

CLAIMS

1. A method of managing hydrocarbons, comprising:
 - obtaining, with a computer, an initial geophysical model;
 - modeling, with the computer, a forward wavefield with viscoacoustic or viscoelastic wave equations, wherein the forward wavefield with viscoacoustic wave equations is based on a first auxiliary variable that is a function of both pressure and a memory variable or the forward wavefield with viscoelastic wave equations is based on a second auxiliary variable that is a function of both stress and a memory variable, respectively, and the first or second auxiliary variable is chosen to eliminate a velocity term from the forward wavefield with viscoacoustic or viscoelastic wave equations;
 - modeling, with the computer, an adjoint wavefield with adjoint viscoacoustic or adjoint viscoelastic wave equations, which are derived from the forward viscoacoustic or the forward viscoelastic wave equations, respectively, and are based on an adjoint of the first or second auxiliary variable, respectively,
 - wherein the modeling includes causing a spatial derivative of the memory variable to not be present in the adjoint viscoacoustic or viscoelastic wave equations by a change of variables comprising replacing the memory variable with the first or second auxiliary variable in the viscoacoustic or viscoelastic wave equations, respectively, thereby causing the adjoint viscoacoustic or viscoelastic wave equations to include spatially-pointwise operations of the first or second auxiliary variable, respectively, instead of the spatial derivative operations of the memory variable;
 - obtaining, with the computer, a gradient of a cost function based on a combination of a model of the forward wavefield and a model of the adjoint wavefield;
 - updating, with the computer, the initial geophysical model with the gradient of the cost function, and obtaining and displaying an updated geophysical model; and
 - managing for hydrocarbons using the updated geophysical model.
2. The method of claim 1, wherein obtaining the initial geophysical model includes obtaining relaxation parameters for a given quality factor using an initial rheological model and acoustic or elastic medium parameters.
3. The method of any one of claims 1-2, wherein the adjoint wavefield is modeled using the adjoint viscoacoustic equations.

4. The method of any one of claims 1-3, wherein the first auxiliary variable is $\zeta_l = p - \frac{1}{\alpha_l} m_l$, where p is pressure, m_l is the memory variable for mechanism l , where l is an integer greater than or equal to 1, and α_l is inversion parameter for mechanism l .

5. The method of any one of claims 1-4, wherein the adjoint viscoacoustic equations are

$$\frac{\partial \bar{p}}{\partial t} + \nabla \cdot \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) + \sum_{l=1}^L \phi_l (-\bar{\zeta}_l + \bar{r}_l) = \frac{\partial \mathcal{F}}{\partial p},$$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \nabla (\kappa \bar{p}) = \frac{\partial \mathcal{F}}{\partial \mathbf{v}},$$

$$\frac{\partial \bar{\zeta}_l}{\partial t} + \phi_l \bar{\zeta}_l - \phi_l \bar{r}_l = 0,$$

$$\bar{r}_l = \alpha_l (\bar{p} + \sum_{k=1}^L \bar{\zeta}_k),$$

where

∇ is a divergence operator,

t is time,

κ is an unrelaxed bulk modulus ($\lim_{\omega \rightarrow \infty} \kappa(\omega) \rightarrow \kappa$),

ρ is a mass density,

$\mathbf{v}$ is a velocity ($\mathbf{v} = \{v_x \ v_y \ v_z\}^T$ in 3D space),

$\bar{p}$ is an adjoint pressure,

$\mathcal{F}(\boldsymbol{\Theta})$ is the cost function, where $\boldsymbol{\Theta}$ is the model parameters (i.e. some combination of Q or α_l , κ and ρ),

$\bar{\mathbf{v}}$ is an adjoint velocity, and

ϕ_l is a relaxation frequency for mechanism l ,

$\partial \mathcal{F} / \partial p$ and $\partial \mathcal{F} / \partial \mathbf{v}$ are derivatives of the objective function $\mathcal{F}$ with respect to pressure and velocity respectively.

6. The method of claim 1, wherein the adjoint wavefield is modeled using the adjoint viscoelastic equations.

7. The method of claim 6, wherein the second auxiliary variable is $\bar{\zeta}_l = \bar{\sigma} - \mathbf{C}\mathbf{D}_l^{-1}\mathbf{m}_l$, where $\bar{\sigma}$ is stress, $\mathbf{m}_l$ is a memory variable for mechanism l , where l is an integer greater than or equal to 1, $\mathbf{C}$ is an elastic constitutive relationship for an isotropic unrelaxed system and $\mathbf{D}_l$ is a constitutive relationship for a memory system.

