

A SHAPE INDEX MODULATION TRANSMITTER FOR A MULTI-MODE ORBITAL ANGULAR MOMENTUM (OAM) SYSTEM

CROSS-REFERENCE TO RELATED APPLICATION

[0001] This application claims the benefit of priority to Singapore patent application no. 10202301348Y which was filed on 15 May 2023, the contents of which are hereby incorporated by reference in its entirety for all purposes.

TECHNICAL FIELD

[0002] This application relates to a shape index modulation transmitter for a multi-mode orbital angular momentum (OAM) system that increases the probability that OAM modes with high reliability are activated during a transmission process of the OAM system.

BACKGROUND

[0003] Multiple-input multiple-output (MIMO) technology is widely recognized as an effective solution for enhancing data transmission reliability and spectral efficiency in wireless communication systems. However, traditional MIMO architectures necessitate complex radio frequency (RF) chains for each antenna element, and this leads to a substantial increase in hardware costs and power consumption. Additionally, MIMO systems require intricate digital signal processing techniques, such as the transmitting and receiving of precoding and equalization algorithms to mitigate inter-channel interference (ICI). A significant challenge for MIMO technology is its limited capacity for further enhancement without the introduction of additional resources, and its suboptimal performance in line-of-sight (LoS) scenarios which are characterized by high channel correlation and low rank. LoS channels are prevalent in various communication contexts, including LoS-dominant intelligent reflecting surfaces (IRS), unmanned aerial vehicles (UAV), satellites, and backhaul networks. To address these limitations while preserving the inherent benefits of MIMO, considerable research efforts have been directed towards the integration of orbital angular momentum (OAM) with mode division multiplexing (MDM) transmission.

[0004] OAM is a phenomenon resulting from the phase variation of electromagnetic (EM) waves with respect to the azimuthal angle around the propagation axis. In EM waves which have OAM, the wavefront takes on a helical structure, represented by the exponential function $\exp(-j\xi\varphi)$, where j is the imaginary unit, ξ is the OAM mode number, and φ is the azimuthal

angle. OAM-assisted mode division multiplexing (MDM) systems utilize EM waves with distinct OAM modes, which exhibit relatively low correlation. The multiplexing and demultiplexing of OAM modes are achieved through RF analog processing technologies, such as phase shift networks, which help to mitigate ICI and reduce computational complexity. Due to these advantageous properties, extensive research efforts have been devoted to developing and optimizing methods for generating OAM EM waves, including the use of spiral phase plates, helicoidal parabolic antennas, meta-surfaces, and uniform circular arrays (UCAs). Among these, UCAs are particularly promising due to their lower hardware costs and simpler implementation. Furthermore, numerous studies have demonstrated the viability of UCA-based OAM in both theoretical and practical contexts.

[0005] Uniform Circular Array-based OAM systems have garnered significant attention in a variety of wireless applications, encompassing multiple access communications and line-of-sight communications. OAM systems often encounter significant inter-mode interference due to misalignments between transmitting and receiving UCAs. To mitigate this issue, those skilled in the art have focused their efforts on a beam steering approach tailored for transceivers, as well as an OAM receiver that combines beam steering with amplitude detection. Moreover, optimization strategies for power allocation and precoding have been developed to eliminate inter-mode interference. To enhance spectral efficiency, an index modulation (IM) technique has been incorporated into the OAM framework, resulting in OAM-IM systems. In parallel with the development of index modulation techniques, those skilled in the art have proposed a coded modulation solution that is able to achieve high reliability-and-throughput requirements of modern communications.

[0006] Despite efforts of those skilled in the art to improve the performance of existing OAM-IM systems, transceivers in existing systems still ignore the difference in reliabilities between different OAM modes and/or mode combinations and as a result, these various OAM modes and/or mode combinations are activated randomly during the transmission of data, and this results in undesirable decoding thresholds and poor bit error rates (BERs).

SUMMARY

[0007] In one aspect, the present application discloses a shape index modulation transmitter for a multi-mode orbital angular momentum (OAM) system. The system comprises a processing unit and a non-transitory media readable by the processing unit, the media storing

instructions that when executed by the processing unit causes the processing unit to: divide, using a shaping encoder module, a binary coded-bit sequence received from a bit-splitter module into a K-number of sub-sequences, whereby K is an integer that is determined based on a total number of OAM modes and a number of active OAM modes. The processing unit then proceeds to process, using a shaping function provided within the shaping encoder module, the sub-sequences to increase a probability that bits in the sub-sequences comprise a predetermined binary value and then formulates, using the shaping encoder module, index symbols based on the processed sub-sequences, whereby a first bit of each of the index symbols is obtained from a first processed sub-sequence. The processing unit then causes the shaping encoder module to provide the index symbols to an OAM mode selector module for selecting corresponding OAM mode combinations for transmission by the transmitter, whereby optimal OAM mode combinations of the transmitter are associated with index symbols that comprise primarily of the predetermined binary value.

[0008] In another aspect, the present application discloses a method for shaping index modulation in a multi-mode orbital angular momentum (OAM) system whereby the system comprises a shape index modulation transmitter having a shaping encoder module communicatively coupled to an OAM mode selector module. The disclosed method then comprises the steps of dividing, using the shaping encoder module, a binary coded-bit sequence received from a bit-splitter module into a K-number of sub-sequences, whereby K is an integer that is determined based on a total number of OAM modes and a number of active OAM modes; processing, using a shaping function provided within the shaping encoder module, the sub-sequences to increase a probability that bits in the sub-sequences comprise a predetermined binary value; formulating, using the shaping encoder module, index symbols based on the processed sub-sequences, whereby a first bit of each of the index symbols is obtained from a first processed sub-sequence; and providing, using the shaping encoder module, the index symbols to an OAM mode selector module for selecting corresponding OAM mode combinations for transmission by the transmitter, whereby optimal OAM mode combinations of the transmitter are associated with index symbols that comprise primarily of the predetermined binary value.

BRIEF DESCRIPTION OF THE DRAWINGS

[0009] Various embodiments of the present disclosure are described below with reference to the following drawings:

Figure 1 illustrates a block diagram of a protograph low-density parity-check (PLDPC)-coded orbital angular momentum (OAM)-index modulation (IM) system over line of sight (LoS) channels in accordance with embodiments of the present disclosure;

Figure 2a illustrates a block diagram representative of a transmitting side of a shape index modulation transceiver for a multi-mode OAM system in accordance with embodiments of the present disclosure;

Figure 2b illustrates a block diagram representative of a receiving side of a shape index modulation transceiver for a multi-mode OAM system in accordance with embodiments of the present disclosure;

Figure 3 illustrates a block diagram of a processing system for performing embodiments of the present disclosure;

Figure 4 illustrates a flowchart of a process for shaping index modulation in a multi-mode OAM system in accordance with embodiments of the present disclosure;

Figure 5 illustrates a flowchart of a process for processing the sub-sequences using a shaping function in accordance with embodiments of the present disclosure;

Figure 6a illustrates histograms of the log-likelihood ratio (LLR) values l_i obtained as output from a channel detector with coded bits $b_i = 0$ and $b_i = 1$ under SNR = 80 dB, and transformed LLR values l_i^{CBCT} , and a regular-(3, 6) PLDPC code with $R_p = 0.5$ and $n_e = 2400$ is considered in OAM-IM systems with $N_A = 1$ over LoS channels: $\theta = 0$;

