

METHOD FOR DESPECKLING OF SINGLE-LOOK DUAL-POLARIZATION
SYNTHETIC APERTURE RADAR (SAR) DATA

The present application relates to a method and a device
5 for despeckling of single-look dual-polarization
Synthetic Aperture Radar (SAR) Data.

Reference 1 and Reference 2 below show that each pixel in
a fully polarimetric Synthetic Aperture Radar (SAR) data
10 can be represented by a covariance matrix. A covariance
matrix based speckle filter was proposed for multi-look
fully polarimetric SAR data by making use of a
multiplicative noise model and linear minimum mean square
error estimation.

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Reference 1: Lee et al. (1999), IEEE Trans on Geo-
science and Remote Sensing, 37(5), pp.
2363-2373

Reference 2: Lee et al. (2008), Proceedings of the IEEE
20 IGARSS, vol. 4, pp. 21-24

It is an objective of this application to provide an
improved method for despeckling single-look dual-
polarization Synthetic Aperture Radar (SAR) Data.

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The application discloses a computer-implemented method
for despeckling single look dual polarization synthetic
aperture radar (SAR) images. A digital SAR image is
received which comprises pixels that are represented by
30 two dimensional complex valued input vectors. Each of the
input vectors comprises a horizontal and a vertical
polarization value.

For each pixel of a multitude of the pixels at least one feature strength parameter is calculated. More specifically, the application discloses the calculation of an edge strength, a line strength and a point strength parameter.

Furthermore, for each pixel of the multitude of the pixels, a dual polarization output matrix Z_{out} is calculated from at least one input vector. The dual polarization output matrix Z_{out} is calculated according to a rule that, if at least one of a predetermined subset of the feature strength parameters has its maximum value, for example the line strength and the point strength parameter, the dual polarization output matrix Z_{out} is set equal to a dual polarization input matrix Z_{in} of an input vector for that pixel. Otherwise, the dual polarization output matrix Z_{out} is set equal to a weighted average of dual polarization matrices. The dual polarization matrices comprise a first dual polarization matrix Z_{struct} and a second dual polarization matrix Z_{radius} . More specifically, the first dual polarization matrix Z_{struct} is disclosed as structure based average of input vectors, and the second dual polarization matrix is disclosed as radius based average of input vectors.

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The calculated dual polarization output matrix Z_{out} is output to an output destination such as a storage medium, an output device or a means for further processing of the data, such as a target recognition means. The output may comprise further processing steps, for example a conversion in a graphical representation of the output matrix Z_{out} , such as a grey scale pixel, which can be represented on the output device.

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The application furthermore discloses a calculation of the weighted average of the dual polarization matrices by an average of the first dual polarization matrix Z_struct and the second dual polarization matrix of the form w
5 $Z_struct + (1-w) Z_radius$, wherein w is a weight factor and the value of the weight factor w is set equal to the maximum value of the feature strength parameters for edge, line and point features.

10 Furthermore, the application discloses a calculation of the first dual polarization matrix Z_struct as structure based average. Average values of input vectors for multiple filter configurations of pixels are calculated, wherein the input vectors represent the pixels. For each
15 filter configuration, a distance of the averaged input vectors of the filter configuration to the original input vectors of the filter configuration is calculated. "Original" refers to the input vectors which represent the pixel of the received SAR image, prior to averaging.

20 The filter configuration for which the distance of the averaged input vectors to the original input vectors is smallest is selected and the dual polarization matrix of the averaged input vectors of the selected filter
25 configuration is output as the structural average Z_struct . More specifically, the filter configurations are disclosed as sets of pixels which are arranged in a straight line and which correspond to a horizontal line, a vertical line, a downwards diagonal line, an upwards
30 diagonal line, respectively.

The calculation of the average values of input vectors comprises forming real valued vectors from complex valued

input vectors by concatenating the real part and the imaginary part of the horizontal polarization value of the input vector and the real part and the imaginary part of the vertical polarization value of the input vectors and calculating the distance between the original input vectors of a filter configuration and the averaged input vectors of a filter configuration as the sum of the distances between the real valued vectors of the original input vectors and of the averaged input vectors.