8. The method of claim 7, wherein the adjoint viscoelastic equations are

$$\frac{\partial \bar{\sigma}}{\partial t} + \kappa^T \left(\frac{1}{\rho} \bar{\mathbf{v}} \right) + \sum_{l=1}^L \phi_l (-\bar{\zeta}_l + \bar{\mathbf{r}}_l) = \frac{\partial \mathcal{F}}{\partial \sigma},$$

$$\frac{\partial \bar{\mathbf{v}}}{\partial t} + \kappa(\mathbf{C}\bar{\sigma}) = \frac{\partial \mathcal{F}}{\partial \mathbf{v}},$$

$$\frac{\partial \bar{\zeta}_l}{\partial t} + \phi_l \bar{\zeta}_l - \phi_l \bar{\mathbf{r}}_l = 0,$$

$$\bar{\mathbf{r}}_l = \mathbf{D}_l \mathbf{C}^{-1} (\bar{\sigma} + \sum_k \bar{\zeta}_k),$$

where

t is time,

$\bar{\sigma}$ is an adjoint stress,

$$\bar{\sigma} = \{\bar{\sigma}_{xx} \quad \bar{\sigma}_{yy} \quad \bar{\sigma}_{zz} \quad \bar{\sigma}_{xz} \quad \bar{\sigma}_{yz} \quad \bar{\sigma}_{xy}\}^T,$$

$\bar{\mathbf{v}}$ is an adjoint velocity,

$\bar{\zeta}_l$ is an adjoint memory field for mechanism l ,

$$\bar{\zeta}_l = \{\bar{\zeta}_{l,xx} \quad \bar{\zeta}_{l,yy} \quad \bar{\zeta}_{l,zz} \quad \bar{\zeta}_{l,xz} \quad \bar{\zeta}_{l,yz} \quad \bar{\zeta}_{l,xy}\}^T,$$

$\bar{\mathbf{r}}_l$ is a variable defined for clarity for mechanism l ,

$$\bar{\mathbf{r}}_l = \{\bar{r}_{l,xx} \quad \bar{r}_{l,yy} \quad \bar{r}_{l,zz} \quad \bar{r}_{l,xz} \quad \bar{r}_{l,yz} \quad \bar{r}_{l,xy}\}^T,$$

ρ is a mass density,

$\partial \mathcal{F} / \partial \bar{\sigma}$ and $\partial \mathcal{F} / \partial \bar{\mathbf{v}}$, are derivatives of the objective function $\mathcal{F}$ with respect to the stress and velocity respectively.

$$\mathbf{C} = \begin{bmatrix} \lambda + 2\mu & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda + 2\mu & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda + 2\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu \end{bmatrix}, \text{ which is the elastic constitutive}$$

relationship for the isotropic unrelaxed system in terms of Lamé constants λ and μ ,

$$\mathbf{D}_i = \begin{bmatrix} \lambda\alpha_i^\lambda + 2\mu\alpha_i^\mu & \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda & 0 & 0 & 0 \\ \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda + 2\mu\alpha_i^\mu & \lambda\alpha_i^\lambda & 0 & 0 & 0 \\ \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda & \lambda\alpha_i^\lambda + 2\mu\alpha_i^\mu & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu\alpha_i^\mu & 0 & 0 \\ 0 & 0 & 0 & 0 & \mu\alpha_i^\mu & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu\alpha_i^\mu \end{bmatrix}, \text{ which is the}$$

constitutive relationship for the memory system ($\alpha_i^{\lambda,\mu}$ are computed from the compressional and shear wave quality factor using Equation (9)), and $\mathbf{\aleph}^T$ = strain

$$\text{operator, } \mathbf{\aleph}^T = \begin{bmatrix} \partial_x & 0 & 0 & \partial_z & 0 & \partial_y \\ 0 & \partial_y & 0 & 0 & \partial_z & \partial_x \\ 0 & 0 & \partial_z & \partial_x & \partial_y & 0 \end{bmatrix}^T.$$

$\mathcal{F}(\boldsymbol{\theta})$ is the cost function, where $\boldsymbol{\theta}$ is the model parameters (i.e. some combination of Q^P , Q^S or $\alpha_i^{\lambda,\mu}$, λ , μ and ρ).

9. The method of any one of claims 1-8, wherein the initial geophysical model includes a velocity model.
10. The method of any one of claims 1-9, wherein the initial geophysical model includes a quality factor model.
11. The method of any one of claims 1-10, wherein the first or second auxiliary variable is based on a plurality of relaxation mechanisms.
12. The method of any one of claims 1-11, wherein the obtaining the initial geophysical model includes obtaining relaxation parameters for a given quality factor using a Generalized-Maxwell rheological model and acoustic or elastic medium parameters.
13. The method of any one of claims 1-12, further comprising causing a well to be drilled at a location derived from the updated geophysical model.
14. The method of any one of claims 1-13, wherein the updated geophysical model resulting from the updating identifies location of structure in earth's subsurface that returned seismic wave to receivers that recorded.