Figure 6b illustrates histograms of the log-likelihood ratio (LLR) values l_i as obtained as output from a channel detector with coded bits $b_i = 0$ and $b_i = 1$ under SNR = 80 dB, and transformed LLR values l_i^{CBCT} , and a regular-(3, 6) PLDPC code with $R_p = 0.5$ and $n_e = 2400$ is considered in OAM-IM systems with $N_A = 1$ over LoS channels: $\theta = 0.8$;

Figure 7a illustrates histograms of the log-likelihood ratio (LLR) values l_i obtained as output from a channel detector with coded bits $b_i = 0$ and $b_i = 1$ under SNR = 82 dB, and transformed LLR values l_i^{CBCT} , and a regular-(3, 6) PLDPC code with $R_p = 0.5$ and $n_e = 2400$ is considered in OAM-IM systems with $N_A = 2$ over LoS channels: $\theta = 0$;

Figure 7b illustrates histograms of the log-likelihood ratio (LLR) values l_i as obtained as output from a channel detector with coded bits $b_i = 0$ and $b_i = 1$ under SNR = 82 dB, and transformed

LLR values l_i^{CBCT} , and a regular-(3, 6) PLDPC code with $R_p = 0.5$ and $n_e = 2400$ is considered in OAM-IM systems with $N_A = 2$ over LoS channels: $\theta = 0.8$;

Figure 8 illustrates a flow diagram of a CBCT-PEXIT algorithm used for analyzing and designing PLDPC-coded OAM-IM systems over LoS channels in accordance with embodiments of the present disclosure;

Figure 9 illustrates protographs for a family of AR4JA codes with a wide range of rate from $\frac{1}{2}$ to $(e+1)/(e+2)$;

Figure 10a illustrates protographs for the family of IP1 PLDPC codes with a wide range of rate from $\frac{1}{2}$ to $(e+1)/(e+2)$ in accordance with embodiments of the present disclosure;

Figure 10b illustrates protographs for the family of IP2 PLDPC codes with a wide range of rate from $\frac{1}{2}$ to $(e+1)/(e+2)$ in accordance with embodiments of the present disclosure;

Figure 11a illustrates channel gains of a 4×4 OAM-IM system without inter-mode interference (i.e., $\theta = 0$);

Figure 11b illustrates channel gains of a 4×4 OAM-IM system with inter-mode interference (i.e., $\theta = 0.8$);

Figure 12 illustrates extrinsic MIs output from detector with and without CBCT in a 4×4 OAM-IM system with $\theta = 0.8$, which correspond to the coded bits transmitted by 4 different modes whereby the regular-(3, 6) PLDPC code and Gray-labeled QPSK modulation are used;

Figure 13 illustrates constellation-constrained capacities of the natural index and MI-based index schemes where Gray-labeled QPSK and 8PSK constellations are considered;

Figure 14a illustrates BER curves of two MIMO LDPC codes (i.e., Code I and Code II) and the regular-(3, 6) code, the AR4JA code, and the IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 1$ and $\theta = 0.8$ whereby a natural index scheme is considered with QPSK modulation;

Figure 14b illustrates BER curves of two MIMO LDPC codes (i.e., Code I and Code II) and the regular-(3, 6) code, the AR4JA code, and the IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 1$ and $\theta = 0.8$ whereby a natural index scheme is considered with 8PSK modulation;

Figure 15a illustrates BER curves of two MIMO LDPC codes (i.e., Code I and Code II) and the regular-(3, 6) code, the AR4JA code, and the IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$ and $\theta = 0.8$ whereby a natural index scheme is considered with QPSK modulation;

Figure 15b illustrates BER curves of two MIMO LDPC codes (i.e., Code I and Code II) and the regular-(3, 6) code, the AR4JA code, and the IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$ and $\theta = 0.8$ whereby a natural index scheme is considered with 8PSK modulation;

Figure 16 illustrates BER curves of a 5G LDPC code, three irregular LDPC codes (i.e., irregular LDPC codes with maximum VN degrees $d_v = 4, 5$, and 50), (3, 6) SC-LDPC code, and the IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$ and $\theta = 0.8$ when a QPSK modulation and natural index scheme are considered;

Figure 17 illustrates BER curves of the high-rate AR4JA code, and proposed IP1 and IP2 PLDPC codes in 4×4 OAM-IM system with $N_A = 2$ and $\theta = 0.8$ when a natural index scheme is considered;

Figure 18a illustrates BER curves of the regular-(3, 6), the AR4JA code, and proposed IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$ and $\theta = 0.8$ when both natural index and proposed shaped index schemes are considered with QPSK modulation;

Figure 18b illustrates BER curves of the regular-(3, 6), the AR4JA code, and proposed IUP, IP1, and IP2 PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$ and $\theta = 0.8$ when both natural index and proposed shaped index schemes are considered with 8PSK modulation;

Figure 19a illustrates BER curves of the proposed improved PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$, $\theta = 0.8$, and two different index schemes (i.e., ED based index and proposed shaped index schemes) when both QPSK and 8PSK modulations are considered with the IUP PLDPC code;

Figure 19b illustrates BER curves of the proposed improved PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$, $\theta = 0.8$, and two different index schemes (i.e., ED based index and proposed shaped index schemes) when both QPSK and 8PSK modulations are considered with the IP1 PLDPC code;

Figure 19c illustrates BER curves of the proposed improved PLDPC codes in 4×4 OAM-IM systems with $N_A = 2$, $\theta = 0.8$, and two different index schemes (i.e., ED based index and proposed shaped index schemes) when both QPSK and 8PSK modulations are considered with the IP2 PLDPC code;

Figure 20 illustrates BER curves of the (3, 6) PLDPC code in the 4×4 MIMO-VLC systems with $N_A = 1$ and 2, and four different index schemes (i.e., the CONSM scheme, ED scheme and their shaped counterparts);

Figure 21a illustrates BER curves of (3, 6) PLDPC code in multi-cell network systems with two different index schemes (i.e., CONSM and shaped CONSM schemes) when the value of multi-user interference is 0 dB;

Figure 21b illustrates BER curves of (3, 6) PLDPC code in multi-cell network systems with two different index schemes (i.e., CONSM and shaped CONSM schemes) when the value of multi-user interference is 6 dB; and

Figure 21c illustrates BER curves of (3, 6) PLDPC code in multi-cell network systems with two different index schemes (i.e., CONSM and shaped CONSM schemes) when the value of multi-user interference is 9 db.

DETAILED DESCRIPTION

[0010] The following detailed description is made with reference to the accompanying drawings, showing details and embodiments of the present disclosure for the purposes of illustration. Features that are described in the context of an embodiment may correspondingly be applicable to the same or similar features in the other embodiments, even if not explicitly described in these other embodiments. Additions and/or combinations and/or alternatives as described for a feature in the context of an embodiment may correspondingly be applicable to the same or similar feature in the other embodiments.

[0011] In the context of various embodiments, the articles “a”, “an” and “the” as used with regard to a feature or element include a reference to one or more of the features or elements.