Furthermore, the application discloses the calculation of the second dual polarization matrix as a radius based average. A dual polarization matrix is calculated for each input vector of a selection of input vectors wherein the selection of input vectors corresponds to an $n \times n$ window of pixels. A sample covariance matrix is estimated from the selection of input vectors. A critical value of a probability distribution of a sample squared radius for a given significance level α is determined. The sample squared radius is a function of the estimated covariance matrix and a quadratic function of input vectors.

The input vectors for which the sample squared radius is not greater than the critical value of the probability distribution is determined. More specifically, the beta distribution with parameters 2 and $n^2 - 2$ is used. If the number of determined input vectors is not greater than the number of pixels of an $m \times m$ subwindow of the $n \times n$ window of pixels, the average of the input vectors which represent the pixels of the $m \times m$ subwindow is calculated. The dual polarization matrix of the average is output as the second average Z_{radius} . Otherwise, the average of

all input vectors of the $n \times n$ window for which the sample squared radius is not greater than the critical value of the probability distribution is calculated. The dual polarization matrix of the average is output as the second average Z_{radius} . More specifically, a calculation using a 5×5 window and a 3×3 is disclosed.

The edge strength parameter is disclosed as a proportion of pixels which are detected as edges. The edges are detected according to a constant false alarm rate edge detector. The edge detection uses predetermined edge templates which comprise two test regions r_1 , r_2 of pixels.

The line strength parameter is disclosed as a proportion of pixels which are detected as lines. The lines are detected according to a constant false alarm rate line detector. The line detection uses predetermined line templates which comprise two test regions r_1 , r_2 of pixels.

The point strength parameter is disclosed as a proportion of pixels which are detected as points. The points are detected according to a constant false alarm rate point detector. The point detection uses a predetermined point template which comprises two test regions r_1 , r_2 of pixels.

The detection of the edge strength, the line strength and the point strength parameters is based on forming a Wilks lambda quotient of a sums of dyadic products of the complex vectors in the first test region r_1 and a sum of dyadic products of the complex vectors in the second test

region r_2 and comparing the Wilks lambda quotient to the value of a Wilks lambda probability distribution for a given significance level α . In the following, the term "cumulative density" is also used for "probability
5 distribution".

Furthermore, the application comprises a computer readable medium with computer readable code, a computing device and a radar image processing system comprising the
10 computing device for executing a method according to the application

The application provides a new filtering technique for single-look dual-polarization SAR data. The method is
15 based on context-sensitive despeckling, which reduces speckle while preserving image features.

The method is characterized in that neither multiple looks (time complexity) nor full polarization (energy
20 requirement) is needed. This is different from performance of most established processing methods for segmentation, classification, automatic target detection, recognition etc. that can strongly depend on the noise level in the input data. In particular, speckle noise
25 (multiplicative noise) can have a severe impact, which generally can be balanced by acquiring multi-look data and/or the full polarization.

The application can reduce speckle in single-look dual-
30 polarization SAR remotely sensed imagery without smearing out meaningful image features, such as edges, lines, and point targets. Therefore, established automatic target detection / recognition algorithm can operate on the

speckle-reduced data. The mathematical formulation of the application is suitable for real-time hardware implementations. It can be generalized to single-look single-polarization and to single-look multi-polarization data.

In the following description, details are provided to describe the embodiments of the application. It shall be apparent to one skilled in the art, however, that the embodiments may be practiced without such details.

Figure 1 illustrates a flow chart of context-sensitive despeckling of single-look dual-polarization data,

Figure 2 illustrates edge templates of 5x5 pixels with different edge orientations, wherein a crossed pixel is the pixel under test and the two test regions r1 and r2 are in light color with shading and dark color, respectively,

Figure 3 illustrates line templates of 5x5 pixels with different line orientations, wherein a crossed pixel is the pixel under test and the two test regions r1 and r2 are in light color with shading and dark color, respectively,

Figure 4 illustrates a point template of 5x5 pixels, wherein a crossed pixel is the pixel under test and two test regions r1 and r2 are in light

color with shading and dark color, respectively,
 Figure 5 illustrates filter configurations used in
 structure-based averaging, and
 Figure 6 illustrates a device for performing steps of
 5 despeckling of Figure 1.

