelizer 16 to produce a model 17. This
25 model can then be used to receive input data 19 and compute a risk score 21.

The risk neutral density of returns unit 6 computes the risk neutral density function from the option asset data and from the underlying asset data. This is made by calculating an implied volatility surface, and by deriving the risk neutral density function of returns from
30 this implied volatility surface.

Formula Math 1 of Annex A is a reminder of the expression of Risk Neutral Density, where S is the underlying asset price, r is the risk-free rate, T is the time at which the evaluation is made, O is the option asset price and X is the option asset strike price.

- 5 By performing a Taylor-Young formula of level 2, one can prove that the second order derivative of the option asset price with respect to underlying asset price can be determined at each point, and can thus be interpolated by the means of splines. According to the invention, the implied volatility is interpolated across maturity values and moneyness values with cubic splines with a convexity constraint.

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- Figures 2 and 3 represent the result of the approximation of the data points by cubic splines. More precisely, in the case of Figure 2, the triplets of option asset data are regrouped by maturity, and the implied volatility is computed across all the option asset strike prices of these triplets and the corresponding underlying asset data. The determination of implied volatility in itself is known. Then cubic splines are used to approximate the couples (implied volatility; maturity value), as shown on Figure 2. In the case of Figure 3, the situation is similar, except that the triplets are grouped by moneyness values to establish implied volatility.

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20 Figure 4 represents the result of combining Figure 2 and Figure 3. Indeed, by combining the splines for each maturity value and those for each moneyness value, one obtains an implied volatility surface for each maturity value and moneyness value.

- Having the implied volatility associated with each option asset strike value in the strike range, it can be translated in the option asset strike-option asset price space. Then, formula Math 1 can be used in order to get the estimation of the risk neutral density at each point.

- Optionally, since the cubic splines with a convexity constraint may introduce discontinuities which may complicate the determination of the second order derivative in the option asset strike-Risk Neutral Density space, a further polynomial fit may be performed as shown on Figure 4. Optionally, the cubic splines may be replaced by splines of degree 2 or 4 or other.

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In order to calculate the relative risk aversion values, the system 2 needs to calculate a historical density of returns function to be combined with the risk neutral density. In order to do so, the historical density of returns unit 8 may apply several existing approaches. One such approach is presented here, along with another approach developed by the
5 Applicant.

According to a first conventional approach, GARCH-like models, such as disclosed by Barone-Adesi in the article by Giovanni Barone-Adesi and Hakim Dall'O, "*Is the price kernel monotone?*", Swiss Finance Institute Research Paper Series 10-03, Swiss Finance
10 Institute, Jan 2010). The main idea is to fit as well as possible past returns, and then simulate different paths (on a length corresponding to the expiry of the used options) in order to obtain a density. To do so, the Barone-Adesi approach teaches to first fit historical daily returns with a GJR-GARCH on a long period of time (12 years). The use of a long period of time allows to capture as much as possible the excess skewness, the excess
15 kurtosis and other extreme returns. In some variants, other types of GARCH models can be used, such as the E-GARCH model described in the article by Luiz F. Felix, Roman Kräussl, and Philip A. Stork, "*Implied volatility sentiment: A tale of two tails*", Tinbergen Institute Discussion Paper 17-002, Tinbergen Institute Amsterdam, Jan 2017.

20 Once the parameters of this model have been estimated, a time series of scales innovations needs (defined as the ratio of the estimated residuals and the estimated volatility of the fitted model) in order to further apply the approach described in the article by Giovanni Barone-Adesi and Kostas Giannopoulos "*Backtesting derivative portfolios with filtered historical simulation*", European Financial Management, 8:31-58, 2002, which allows to
25 perform Filtered Historical Simulations. By applying these simulations to the scaled innovations, a probability density according to the final values of the asset price may be constructed.

According to a preferred embodiment, the historical density of returns unit 8 represent
30 past returns with Gaussian Mixture Models (GMM) instead of GARCH-like models. This approach does not rely only on a Gaussian distribution as the possible high kurtosis needs

to be taken into account, and as the simulation needs to be as dynamic as possible in order to get the current regime in the market.

As a consequence, the sample of data used to estimate the parameters of the GMM is proportional to the maturity of the options used to infer the Risk Neutral Density. The longer the maturity, the deeper the dive in history. Typically, the history used is 20 times the maturity, e.g. 600 days for the 30 days maturity.

The Gaussian Mixture Model contains two separate Gaussians (examples of which are shown on Figure 6 and 7), and is not calibrated on daily returns, but on rolling returns on periods similar to the used maturity. The Applicant has found that the estimation is very stable as long as the starting values in the estimation-maximization algorithm used to calibrate the GMM are not extreme. This has allowed to identify that, on average, most single stocks have one important “quiet” distribution with a small expected positive return, and conversely one smaller “crash” distribution with an important expected negative return. In the example described here, this density is estimated on the same maturities as that of the option assets: 30, 60 and 90 days.