[0012] In the context of various embodiments, the term “about” or “approximately” as applied to a numeric value encompasses the exact value and a reasonable variance as generally understood in the relevant technical field, e.g., within 10% of the specified value.

[0013] As used herein, the term “and/or” includes any and all combinations of one or more of the associated listed items.

[0014] As used herein, “comprising” means including, but not limited to, whatever follows the word “comprising”. Thus, use of the term “comprising” indicates that the listed elements are required or mandatory, but that other elements are optional and may or may not be present.

[0015] As used herein, “consisting of” means including, and limited to, whatever follows the phrase “consisting of”. Thus, use of the phrase “consisting of” indicates that the listed elements are required or mandatory, and that no other elements may be present.

[0016] As used herein, the term “distinct binary sequence” or “distinct bits” refers to a sequence of binary digits or bits that are unique and are not repeated within a given set of binary numbers.

[0017] Further, one skilled in the art will recognize that certain functional units in this description have been labelled as modules throughout the specification. The person skilled in the art will also recognize that a module may be implemented as circuits, logic chips or any sort of discrete component. Still further, one skilled in the art will also recognize that a module may be implemented in software which may then be executed by a variety of processor architectures. In embodiments of the disclosure, a module may also comprise computer instructions or executable code that may instruct a computer processor to carry out a sequence of events based on instructions received. The choice of the implementation of the modules is left as a design choice to a person skilled in the art and does not limit the scope of the claimed subject matter in any way.

[0018] A block diagram of a protograph low-density parity-check (PLDPC)-coded orbital angular momentum (OAM)-index modulation (IM) system over line of sight (LoS) channels is illustrated in Figure 1. At the transmitter side of PLDPC-coded OAM-IM system 100, an information bit sequence $\mathbf{n}_S$ of length n_S is first passed to PLDPC encoder 102. PLDPC encoder 102 converts information bit sequence $\mathbf{n}_S$ into a binary coded-bit sequence $\mathbf{n}_E$ of length n_E . Then, $\mathbf{n}_E$ is permuted by bit-level random interleaver 104 to yield a bit-interleaved sequence $\tilde{\mathbf{n}}_E$. Subsequently, a bit-splitting operation is performed on $\tilde{\mathbf{n}}_E$ by bit-splitter module 106, and this bit-interleaved sequence $\tilde{\mathbf{n}}_E$ is divided into two subsequences, i.e., $\tilde{\mathbf{n}}_{E1}$ of length n_{e1} and $\tilde{\mathbf{n}}_{E2}$ of length n_{e2} , where $n_e = n_{e1} + n_{e2}$, which are then provided to OAM-based index module 110 and modulator 108 respectively.

[0019] OAM-based index module 110 is configured to have an OAM mode set Φ of size N and an active N_A number of OAM modes between 1 and N , i.e., $(1 \leq N_A \leq N)$. In embodiments of the disclosure, the number of effective OAM mode combinations used for the index is

defined as $2^{\lfloor \log_2 \binom{N}{N_A} \rfloor}$, where $\lfloor \cdot \rfloor$ is a floor operation. During each signal transmission slot, $\lfloor \log_2 \binom{N}{N_A} \rfloor$ bits are extracted from $\tilde{\mathbf{n}}_{E1}$ to form a vector $\mathbf{b}^{\text{Ind}}$ and are sent to OAM-based index module 110 for selecting the corresponding OAM mode combination $\phi_k \subset \Phi$ ($k = 1, 2, \dots, 2^{\lfloor \log_2 \binom{N}{N_A} \rfloor}$) that is be used by the transmitter in system 100. In particular, $\phi_k \triangleq \{m_1^k, m_2^k, \dots, m_{N_A}^k\}$ and $m_i^k (i = 1, 2, \dots, N_A) \in \Phi$.

[0020] As for modulator 108, module 108 is configured to utilize modulation schemes such as, but not limited to, the M -ary phase shift keying (PSK) or quadrature amplitude modulation (QAM) constellation set χ . During each signal transmission slot, $N_A \log_2 M$ bits are extracted from $\tilde{\mathbf{n}}_{E2}$ to form a vector $\mathbf{b}^{\text{Mod}}$ and sent to modulator 108 for determining a modulated symbol sequence $X_{N_A} \triangleq \{x_1, x_2, \dots, x_{N_A}\}$, where $x_i (i = 1, 2, \dots, N_A) \in \chi$ and M is defined as the size of the modulation scheme. The outputs from OAM-based index module 110 and modulator 108, i.e., ϕ_k and x_{N_A} , are respectively passed to OAM multiplexing module 112 which in turn is communicatively coupled to power allocation module 113. Power allocation module 113 is configured to determine the optimal distribution of power between different OAM modes in order to guarantee that all possible active OAM modes can share the same energy intensity. Based on these received values, OAM multiplexing module 112 is then configured to compute the spectral efficiency, η , (i.e., bits per channel use (bpcu)) of a PLDPC-coded OAM-IM system as:

$$\eta = \left\lfloor \log_2 \binom{N}{N_A} \right\rfloor + N_A \log_2 M \quad \dots (1)$$

[0021] In order to transmit an OAM signal with the mode number ξ , OAM multiplexing module 112 is then configured to communicate its corresponding modulated symbol x_ξ to all array elements in transmitting UCA 114 and this is implemented with a successive phase shift between array elements. As a result, the signal on the n -th ($n = 0, 1, \dots, N-1$) array element may be defined as:

$$s_n^\xi = \frac{1}{\sqrt{N}} p_\xi x_\xi \exp(-j\xi \frac{2\pi n}{N}) \quad \dots (2)$$

where p_ξ is defined as a power allocation factor for the signal with a mode number ξ .

[0022] Based on the above, a composite signal, s_n , (i.e., an OAM signal) emitted by the n -th array element may be defined as a linear superposition of the signals with different active modes and as such may be defined as:

$$s_n = \sum_{\xi \in \phi_k} s_n^\xi = \frac{1}{\sqrt{N}} \sum_{\xi \in \phi_k} p_\xi x_\xi \exp(-j\xi \frac{2\pi n}{N}) \quad \dots \text{equation (3)}$$

[0023] For simplicity, the transmitted OAM signal vector may be denoted as $\mathbf{s} = \mathbf{W}\mathbf{P}\mathbf{x}$ where $\mathbf{s} = [s_0, s_1, \dots, s_{N-1}]^T$ where the superscript “T” denotes a transpose operation; $\mathbf{W} \in \mathbb{C}^{N \times N}$ is defined as a unitary discrete Fourier transform (DFT) matrix with elements $\frac{1}{\sqrt{N}} \exp(-j\xi \frac{2\pi n}{N})$; $\mathbf{P} \triangleq \text{diag}([p_0, p_1, \dots, p_{N-1}])$ is defined as a power allocation matrix, while the total transmission power equals $P_t = \sum_{\xi=0}^{N-1} p_\xi^2$; $\mathbf{x} = [x_0, x_1, \dots, x_{N-1}]^T$ and comprises a complex data vector containing N_A modulated symbols and $(N - N_A)$ zero entries where the indices of N_A modulated symbols within x are determined by the entries (i.e., mode numbers) within ϕ_k .