1. Introduction

In single-look slant-range dual-polarization (HH and VV
 10 polarizations) spotlight data, each pixel can be
 represented by a two-dimensional complex vector, that is

$$z = \begin{bmatrix} HH \\ VV \end{bmatrix} = \begin{bmatrix} \Re(HH) + j\Im(HH) \\ \Re(VV) + j\Im(VV) \end{bmatrix} \quad (1)$$

where the symbols $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and
 imaginary components, respectively. In this application,
 15 the complex vector z of a homogeneous area in the data is
 assumed to be independent and identically distributed as
 a zero-mean bivariate complex Gaussian (Goodman, 1963),
 that is

$$f(z) = \pi^{-2} |\mathbf{S}|^{-1} \exp(-z^{*T} \mathbf{S}^{-1} z) \quad (2)$$

20 where $\mathbf{S}$ is an estimate of the population covariance matrix
 Σ . The symbols $*$ and T denote the complex conjugate and
 transpose, respectively. The notation $|\cdot|$ refers to the
 matrix determinant. In an Appendix below, the population

squared radius $2\mathbf{z}^{*T}\Sigma^{-1}\mathbf{z}$ is shown to follow a χ^2 distribution with four degrees of freedom. In most cases, the population covariance matrix Σ remains unknown. This parameter can, however, be estimated from the samples
5 based on the maximum likelihood method. As shown below, the exact distribution of the sample squared radius $N^{-1}\mathbf{z}^{*T}\mathbf{S}^{-1}\mathbf{z}$ is a beta distribution, which depends on the vector dimension and the number of samples.

10 2. Context-sensitive Despeckling

An objective of the context-sensitive despeckling is to reduce the inherent speckle in single-look dual-polarization data while preserving the meaningful image
15 features, such as edges, lines, and point targets. To achieve this objective, the detections of edges, lines, and point targets are first carried out, followed by a final despeckling process.

20 Figure 1 shows the flow chart 10 of the context-sensitive despeckling.

The flow chart 10 includes a step 12 of providing data that may be speckled. Three treatment steps 14, 15, and

16 follow the step 12 of providing data.

The first treatment step 14 detects edges. The step 14 is followed by a step 20 of calculating edge strength. The
 5 second treatment step 15 detects lines. This step 15 is followed by a step 22 of calculating line strength. The third treat step 16 detects points. This step 16 is followed by a step 24 of calculating point strength.

10 The steps 12 as well as the steps 20, 22, and 24 can be followed by a step 26 of despeckling data. A step 28 of filtering data follows the step 26 of despeckling data.

Each above step is explained in more detail below.

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2.1 Edge Detection and Edge Strength Computation

In order to detect edges in single-look dual-polarization data, a constant false alarm rate edge detector is
 20 introduced based on the Wilks' lambda distribution, i.e. a distribution of the type/ according to

$$\Lambda = \frac{|X|}{|X+Y|} \quad (3)$$

wherein random matrices X and Y have independent p -variate central complex Wishart distributions $CW_p(n, \Sigma)$

and $CW_p(m, \Sigma)$, respectively. The number of samples for X and Y is denoted separately by n and m . Anderson et al. (1995, p. 61) showed that the Wilks' lambda distribution is a distribution of a product of p independent beta-distributed random variables. Since $p=2$ for dual-polarization data, the Wilks's lambda is distributed as the product of two independent beta-distributed random variables β_1 and β_2 with beta $(n-1, m)$ and beta (n, m) , respectively. The exact distribution for the product $w = B_1 B_2$ was derived by Steece (1976, p. 189), where the cumulative density function (cdf) is given by

$$F(w) = \frac{\Gamma(m+n-1)\Gamma(m+n)}{\Gamma(n)\Gamma(m)\Gamma(m-1)} \sum_{k=0}^{\infty} \frac{\Gamma(k+m)\Gamma(k+m-1)I_w(n-1, k+2m)}{\Gamma(k+2m+n-1)k!} \quad (4)$$

Both $\Gamma(\cdot)$ and $I_w(\cdot)$ are, respectively, the gamma and regularized incomplete beta functions.

For edge detection, the processing steps are outlined below:

- 1) Place an edge template over a pixel i . Figure 2 shows a set of edge templates 30 of 5x5 pixels with different orientations. In each template, there are two test regions, namely $r1$ and $r2$ containing n and m pixels, i.e.

$n = m = 10$.

2) Compute X and Y :

$$X = \sum_{k=1}^n z_k z_k^{*T} \text{ and } Y = \sum_{l=1}^m z_l z_l^{*T} , \quad (5)$$

5 where z_k refers to the complex vector of a pixel k in $r1$, while the complex vector of a pixel l in $r2$ is denoted by z_l .