Once the risk neutral density function of returns 7 and the historical density of returns 9 have been calculated, the relative risk aversion unit 10 can calculate the relative risk aversion values 11.

One of the methods to assess relative risk aversion is to use the Arrow-Pratt measure of Relative Risk Aversion, which is robust for any type of utility function.

Formula Math 2 of Annex A provides the definition of the Arrow-Pratt measure of Relative Risk Aversion. In this formula, S is the price of the underlying asset, and $DF(S)$ is the discount factor DF , which summarizes risk preferences of investors. Simultaneously, formula Math 3 of Annex A provides that the discount factor DF can also be defined as the ratio of risk neutral density and historical density. As a result, relative risk aversion values can be calculated as per formula Math 4 of Annex A. This formula can then be approximated using the Taylor-Young formula since the risk neutral

density and the historical density distributions are known. Relative risk aversion unit 10 thus calculates relative risk aversion values as a function of both the maturity values and the moneyiness values.

- 5 The output of the relative risk aversion unit 10 are relative risk aversion values 11 for specific moneyiness values and for specific maturity values. Advantageously, the moneyiness values can be chosen typically within the range from 95% to 105% (each separated by 1%), and the maturity values can usually be chosen between 30 days, 60 days and 90 days.

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The relative risk aversion values 11 contain a significant amount of noise. However, in view of the only the size of the dataset which is used – going close to 20 years – it is the structural change in risk aversion, beyond the short-term shifts and noise which are interesting. This part of the signal corresponds to the lowest possible frequencies.

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- Conventionally, Fourier transform is used for this type of analysis. However, it fails in the current case because it fails to provide information for non-stationary signals. As a result, the denoiser 12 uses wavelet transforms, which are filtered to keep only the lowest frequencies, and detransformed to provide the filtered relative risk aversion values 13 (which is a continuous function due to the reverse transform), as shown on Figure 6. As the relative risk aversion values 11 are arranged in time series concerning a specific maturity value and a specific moneyiness value, the denoiser 12 produces corresponding filtered relative risk aversion data 13 time series which are specific to a maturity value and a moneyiness value. Figure 8 represents an example of relative risk aversion values 11 fed to the denoiser 12, and the resulting filtered relative risk aversion values 13.

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- Based on the filtered relative risk aversion values 13, the aggregator 14 produces daily distributions for each maturity value, and computes the four first statistical moments of this daily distribution and the drift from one day to the next on the filtered relative risk aversion values 13 for a given maturity value and a given moneyiness value.

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The resulting aggregates 15 constitute individual features which are relative to a given option asset, a given date and a given maturity. Because the aggregator calculates 5 aggregates for each maturity value, in the described examples where there are 3 retained maturity values, the option asset risk profile is quantified by a 15 rows vector, each row constituting a feature, with one vector for each day. The aggregator may of course calculate another number of aggregates to define the vectors.

These vectors can then be used by the modelizer 16 over a long period to train one or several models which allows to predict future performance of the option asset.

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The modelizer 16 can put in place several types of models. One of these models is based on a Gaussian Naive Bayes Classifier. According to a preferred embodiment, the modelizer 16 trains three models 17:

- a first model where a given vector is associated with a first binary value if the daily return over the following day is lower than -1%, and a second binary value otherwise,
- 15 - a second model where a given vector is associated with a first binary value if the daily return over the following day is lower than +1%, and a second binary value otherwise, and
- a third model where a given vector is associated with a first binary value if the daily absolute return value over the following day is more than +1%, and a second binary value otherwise.

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The models are then trained over the vectors of the past 5 years, and, given an input vector 19, each returns a probability for the option asset corresponding to the input vector 19 to be associated with the first binary value for each model. Of course, the input vector 19 has to undergo the same treatment as the rest of the data in order for it to be an aggregate based vector. These probabilities are obtained by considering that each feature follows the normal distribution, and by determining the sample mean and sample standard deviation for the dates labeled as 1, for each feature.

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The resulting probability can then be classified in a quintile for each model, and the triplet of quintiles constitutes a risk score 21 which allows to classify the current day in terms of expected returns in view of the market risk aversion.

- 5 The modelizer 16 can also be implemented as a neural network to provide the model 17. The fact that the system 2 allows to compress the market risk profile into a 15-row vector opens up a new realm of possibilities. Hidden Markov chains may also be used.

Annex A

$$[\text{Math 1}] f(S) = e^{rT} \frac{\partial^2 O}{\partial X^2}$$

$$5 \quad [\text{Math 2}] RRA(S) \stackrel{\text{def}}{=} -S * \frac{DF(S)}{DF(S)}$$

$$[\text{Math 3}] DF(S) = rnd(S) / hd(S)$$

$$[\text{Math 4}] RRA(S) = S * \frac{\frac{\partial rnd}{\partial S}}{rnd} - \frac{\frac{\partial hd}{\partial S}}{hd}$$