[0024] At the receiver side of system 100, receiving UCA 116 is configured to have the same number of array elements as transmitting UCA 114. When both UCAs are aligned with each other, the received OAM signal vector $\mathbf{y} = [y_0, y_1, \dots, y_{N-1}]^T$ may be defined as:

$$\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n} \quad \dots (4)$$

where $\mathbf{H} \in \mathbb{C}^{N \times N}$ is the channel response matrix and characterizes the feature of an LoS transmission environment between both UCAs; $\mathbf{n} \in \mathbb{C}^{N \times 1}$ is an additive white Gaussian noise (AWGN) vector, and each entry is a complex Gaussian variable with zero mean and variance σ_n^2 . The channel response, h_{pn} , (i.e., channel gain) between the n -th transmitting (OAM) mode and the p -th receiving (OAM) mode may then be computed as:

$$h_{pn} = \frac{\beta \lambda \exp(-j \frac{2\pi}{\lambda} d_{pn})}{4\pi d_{pn}} \quad \dots (5)$$

where β is the antenna gain, λ is the wavelength, and d_{pn} is the distance between the n -th and p -th array elements. If both UCAs are aligned perfectly, d_{pn} may be computed as:

$$d_{pn} = \sqrt{D^2 + R_{tx}^2 + R_{rx}^2 - 2R_{tx}R_{rx}\cos\left(\frac{2\pi(n-p)}{N}\right)} \quad \dots (6)$$

where D is the distance between both the centers of transmitting UCA 114 and receiving UCA 116, and R_{tx} (R_{rx}) denotes the radius of the transmitting (receiving) UCA.

[0025] The OAM signal vector $\mathbf{y}$ as obtained by receiving UCA 116 will then be provided to demultiplexing module 118, where it is then processed by an inverse DFT (IDFT) operation (i.e., an IDFT matrix $\mathbf{W}^H$), which yields a demultiplexing-aided OAM signal vector $\tilde{\mathbf{y}} \in \mathbb{C}^{N \times 1}$ that may be defined as:

$$\begin{aligned} \tilde{\mathbf{y}} &= \mathbf{W}^H \mathbf{y} \\ &= \mathbf{W}^H \mathbf{H} \mathbf{s} + \mathbf{W}^H \mathbf{n} \\ &= \mathbf{W}^H \mathbf{H} \mathbf{W} \mathbf{p} + \mathbf{W}^H \mathbf{n} \end{aligned} \quad \dots (7)$$

[0026] In a scenario of perfect alignment between both UCAs, $\mathbf{H}$ may be defined as a circulant matrix and its N eigenvalues (i.e., $\zeta_0, \zeta_1, \dots, \zeta_{N-1}$) formulate a diagonal matrix $\Lambda \in \mathbb{C}^{N \times N}$, as:

$$\begin{aligned} \Lambda &= \text{diag}(\text{eig}(\mathbf{H})) \\ &= \mathbf{W}^H \mathbf{H} \mathbf{W} \end{aligned} \quad \dots (8)$$

[0027] In view of equation (8), equation (7) above may then be simplified to:

$$\tilde{\mathbf{y}} = \Lambda \mathbf{P} \mathbf{x} + \mathbf{W}^H \mathbf{n} \quad \dots (9)$$

[0028] As the radiation energies of different active OAM modes are different due to the characteristics of a Bessel function, the power allocation factor p_i ($i = 0, 1, \dots, N-1$) in $\mathbf{P}$ is used to ensure that all possible active OAM modes can share the same energy intensity. In particular, the power allocation factor p_i may be defined as:

$$p_i = \sqrt{\frac{p_t}{|\zeta_i|^2} / \left(\frac{1}{|\zeta_0|^2} + \frac{1}{|\zeta_1|^2} + \dots + \frac{1}{|\zeta_{N-1}|^2} \right)} \quad \dots (10)$$

[0029] Unlike ideal LoS transmission environments, both UCAs are usually misaligned in actual applications and as a result, inter-mode interference will occur. Under the assumption that the plane of the receiving UCA is tilted towards that of the transmitting UCA with an angle $\theta \in [0, \pi/2)$, the distance between two array elements for such two UCAs (i.e., d_{pn} in equation (6)) may be computed as:

$$d_{pn}^\theta = \sqrt{\tilde{D}^2 + R_{tx}^2 + \tilde{R}_{rx}^2 - 2R_{tx}\tilde{R}_{rx}\cos\left(\frac{2\pi(n-p)}{N}\right)} \quad \dots (11)$$

where $\tilde{D} = D - R_{rx} \cos \theta_p \sin \theta$, while $\theta_p = 2\pi p/N$; $\tilde{R}_{rx} = R_{rx} \sqrt{\cos^2 \theta \sin^2 \theta_a + \cos^2 \theta_a}$ and $\theta_a = \arctan\left(\frac{\cot \theta_p}{\cos \theta}\right)$.

[0030] When all received OAM signals are obtained using a demultiplexing operation, the corresponding extrinsic log-likelihood ratio (LLR) sequence $\tilde{\mathbf{L}}_E$ can be obtained from detector 120. Based on a specific deinterleaving procedure, deinterleaver 122 may transform $\tilde{\mathbf{L}}_E$ into $\mathbf{L}_E$ and this information may then be passed to decoder 124, which will use this information to produce the decoded information-bit sequence $\hat{\mathbf{n}}_s$. In an embodiment of the disclosure, detector 120 and decoder 124 may each be implemented by the maximum-likelihood (ML) and belief-propagation (BP) algorithms, respectively. The full channel state information (CSI) is assumed for the detection because a well-designed channel estimator may be adopted by the receiver to perfectly acquire the CSI in practical applications. The transmitted OAM signal vector $\mathbf{s}$ in each transmission slot may then be determined by the coded-bit vector $\mathbf{b} = [\mathbf{b}^{\text{Ind}}, \mathbf{b}^{\text{Mod}}]$ where $\mathbf{b}^{\text{Ind}}$ denotes a vector consisting of $q = \left\lceil \log_2 \left(\frac{N}{N_A} \right) \right\rceil$ coded bits that are used for the index, while $\mathbf{b}^{\text{Mod}} = [\mathbf{b}_1^{\text{Mod}}, \mathbf{b}_2^{\text{Mod}}, \dots, \mathbf{b}_{N_A}^{\text{Mod}}]$ denotes a vector composed of N_A sub-vectors $\mathbf{b}_z^{\text{Mod}}$ ($z = 1, 2, \dots, N_A$), where each sub-vector consists of $m = \log_2 M$ coded bits used for modulation. Under the assumption that all effective OAM mode combinations constitute a set Φ^{eff} , the soft outputs of detector 120, i.e., the LLRs l_i^{Ind} and $l_{q+m(z-1)+j}^{\text{Mod}}$ for the i -th and $(q+m(z-1)+j)$ -th coded bits, respectively, in $\mathbf{b}$ may be computed as:

$$l_i^{Ind} = \log \frac{\sum_{\mathbf{s}, \mathbf{b}^{Ind} \in \Phi_{i,0}^{eff}, \mathbf{b}^{Mod} \in \chi_{j,b}^z} \exp\left(\frac{-\|\bar{\mathbf{y}} - \mathbf{W}^H \mathbf{H} \mathbf{s}\|^2}{2\sigma_n^2}\right)}{\sum_{\mathbf{s}, \mathbf{b}^{Ind} \in \Phi_{i,1}^{eff}, \mathbf{b}^{Mod} \in \chi_{j,b}^z} \exp\left(\frac{-\|\bar{\mathbf{y}} - \mathbf{W}^H \mathbf{H} \mathbf{s}\|^2}{2\sigma_n^2}\right)} \quad \dots (12)$$

$$l_{q+m(z-1)+j}^{Mod} = \log \frac{\sum_{\mathbf{s}, \mathbf{b}^{Ind} \in \Phi_{i,b}^{eff}, \mathbf{b}^{Mod} \in \{\chi_{j,0}^z, \chi_{j',b}^{z'}\}} \exp\left(\frac{-\|\bar{\mathbf{y}} - \mathbf{W}^H \mathbf{H} \mathbf{s}\|^2}{2\sigma_n^2}\right)}{\sum_{\mathbf{s}, \mathbf{b}^{Ind} \in \Phi_{i,b}^{eff}, \mathbf{b}^{Mod} \in \{\chi_{j,1}^z, \chi_{j',b}^{z'}\}} \exp\left(\frac{-\|\bar{\mathbf{y}} - \mathbf{W}^H \mathbf{H} \mathbf{s}\|^2}{2\sigma_n^2}\right)} \quad \dots (13)$$

where $\Phi_{i,b}^{eff}$ denotes a subset of Φ^{eff} , and it is mapped by $\mathbf{b}^{Ind}$ with the i -th ($i = 1, 2, \dots, q$) coded bit being $b \in \{0, 1\}$. Also, $\chi_{j,b}^z$ denotes the z -th constellation subset mapped by $\mathbf{b}_z^{Mod}$ with the j -th ($j = 1, 2, \dots, m$) coded bit being $b \in \{0, 1\}$, while $\chi_{j',b}^{z'}$ denotes the z' -th ($z' = 1, 2, \dots, N_A$ and $z' \neq z$) constellation subset mapped by $\mathbf{b}_{z'}^{Mod}$ with the j' -th ($j' = 1, 2, \dots, m$) coded bit being $b \in \{0, 1\}$, where $\|\cdot\|$ denotes the L_2 -norm, and $\log(\cdot)$ denotes the natural logarithm operation. From the above, it is understood that the LLRs of the n_{el} bits used for index can be estimated even if they are not transmitted. The SNR may be defined as $P_t/\eta/\sigma_n^2$, which equals E_b/N_0 , where E_b and N_0 are the average energy per bit and noise power spectral density respectively. Furthermore, it should be noted that the inter-mode interference has been considered in the procedure of computing the LLR (see equations (12) and (13)).

[0031] A block diagram representative of a transmitting side of a shape index modulation transceiver for a multi-mode OAM system in accordance with embodiments of the present disclosure is disclosed in Figure 2a. As shown, shape index modulation transmitter 200 includes many of the same modules as the transmitter section of PLDPC-coded OAM-IM system 100 as previously described in Figure 1. Additionally, two new modules are incorporated into transmitter 200: (i) shaping encoder module 202, which is connected to the output of bit-splitter 106; and (ii) an interleaver-2 module 204, which is positioned between an output of the shaping encoder module 202 and an input of OAM-based index module 110. In embodiments of the disclosure, the output of interleaver-2 module 204 is concatenated with OAM-based index module 110 and is used to process the output bits from shaping encoder module 202. Further, it should be noted that the output bits from interleaver-2 module 204 are used to activate the transmitting modes/antennas/beams of transmitter 200 and that these index bits are not transmitted with the modulation signals.

[0032] In embodiments of the disclosure, optimal OAM mode combinations are pre-computed for shape index modulation transmitter 200. In further embodiments of the disclosure, the optimal OAM mode combinations of the transmitter may comprise four optimal mode combinations that were selected by channel estimation of a 4×4 orbital angular momentum-index modulation system that has two active OAM modes. In another embodiment of the disclosure, the optimal OAM mode combinations of the transmitter may comprise optimal mode combinations that were selected using a mutual information (MI)-based index scheme. The detailed steps to compute these optimal OAM mode combinations will be described in greater detail in the later sections in this description. Furthermore, these pre-computed mode combinations are configured such that each of these optimal OAM mode combinations are associated with index symbols that comprise primarily of a predetermined binary value – which may comprise ‘0’ bits or ‘1’ bits.

[0033] **Embodiment 1 of the Transmitter**

[0034] In embodiments of the disclosure, when the optimal OAM mode combinations of the transmitter comprises four optimal mode combinations that were selected by channel estimation of a 4×4 orbital angular momentum-index modulation system having two active OAM modes (i.e., $N = 4$ OAM modes and $N_A = 2$ active OAM modes), it was found that the index modulation (IM) scheme may comprise of $C_e = 2^{\lfloor \log_2 \binom{N}{N_A} \rfloor}$ mode combinations (i.e., $\phi_1, \phi_2, \dots, \phi_{C_e}$). The reliabilities of these C_e mode combinations may then be estimated with the aid of a standard channel estimation process. The detailed workings of the standard channel estimation process are omitted for brevity in this description as it is well known to one skilled in the art.

[0035] As an example, with the aid of the channel estimation process, it is assumed that four (4) mode combinations which provide the optimum system performance for 4×4 OAM-IM systems with $N_A = 2$ were obtained as: $\{(m1, m2), (m1, m3), (m1, m4), (m2, m3)\}$ – and was arranged in descending order of reliability from the left to the right. For this example, it was determined that the optimal OAM mode combinations are associated with index symbols that comprise primarily of ‘0’ bits, i.e., the predetermined binary value is ‘0’.

[0036] Specifically, it was set that the smaller the number k is, the higher the reliability of ϕ_k that can be achieved ($k = 1, 2, \dots, C_e$). Each of the C_e index symbols, of which each comprises of a binary sequence composed of $\left\lceil \log_2 \left(\frac{N}{N_A} \right) \right\rceil$ index bits is then defined to have an one-to-one correspondence between the C_e index symbols and C_e mode combinations. Moreover, the k -th index symbol corresponding to ϕ_k was set to be a binary representation of the decimal number $(k - 1)$. An exemplary representation of this is shown in Table 1.

Index bits		Shaped Natural Index Scheme
n_1	n_2	$\Phi: m_1, m_2, m_3, m_4$
0	0	$\phi_1 (P_{\phi_1} = 0.40625): m_1, m_2$
0	1	$\phi_2 (P_{\phi_2} = 0.40625): m_1, m_3$
1	0	$\phi_3 (P_{\phi_3} = 0.09375): m_1, m_4$
1	1	$\phi_4 (P_{\phi_4} = 0.09375): m_2, m_3$

TABLE 1

[0037] Table 1 shows the shaped Natural Index Modulation scheme for a 4×4 OAM system with $N_A = 2$ and $\theta = 0.8$. It is useful to note that for a conventional Natural Index Modulation scheme (not shown), the occurrence probabilities of all OAM mode combinations are usually set to be the same (e.g., $P_{\phi_1} = P_{\phi_2} = P_{\phi_3} = P_{\phi_4} = 0.25$). However, for the shaped Natural index scheme shown in Table 1 above, as it was determined that the OAM mode combinations ϕ_1 and ϕ_2 have better reliabilities as compared to the other two mode combinations, the OAM mode combinations ϕ_1 and ϕ_2 were set to have higher occurrence probability than other mode combinations ϕ_3 and ϕ_4 ($P_{\phi_1} = P_{\phi_2} = 0.40625 > P_{\phi_3} = P_{\phi_4} = 0.09375$).