3) Compute the Wilks' lambda as given in (3). Its value
10 is 0.25 for a perfectly homogeneous area with $X=Y$.

4) Mark the pixel i as an edge pixel if one of the following criteria is fulfilled:

$$\Lambda \leq U_{1-\alpha/2} \text{ or } \Lambda > U_{\alpha/2} \quad (6)$$

15 The critical value U is obtained from the cdf in (4) with a desired significance level α . If the pixel is an edge pixel, go to the next Step 5. Otherwise, employ another edge template and repeat Steps 1-4.

20 5) Move the edge template to the next pixel and repeat Steps 1-4. Terminate the execution if there are no more pixels to be processed and an edge map is obtained in final. The edge map is a binary map in which the detected

edges are normally represented by black-colored pixels against a white background, i.e. non-edge pixels.

From the obtained edge map, the so-called edge strength

5 s_{edge} is computed by using a 3x3 window for each pixel,

which is defined as

$$s_{edge} = \frac{1}{9} \times \text{total number of pixels which are detected as edges.} \quad (7)$$

The edge strength value is bounded between zero and one.

10 It equals to zero if all pixels within the 3x3 window are non-edge pixels. In contrast, it is unity if all pixels within the test window are edge pixels. Furthermore, it is clear from (7) that the edge strength can have only 10 known or fixed values. This property makes the edge
15 strength computation suitable for hardware implementation. The computed edge strength is used for defining the filtering weight in the subsequent despeckling process.

2.2 Line Detection and Line Strength Computation

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The Wilks' lambda-based edge detector can be applied for line detection by using the 5x5 line templates 50 given in Figure 3, where $n=20$ and $m=5$. The computation of line strength from the obtained line map is similar to

that outlined in Section 2.1. The line strength of a pixel is given by

$$s_{line} = \frac{1}{9} \times \text{total number of pixels which are detected as lines,} \quad (8)$$

5 using a 3x3 window for each pixel.

2.3 Point Detection and Point Strength Computation

For point detection, the Wilks' lambda-based detector can
10 be employed with the use of a point template. Figure 4 shows the point template 70 of 5x5 pixels, where $n=16$ and $m=9$. The processing steps are given below:

1) Place the point template over a pixel i . Compute X
15 and Y :

$$X = \sum_{k=1}^n z_k z_k^{*T} \quad \text{and} \quad Y = \sum_{l=1}^m z_l z_l^{*T} \quad (9)$$

where z_k refers to the complex vector of a pixel k in $r1$, while the complex vector of a pixel k in $r2$ is denoted by z_l .

20

2) Compute the Wilks' lambda in (3).

3) Mark the pixel i as a point pixel if one of the

following criteria is fulfilled:

$$\Lambda \leq U_{1-\alpha/2} \text{ or } \Lambda > U_{\alpha/2}. \quad (10)$$

The critical value U is obtained from the *cdf* in (4) for a desired significance level α with $n=16$ and $m=9$.

5

4) Move the point template to the next pixel and repeat Steps 1-3. Terminate the execution if there are no more pixels to be processed and a point map is generated in final.

10

Similarly, the so-called point strength of each pixel is computed from the obtained point map by using a 3x3 window as

$$s_{point} = \frac{1}{9} \times \text{total of pixels which are detected as points.}$$

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(11)

2.4 Despeckling

With the computed feature strengths s_{edge} , s_{line} as well as

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s_{point} , the resultant dual-polarization matrix Z_{out} for a given pixel is solved by

$$Z_{out} = \begin{cases} Z & \text{if } s_{point} = 1 \text{ or } s_{line} = 1 \\ wZ_{structure} + (1-w)Z_{radius} & \text{otherwise} \end{cases} \quad (12)$$

wherein w is a filtering weight, Z is a dual-

polarization matrix of the given pixel, $Z_{structure}$ is a structure based average of Z , and Z_{radius} is a radius based average of Z . The matrix Z is the dual-polarization matrix of the given pixel, which is defined by

5

$$Z = ZZ^{*T} = \begin{bmatrix} |HH|^2 & HHVV^* \\ VVHH^* & VV^2 \end{bmatrix}. \quad (13)$$

The above matrix is Hermitian, where the diagonal elements are the HH and VV intensities. The phase

10 difference between the HH and VV polarizations can be derived from the off-diagonal elements. The filtering weight w in (12) is defined as

$$w = \max\{s_{edge}, s_{line}, s_{point}\} \quad (14)$$

where max refers to the maximum operator,

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From (12), it is clear that no filtering is applied to the given pixel if the pixel has its unit point or line strength, thus the point or line pixel can be well-retained. Moreover, the weight can be unity if and only
20 if s_{edge} equals to one. In this case, the resultant output is the dual-polarization matrix generated from structure-based averaging, i.e. $Z_{structure}$. On the other hand, it is

obvious from (12) that $Z_{out} = Z_{radius}$ if the weight is zero.

This implies that the currently processed pixel is part of a homogeneous area. Hence, the filtering is performed based on the beta-distributed sample squared radius.

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As discussed in Section 1, the sample squared radius

$N^{-1}z^{*T}S^{-1}z$ follows a beta distribution. To compute Z_{radius} ,

the steps are as follows:

10 1) Construct the dual-polarization matrix for each pixel,

i.e. $Z = zz^{*T}$.

2) Estimate the sample covariance matrix S within a 5x5 window.

15

3) Determine the critical value $\beta_{\alpha;2,23}$ from the beta distribution with a desired significance level α .

4) Compute the average dual-polarization matrix Z_{radius} by
20 using only those pixels within the window, which fulfills the following criterion:

$$z^{*T}S^{-1}z \leq 25\beta_{\alpha;2,23}. \quad (15)$$

If the number of pixels which fulfills the above criterion is less than or equal to nine pixels, then the

average dual-polarization matrix Z_{radius} is completed by using a 3x3 window is computed as the output.

In the structure-based averaging, a set of four filter
 5 configurations is employed to compute the average dual polarization matrix $Z_{structure}$, where each configuration contains the central pixel and its two neighboring pixels. Figure 5 shows the four filter configurations 90. The use of these filter configurations can assist in preserving
 10 edges in the filtered output. To select a proper filter configuration for a given pixel, the procedures are outlined below.

For a filtering configuration along a line, the line can
 15 be horizontal, vertical, downwards diagonal, or upwards diagonal. See Fig. 5.

To compute $Z_{structure}$, the steps are as follows:

1) Express the complex vectors of all pixels in each
 20 filter configuration in form of real vectors, i.e.

$$r = [\Re(HH) \quad \Im(HH) \quad \Re(VV) \quad \Im(VV)]^T. \quad (16)$$

2) Compute the average real vector $r_{average}$ of each filter configuration.

3) For each filter configuration, compute the sum of Euclidean distances:

$$d = \|r_0 - r_{average}\|_2 + \|r_1 - r_{average}\|_2 + \|r_2 - r_{average}\|_2 \quad (17)$$

5 where $\|\cdot\|_2$ denotes the Euclidean norm. The vectors r_0 , r_1 , and r_2 refer separately to the real vectors of the central pixel and its two neighboring pixels.

4) Identify the filter configuration, which minimizes the
10 sum of Euclidean distances. Then, compute the average dual-polarization matrix $Z_{structure}$ using all pixels within the filter configuration.

Figure 6 shows a device 100 for performing steps of
15 despeckling of Figure 1.

The despeckling device 100 includes a data unit 102. The data unit 102 is connected to edge detection unit 104, to a line detection unit 106, and to a point detection unit
20 108.

The edge detection unit 104 is connected to an edge strength unit 110 whilst the line detection unit 106 is connected to a line strength unit 112. The point

detection unit 108 is connected to a point strength unit 114.

The data unit 102, the edge strength unit 110, the line
5 strength unit 112, and the point strength unit 114 are
connected to a despeckling unit 116, which is connected
to a data filtering unit 118.

Functionally, the data unit 102 is used for performing
10 the step 12 of Figure 1. The edge detection unit 104 and
the edge strength unit 110 are used for performing
respectively the steps 14 and 20 of Figure 1. The line
detection unit 106 and the line strength unit 112 are
used for performing respectively the steps 15 and 22 of
15 Figure 1. The point detection unit 108 and the point
strength unit 114 are used for performing respectively
the steps 16 and 24 of Figure 1. The despeckling unit 116
is used for performing the step 26 of Figure 1 whilst the
data filtering unit 118 is used for performing the step
20 28 of Figure 1.