[0038] Based on the above process, for the $C_e/2$ mode combinations with the highest reliabilities (i.e., $\phi_1, \phi_2, \dots, \phi_{C_e/2}$), it was determined that their corresponding index symbols have one thing in common, which is that their first coded bits are $n_1 = 0$. In view of the above, it can be seen that a significant system performance gain may be achieved if more index symbols with $n_1 = 0$ exist in the index procedure (i.e., the mode combinations ϕ_k have a high occurrence probability P_{ϕ_k} , where $k = 1, 2, \dots, C_e/2$).

[0039] With reference to Figure 2a, in operation, it can be seen that an interleaved coded-bit sequence $\tilde{\mathbf{n}}_E$ is first divided into two sub-sequences $\tilde{\mathbf{n}}_{E1}$ and $\tilde{\mathbf{n}}_{E2}$ by bit splitter 106. Sub-

sequence $\tilde{\mathbf{n}}_{E1}$ is then provided to shaping encoder 202 having the parameters (n_s, k_s) , while $\tilde{\mathbf{n}}_{E2}$ is provided to modulator 108 to be used in the generation of modulation symbols. Shaping encoder 202 is configured to first divide $\tilde{\mathbf{n}}_{E1}$ into $k = \left\lfloor \log_2 \left(\frac{N}{N_A} \right) \right\rfloor$ sub-sequences (i.e., $\tilde{\mathbf{n}}_{E11}, \tilde{\mathbf{n}}_{E12}, \dots, \tilde{\mathbf{n}}_{E1k}$) whereby the length of $\tilde{\mathbf{n}}_{E11}$ is k_s , while the length of the remaining $(k - 1)$ sub-sequences is defined to be n_s ($k_s < n_s$). Sub-sequence $\tilde{\mathbf{n}}_{E11}$ is then processed by a shaping function and converted to a new sub-sequence $\tilde{\mathbf{n}}'_{E11}$ of length n_s . The detailed (n_s, k_s) shaping function is illustrated in Algorithm 1 below. Both k_s ($k_s \geq 1$) and n_s ($n_s > k_s$) are non-negative integers, and they satisfy the condition $(k_s + (k - 1)n_s) \times i = n_{e1}$, where i is a non-negative integer (i.e., $i = 1, 2, \dots$) and n_{e1} is the length of a coded-bit subsequence used for OAM mode selection. It is worth noting that the larger the value n_s/k_s is, the better the system performance can be achieved, but at the expense of spectral efficiency. Therefore, the values k_s and n_s can be determined based on the actual communication requirements. It should be noted that the aim of the shaping encoder is to increase the occurrence of the predetermined binary value, i.e., bit “0”, within $\tilde{\mathbf{n}}'_{E11}$ (i.e., increase the probability of a bit being 0). Finally, shaping encoder 204 formulates each index symbol by extracting k bits from k different sub-sequences of length n_s in a sequential order. As a result, the bits within $\tilde{\mathbf{n}}'_{E11}$ will determine the bit n_1 within each index symbol. It should be noted that $k_s(n_s)$ denotes the number of bits sent to (output from) the shaping encoder.

[0040] In summary, Algorithm 1 outlines a shaping principle for a sequence $\tilde{\mathbf{n}}_{E1}$, which is divided into $k = \left\lfloor \log_2 \left(\frac{N}{N_A} \right) \right\rfloor$ sub-sequences, with the first sub-sequence $\tilde{\mathbf{n}}_{E11}$ having a length of k_s and the remaining $(k - 1)$ sub-sequences each having a length of n_s ($k_s < n_s$). A codebook ensemble C_{k_s} is created, containing 2^{k_s} different binary sequences, each formulated by k_s binary bits. The i -th binary sequence in C_{k_s} represents the binary version of the decimal number $i - 1$, where i ranges from 1 to 2^{k_s} .

[0041] The shaping process for $\tilde{\mathbf{n}}_{E11}$ involves mapping the i -th binary sequence within C_{k_s} to a length- n_s binary sequence. If $i = 1$, the sequence is mapped to an all-zero sequence of length n_s . For values of $i > 1$, the sequence is mapped to a length- n_s binary sequence having a Hamming distance d_h , which should be as small as possible while ensuring that the current i -th sequence is distinct from all previous sequences. As a result, a codebook ensemble C_{n_s} is formed, comprising of 2^{k_s} length- n_s binary sequences, with a one-to-one correspondence

between the sequences in C_{k_s} and C_{n_s} . Based on the mappings between these two codebooks, the sub-sequence $\tilde{n}_{E11}$ is then converted to a new sub-sequence $\tilde{n}'_{E11}$ of length n_s . Finally, each index symbol is formed by extracting k bits from k different sub-sequences of length n_s , in a sequential order.

Algorithm 1: (n_s, k_s) shaping principle

- 1 **Initialization:** 1) $\tilde{n}_{E11}$ is divided into $k := \lfloor \log_2 \binom{N}{N_A} \rfloor$ sub-sequences (i.e., $\tilde{n}_{E11}, \tilde{n}_{E12}, \dots, \tilde{n}_{E1k}$). The length of $\tilde{n}_{E11}$ is k_s , while that of the remaining $(k - 1)$ sub-sequences is n_s ($k_s < n_s$);
 2) Set a codebook ensemble C_{k_s} consisting of 2^{k_s} different binary sequences. Each binary sequence is formulated by k_s binary bits, and the i -th binary sequence is considered as a binary representation of the decimal number $(i - 1)$, where $i = 1, 2, \dots, 2^{k_s}$.
Shaping for $\tilde{n}_{E11}$:
for $i = 1, 2, \dots, 2^{k_s}$ do
 2 **if $i = 1$ then**
 3 The i -th binary sequence within C_{k_s} is mapped to a length- n_s all-zero sequence;
 4 **if $i \neq 1$ then**
 5 The i -th binary sequence within C_{k_s} is mapped to a length- n_s binary sequence with a Hamming distance d_h . In particular, d_h should be as small as possible while ensuring that the current (i.e., the i -th) length- n_s binary sequence is different from the j -th ($j < i$) length- n_s binary sequence;
 ...
 6 **Shaping Result:** A codebook ensemble C_{n_s} , which is composed of 2^{k_s} length- n_s binary sequences, is formulated. Moreover, there is an one-to-one correspondence between the binary sequences of C_{k_s} and those of C_{n_s} . $\tilde{n}_{E11}$ is converted to a new sub-sequence $\tilde{n}'_{E11}$ of length n_s .
Finalization: Each index symbol is formulated by extracting k bits from k different sub-sequences of length n_s (i.e., $\tilde{n}'_{E11}, \tilde{n}_{E12}, \dots, \tilde{n}_{E1k}$) in a sequential order.
-

[0042] As it was determined that the optimal OAM mode combinations are associated with index symbols that comprise primarily of '0' bits, the formed index symbols may then be used to increase the probability of desirable transmitting modes/antennas/beams being activated for transmitting the signals, which is beneficial to avoid signal fading issues during transmission of the signals. More specifically, the proposed technique considers choosing the optimal transmission path for signals in free space based on previously obtained extrinsic channel feedback.