Reference documents

Andersen, H.H., Højbjerg, M., Sørensen, D., and Eriksen,
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Multivariate Complex Normal Distribution. New York:
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5 certain multivariate complex Gaussian distribution (an
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Steece, B.M. (1976). On the exact distribution for the
10 product of two independent beta-distributed random
variables. Metron, 34(1-2), pp. 187-190.

Although the above description contains much specificity,
these should not be construed as limiting the scope of
15 the embodiments but merely providing illustration of the
foreseeable embodiments. Especially the above stated
advantages of the embodiments should not be construed as
limiting the scope of the embodiments but merely to
explain possible achievements if the described
20 embodiments are put into practice. Thus, the scope of the
embodiments should be determined by the claims and their
equivalents, rather than by the examples given.

Appendix: Solution of the Problem - Analysis of Squared Radius

1. Probability Density Function of Population Squared

5 Radius

Theorem 1: If z is a two-dimensional zero-mean complex Gaussian vector, the probability density function of z is given by

$$10 \quad f(z) = \pi^{-2} |S|^{-1} \exp(-z^{*T} S^{-1} z). \quad (1)$$

The population squared radius $2z^{*T} \Sigma^{-1} z$ has a χ^2 distribution with four degrees of freedom.

15 Proof: The complex vector z can also be written in form of a four-dimensional real vector as

$$r = [\Re(HH) \quad \Im(HH) \quad \Re(VV) \quad \Im(VV)]^T. \quad (2)$$

The probability density function of r is

$$f(r) = (2p)^{-2} |S_r|^{\frac{1}{2}} \exp\left\{-\frac{1}{2} r^T S_r^{-1} r\right\} \quad (3)$$

20 where S_r is an estimate of a 4x4 dimensional positive definite real symmetric covariance matrix Σ_r (Andersen et al., 1995; Hagedorn et al., 2006).

Let $y = S_r^{-\frac{1}{2}} r$, then the inverse linear transformation is $r = S_r^{\frac{1}{2}} y$ and the Jacobian is $J = S_r^{\frac{1}{2}}$. Hence, the probability density function of y is given by

$$f(y) = \text{abs} \left(S_r^{\frac{1}{2}} \right) (2p)^{-2} |S_r|^{\frac{1}{2}} \exp \left\{ -\frac{1}{2} \left(S_r^{\frac{1}{2}} y \right)^T S_r^{-1} \left(S_r^{\frac{1}{2}} y \right) \right\}. \quad (4)$$

5 Since S_r is a positive definite matrix, its eigenvalues λ are always positive. Thus,

$$|S_r|^{-\frac{1}{2}} = \left(\sqrt{\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4} \right)^{-1}. \quad (5)$$

Similarly, the eigenvalues of $S_r^{\frac{1}{2}}$ are also positive, i.e.

$\sqrt{\lambda_1}, \dots, \sqrt{\lambda_4}$, Hence,

$$10 \quad \text{abs} \left(S_r^{\frac{1}{2}} \right) = \sqrt{\lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4}. \quad (6)$$

Based on these facts, Equation (4) can be simplified as

$$f(y) = (2p)^{-2} \exp \left\{ -\frac{1}{2} \left(S_r^{\frac{1}{2}} y \right)^T S_r^{-1} \left(S_r^{\frac{1}{2}} y \right) \right\}. \quad (7)$$

Moreover, $\left(S_r^{\frac{1}{2}} y \right)^T$ can be written as $y^T S_r^{\frac{1}{2}}$. Hence,

$$f(y) = (2p)^{-2} \exp \left\{ -\frac{1}{2} y^T S_r^{\frac{1}{2}} S_r^{-1} S_r^{\frac{1}{2}} y \right\} \quad (8)$$

15 and further

$$f(y) = (2p)^{-2} \exp \left\{ -\frac{1}{2} y^T y \right\}. \quad (9)$$

From the above equation, it is clearly seen that y follows a quad-variate standard Gaussian distribution, i.e. $N_4(0, I)$. The elements of y are independent random variables with standard univariate Gaussian distribution since the covariance matrix is an identity matrix. Thus,

$$(x - \mu_r)^T S_r^{-1} (x - \mu_r) = y^T y = \sum_{i=1}^4 y_i^2 \quad (10)$$

is a sum of squares of independent random variables with standard univariate Gaussian distribution and therefore $r^T S_r^{-1} r = 2z^{*T} S^{-1} z$ has a χ^2 distribution with four degrees of freedom.