[0043] As the technique described above utilizes a Multiple-Input Multiple-Output (MIMO) framework, this enables the transmitter to be used in MIMO Line-of-Sight (LoS) scenarios,

Visible Light Communication (VLC) scenarios, and multi cell scenarios due to the high throughput that may be achieved by transmitter 200. In particular, the throughput may be defined as:

$$\left\lceil \log_2 \binom{N}{N_A} \right\rceil + N_A \log_2 M - (1 - k_s/n_s) \quad 1 \leq N_A < N \quad \dots (14)$$

[0044] **Embodiment 2 of the Transmitter**

[0045] In another embodiment of the disclosure, when the optimal OAM mode combinations of the transmitter comprises optimal mode combinations that were selected using a mutual information (MI)-based theoretical-analysis method, a B_e number of index symbols may be defined such that each symbol comprises a binary coded-bit sequence composed of $\left\lceil \log_2 \binom{N}{N_A} \right\rceil$ coded bits (i.e., $\{b_1, b_2, \dots, b_{\left\lceil \log_2 \binom{N}{N_A} \right\rceil}\}$, where b_i is the i -th coded bit). Moreover, the binary coded-bit sequence of the k -th index symbol is a binary representation of the decimal number $(k-1)$. A one-to-one correspondence is then defined between the k -th index symbol and the k -th mode combination ($k = 1, 2, \dots, B_e$) where the first $B_e/2$ index symbols (i.e., $\{0, 0, \dots, 0\}, \{0, 0, \dots, 1\}, \dots, \{0, 1, \dots, 1\}$) correspond to the $B_e/2$ mode combinations with highest reliabilities (i.e., $\phi_1, \phi_2, \dots, \phi_{B_e}$). Also, in this embodiment, the first $B_e/2$ index symbols we set to have one thing in common: which is that their first coded bits are $b_1 = 0$. An example of such a relationship is illustrated in Table 2 below.

Coded Bits (Index Symbols)		MI-based Index				
b_1	b_2	$\Phi:$	m_1	m_2	m_3	m_4
0	0	$\phi_1:$		✓		✓
0	1	$\phi_2:$	✓			✓
1	0	$\phi_3:$			✓	✓
1	1	$\phi_4:$	✓	✓		

TABLE 2

[0046] Table 2 above shows optimal mode combinations that were selected using a mutual information (MI)-based theoretical-analysis method. As can be seen, if more index symbols with $b_1 = 0$ exist in the index procedure (i.e., the mode combinations ϕ_j have a high occurrence probability P_{ϕ_j} , where $j = 1, 2, \dots, B_e/2$), more modulated symbols can be transmitted by the

OAM modes with relatively high reliabilities. In other words, the OAM-IM system can achieve better performance.

Index bits		Shaped MI-based Index Scheme
b ₁	b ₂	$\Phi: m_1, m_2, m_3, m_4$
0	0	$\phi_1 (P_{\phi_1} = 0.40625): m_2, m_4$
0	1	$\phi_2 (P_{\phi_2} = 0.40625): m_1, m_4$
1	0	$\phi_3 (P_{\phi_3} = 0.09375): m_3, m_4$
1	1	$\phi_4 (P_{\phi_4} = 0.09375): m_1, m_2$

TABLE 3

[0047] Table 3 above shows a shaped MI-based Index Modulation scheme for a 4×4 OAM-IM system with $N_A = 2$, $\theta = 0.8$, and a shaping encoder (4,2) where the reliabilities of these mode combinations were estimated using the mutual information (MI)-based theoretical-analysis method and these mode combinations were ranked accordingly with OAM mode combinations ϕ_1 and ϕ_2 having better reliabilities as compared to the other two mode combinations. As such, the OAM mode combinations ϕ_1 and ϕ_2 were set to have higher occurrence probability than other mode combinations ϕ_3 and ϕ_4 ($P_{\phi_1} = P_{\phi_2} = 0.40625 > P_{\phi_3} = P_{\phi_4} = 0.09375$).

[0048] With reference to Figure 2a, in operation, it can be seen that an interleaved coded-bit sequence $\tilde{\mathbf{n}}_E$ is first divided into two sub-sequences $\tilde{\mathbf{n}}_{E1}$ and $\tilde{\mathbf{n}}_{E2}$ by bit splitter 106. Sub-sequence $\tilde{\mathbf{n}}_{E1}$ is then provided to shaping encoder 202 having the parameters (z_s, k_s) , while $\tilde{\mathbf{n}}_{E2}$ is provided to modulator 108 to be used in the generation of modulation symbols. Shaping encoder 202 is configured to first divide $\tilde{\mathbf{n}}_{E1}$ into $q = \left\lfloor \log_2 \left(\frac{N}{N_A} \right) \right\rfloor$ sub-sequences (i.e., $\tilde{\mathbf{n}}_{E11}, \tilde{\mathbf{n}}_{E12}, \dots, \tilde{\mathbf{n}}_{E1q}$) whereby the length of $\tilde{\mathbf{n}}_{E11}$ is jk_s , while the length of the remaining $(q - 1)$ sub-sequences is defined to be jz_s ($k_s < z_s$) where j is defined as a scaling factor and may comprise any integer $j = 1, 2, \dots$. For this embodiment, the detailed (z_s, k_s) shaping functions performed by shaping encoder 202 is set out in Algorithm 2 below. The function of shaping encoder 202 is to increase the number of bit 0 (i.e., the predetermined binary value) within $\tilde{\mathbf{n}}_{E11}'$ (i.e., to increase the probability that's the bits are '0'). Finally, each index symbol is formulated by extracting q bits from q sub-sequences of length jz_s in a sequential manner. As

the bits within $\tilde{\mathbf{n}}'_{E11}$ determine the b_I bit within each index symbol, more index symbols with $b_I = 0$ will exist upon completion of the shaping function described above.

Algorithm 2 (z_s, k_s) Shaping Procedure

Input: k_s, z_s ;

Output: Two codebook ensembles $C_{k_s} = \{c_{k_s}^1, c_{k_s}^2, \dots, c_{k_s}^{2^{k_s}}\}$ and $C_{z_s} = \{c_{z_s}^1, c_{z_s}^2, \dots, c_{z_s}^{2^{z_s}}\}$, and an one-to-one correspondence between the binary sequences of C_{k_s} and those of C_{z_s} .

- 1: **for** $i = 1, 2, \dots, 2^{k_s}$ **do**
 - 2: $c_{k_s}^i = \text{dec2bin}(k_s, (i-1))$, where $\text{dec2bin}(\alpha, e)$ is a length- α binary sequence representation of the decimal number e ;
 - 3: $c_{z_s}^i$ is a length- z_s binary sequence with the smallest possible weight w_{min} , while $c_{z_s}^i \neq c_{z_s}^j$ ($i > j$). In particular, $w_{min} = \|c_{z_s}^i\|_0$ and $\|\cdot\|_0$ denotes an L_0 -norm;
 - 4: $c_{k_s}^i$ is mapped to $c_{z_s}^i$.
 - 5: **end for**
-

[0049] In summary, Algorithm 2 outlines a shaping procedure for creating two codebook ensembles C_{k_s} and C_{z_s} , each containing 2^{k_s} binary sequences, with one-to-one mapping between the binary sequences of C_{k_s} and those of C_{z_s} . The parameters for Algorithm 2 are k_s and z_s , which are utilized to determine the lengths of the binary sequences in the respective codebooks.