2. Probability Density Function of Sample Squared Radius

Lemma 1: Let $z_1, z_2, \dots, z_N$ be independent and identically distributed as p -dimensional zero-mean complex Gaussian, i.e. $CN_p(0, \Sigma)$. The maximum likelihood estimate $\hat{S}$ of S is

$$\hat{S} = S = \frac{1}{N} \sum_{k=1}^N z_k z_k^{*T}.$$

The proof is given in Goodman (1963, pp. 160-161).

Lemma 2: Let A and B be $p \times p$ complex random matrices having central complex Wishart distributions, namely

$CW_p(n, \Sigma)$ and $CW_p(1, \Sigma)$, respectively. If A and B are independent and $n \geq p$, then $|A|/|A+B|$ has a beta distribution, i.e. $\text{beta}(n-p+1, p)$. Note that $|\cdot|$ denotes the matrix determinant.

5

The proof is given in Andersen et al. (1995, pp. 65-66).

Lemma 3: Let x have a beta distribution, i.e. $\text{beta}(a, b)$.

Then, the distribution of $1-x$ is $\text{beta}(b, a)$.

10

Proof: The probability density function of x is

$$f_x(x) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1} (1-x)^{b-1} \quad (11)$$

Let $y=1-x$ and thus $\frac{dx}{dy} = -1$. Hence,

$$f_y(y) = \left| \frac{dx}{dy} \right| f_x(y) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} (1-y)^{a-1} (y)^{b-1} \quad (12)$$

15

Theorem 2: Let $z_1, z_2, \dots, z_N$ be independent and identically distributed as p -dimensional zero-mean complex Gaussian, i.e. $CN_p(0, \Sigma)$. Then, $N^{-1} z^{*T} S^{-1} z$ is distributed as $\text{beta}(p, N-p)$,

where $S = \frac{1}{N} \sum_{k=1}^N z_k z_k^{*T}$ is the sample covariance matrix (See

20 Lemma 1).

Proof: Let $A = \sum_{k=1, k \neq i}^{N-1} z_k z_k^{*T} \sim CW_p(N-1, \Sigma)$ and $B = z_i z_i^{*T}$. The symbol

" $\sim$ " means is distributed according to. Both A and B are independent since A does not include z_i . From Lemma 2, it

5 is known that $|A|/|A+B|$ has a beta distribution, i.e.
 $beta(N-p, p)$.

Let $C = \sum_{k=1}^N z_k z_k^{*T} = A + B = NS$, then

$$\frac{|A|}{|A+B|} = \frac{|C-B|}{|C|} = 1 - \frac{1}{N} z_i^{*T} S^{-1} z_i. \quad (13)$$

10 As a result,

$$1 - \frac{1}{N} z_i^{*T} S^{-1} z_i \sim beta(N-p, p). \quad (14)$$

Based on Lemma 3,

$$\frac{1}{N} z_i^{*T} S^{-1} z_i \sim beta(p, N-p). \quad (15)$$

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Reference documents

Andersen, H.H., Højbjerg, M., Sørensen, D., and Eriksen,
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Reference number

	10	flow chart
	12	step
5	14	step
	15	step
	16	step
	20	step
	22	step
10	24	step
	26	step
	28	step
	30	edge template
	50	line template
15	70	point template
	90	filter configuration

Claims

1. A method for despeckling synthetic aperture radar (SAR) images comprising
 - 5 - receiving an SAR image, wherein the SAR image comprises pixels, the pixels being represented by two dimensional complex valued input vectors, each input vector comprising a horizontal and a vertical polarization value,
 - 10 - calculating, for each pixel of a multitude of the pixels, at least one feature strength parameter,
 - calculating, for each pixel of the multitude of the pixels, a dual polarization output matrix Z_{out} from at least one input vector, wherein
 - 15 if at least one of a predetermined subset of the feature strength parameters has its maximum value, the dual polarization output matrix Z_{out} is set equal to a dual polarization input matrix Z_{in} of a corresponding input vector and wherein otherwise,
 - 20 the dual polarization output matrix Z_{out} is set equal to a weighted average of dual polarization matrices, the dual polarization matrices comprising a first dual polarization matrix Z_{struct} and comprising a second dual polarization matrix
 - 25 Z_{radius} ,
 - outputting the calculated dual polarization output matrix Z_{out} to an output destination.
2. Method according to claim 1, wherein the weighted
 - 30 average of dual polarization matrices is calculated by an average of the first dual polarization matrix Z_{struct} and the second dual polarization matrix of the form $w Z_{struct} + (1-w) Z_{radius}$,

wherein w is a weight factor and the value of the weight factor w is set equal to the maximum value of the feature strength parameters.

- 5 3. Method according to claim 2, wherein the first dual polarization matrix Z_struct is derived from a calculation, the calculation comprising computing average values of input vectors for multiple filter configurations of pixels wherein the input vectors represent the pixels,
- 10 calculating, for each filter configuration, a distance of the averaged input vectors of the filter configuration to the original input vectors of the filter configuration,
- 15 selecting the filter configuration for which the distance of the averaged input vectors to the original input vectors is smallest
- 20 outputting the dual polarization matrix of the averaged input vectors of the selected filter configuration as the structural average Z_struct .
- 25 4. Method according to claim 3, wherein the filter configurations are sets of pixels which are arranged in a straight line.
- 30 5. Method according to claim 4, wherein the filter configurations are sets of three pixels which correspond to a horizontal line, a vertical line, a downwards diagonal line, an upwards diagonal line, respectively.

6. Method according to one of claims 3 to 5, wherein the calculation of the second dual polarization matrix Z_{struct} comprises forming real valued vectors from complex valued input vectors by concatenating the real part and the imaginary part of the horizontal polarization value of the input vector and the real part and the imaginary part of the vertical polarization value of the input vectors
- 5 calculating the distance between the original input vectors of a filter configuration and the averaged input vectors of a filter configuration as the sum of the distances between the real valued vectors of the original input vectors and of the averaged input vectors.
- 10
- 15
7. Method according to one of the claims 2 to 6, wherein the second dual polarization matrix Z_{radius} is derived from a calculation, the calculation comprising
- 20
- calculating a dual polarization matrix for each input vector of a selection of input vectors wherein the selection of input vectors corresponds to an $n \times n$ window of pixels,
 - 25 - estimating a sample covariance matrix from the selection of input vectors,
 - determining a critical value of a probability distribution of a sample squared radius for a given significance level α wherein the sample squared radius is a function of the covariance matrix and a quadratic function of input vectors,
- 30

- determining the input vectors for which the sample squared radius is not greater than the critical value of the probability distribution,
if the number of determined input vectors is not
5 greater than the number of pixels of an $m \times m$ subwindow of the $n \times n$ window of pixels
 - calculating the average of the input vectors which represent the pixels of the $m \times m$ subwindow and outputting the dual polarization matrix of the
10 average as the second average Z_radius , otherwise
 - calculating the average of all input vectors of the $n \times n$ window for which the sample squared radius is not greater than the critical value of the probability distribution and outputting the dual
15 polarization matrix of the average as the second average Z_radius .
8. Method according to claim 7, wherein the $n \times n$ window of pixels is a 5×5 window of pixels and the
20 $m \times m$ subwindow of pixels is a 3×3 window of pixels.
9. Method according to one of the previous claims, wherein the feature strength parameters comprise an
25 edge strength parameter, a line strength parameter and a point strength parameter, and wherein the subset of feature strength parameters comprises a line strength parameter and a point strength parameter.
- 30
10. Method according to claim 9, wherein the edge strength parameter is equal to a proportion of pixels which are detected as edges and wherein edges

are detected according to a constant false alarm rate edge detector and the edge detection comprises the use of predetermined edge templates which comprise two test regions r1, r2 of pixels.

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11. Method according to claim 9, wherein the line strength parameter is equal to a proportion of pixels which are detected as lines and wherein lines are detected according to a constant false alarm rate line detector and the line detection comprises the use of predetermined line templates which comprise two test regions r1, r2 of pixels.

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12. Method according to claim 9, wherein the point strength parameter is equal to a proportion of pixels which are detected as points and wherein points are detected according to a constant false alarm rate point detector and the point detection comprises the use of a predetermined point template which comprises two test regions r1, r2 of pixels.

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13. Computer readable medium comprising computer executable program code for executing the steps of a method according to one of the claims 1 to 12.

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14. Computing device for executing the steps of a method according to one of the claim 1 to 12.

15. Radar image processing system comprising the computing device according to claim 14

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