[0050] The algorithm proceeds as follows: For each index i ranging from 1 to 2^{k_s} , a binary sequence $c_{k_s}^i$ is generated by converting the decimal number $i-1$ to a binary sequence of length k_s . Then, a corresponding binary sequence $c_{z_s}^i$ of length z_s is created with the smallest possible weight w_{min} , ensuring that $c_{z_s}^i$ is distinct from all previously generated sequences $c_{z_s}^j$ for values of $i > j$. The weight w_{min} is defined as the L_0 -norm of $c_{z_s}^i$, which counts the number of non-zero elements in the sequence. Finally, each sequence $c_{k_s}^i$ in codebook C_{k_s} is mapped to a unique sequence $c_{z_s}^i$ in C_{z_s} , establishing the one-to-one correspondence between the two codebooks.

[0051] In further embodiments, when the parameters (z_s, k_s) were set to be (4, 3) and (5, 3), it was found that the occurrence probabilities of bit '0' in the index symbols was 0.687 and 0.755 respectively.

[0052] A block diagram representative of a receiving side of a shape index modulation transceiver for a multi-mode OAM system in accordance with embodiments of the present disclosure is illustrated in Figure 2b. As shown, shape index modulation receiver 210 includes many of the same modules as the receiver section of PLDPC-coded OAM-IM system 100 as previously described in Figure 1. Additionally, three new modules are incorporated into receiver 210: LLR splitter module 212, deinterleaver-2 module 214 and shaping decoder module 216.

[0053] LLR splitter module 212 is configured to divide the LLR sequence output from detector 120 into two distinct sub-sequences, denoted as $\tilde{\mathbf{L}}_{E1}$ and $\tilde{\mathbf{L}}_{E2}$. Specifically, $\tilde{\mathbf{L}}_{E1}$ corresponds to the data stream $\tilde{\mathbf{n}}_{E3}$, which is the output from the interleaver-2 module 204 in the transmitter; while $\tilde{\mathbf{L}}_{E2}$ is linked to the data stream $\tilde{\mathbf{n}}_{E2}$, which is generated from the bit splitter 106 and is utilized for modulation purposes.

[0054] Deinterleaver-2 module 214, which is matched to interleaver-2 module 204 as used in the transmitter, is designed to permute (i.e., deinterleave) the LLR sequence $\tilde{\mathbf{L}}_{E1}$ and subsequently output a new LLR sequence $\mathbf{L}_{E1}$. This new sequence $\mathbf{L}_{E1}$ is directly related to the data stream $\mathbf{n}_{E3}$ in the transmitter, ensuring a coherent data flow between the transmitter and receiver components.

[0055] As for shaping decoder 216, this module is tasked with executing a shaping decoding procedure on the LLR sequence $\mathbf{L}_{E1}$ output from the deinterleaver-2 module 214. This shaping decoding process can be regarded as the inverse of the shaping encoding procedure carried out in the transmitter. As a result of this decoding, a new LLR sequence $\tilde{\mathbf{L}}_{E3}$ is obtained, which corresponds to the original data stream $\tilde{\mathbf{n}}_{E1}$ in the transmitter, thereby completing the decoding process and ensuring data integrity.

[0056] In accordance with embodiments of the present disclosure, a block diagram representative of components of processing system 300 that may be provided within shaping encoder module 202, OAM-based index module 110, interleaver-2 module 204 and/or any of the modules shown in Figures 2a and 2b to carry out the digital signal processing functions in accordance with embodiments of the disclosure. One skilled in the art will recognize that the exact configuration of each processing system provided within these modules may be different

and the exact configuration of processing system 300 may vary and the arrangement illustrated in Figure 3 is provided by way of example only.

[0057] In embodiments of the disclosure, processing system 300 may comprise controller 301 and user interface 302. User interface 302 is arranged to enable manual interactions between a user and the computing module as required and for this purpose includes the input/output components required for the user to enter instructions to provide updates to each of these modules. A person skilled in the art will recognize that components of user interface 302 may vary from embodiment to embodiment but will typically include one or more of display 340, keyboard 335 and optical device 336.

[0058] Controller 301 is in data communication with user interface 302 via bus 315 and includes memory 320, processing unit or processor 305 mounted on a circuit board that processes instructions and data for performing the method of this embodiment, an operating system 306, an input/output (I/O) interface 330 for communicating with user interface 302 and a communications interface, in this embodiment in the form of a network card 350. Network card 350 may, for example, be utilized to send data from these modules via a wired or wireless network to other processing devices or to receive data via the wired or wireless network. Wireless networks that may be utilized by network card 350 include, but are not limited to, Wireless-Fidelity (Wi-Fi), Bluetooth, Near Field Communication (NFC), cellular networks, satellite networks, telecommunication networks, Wide Area Networks (WAN) and etc.

[0059] Memory 320 and operating system 306 are in data communication with processor 305 via bus 310. The memory components include both volatile and non-volatile memory and more than one of each type of memory, including Random Access Memory (RAM) 323, Read Only Memory (ROM) 325 and a mass storage device 345, the last comprising one or more solid-state drives (SSDs). One skilled in the art will recognize that the memory components described above comprise non-transitory computer-readable media and shall be taken to comprise all computer-readable media except for a transitory, propagating signal. Typically, the instructions are stored as program code in the memory components but can also be hardwired. Memory 320 may include a kernel and/or programming modules such as a software application that may be stored in either volatile or non-volatile memory.

[0060] Herein the term “processor” or “processing unit” is used to refer generically to any device or component that can process such instructions and may include: a microprocessor, a processing unit, a microcontroller, a programmable logic device or other computational device. That is, processor 305 may be provided by any suitable logic circuitry for receiving inputs, processing them in accordance with instructions stored in memory and generating outputs (for example to the memory components or on display 340). In this embodiment, processor 305 may be a single core or multi-core processor with memory addressable space. In one example, processor 305 may be multi-core, comprising—for example—an 8 core CPU. In another example, it could be a cluster of CPU cores operating in parallel to accelerate computations.

[0061] A flowchart which sets out the process for shaping index modulation in a multi-mode OAM system in accordance with embodiments of the present disclosure is illustrated in Figure 4. In embodiments of the disclosure, process 400 as illustrated in Figure 4 may be performed by shaping encoder module 202 and/or OAM-based index module 110.

[0062] Process 400 begins at step 402, with process 400 dividing a binary coded-bit sequence received from a bit-splitter module into a K -number of sub-sequences, whereby K is an integer that is determined based on a total number of OAM modes and a number of active OAM modes. Process 400 then proceeds to step 404 where it processes, using a shaping function, the sub-sequences to increase the probability that bits in the sub-sequences comprise a predetermined binary value.

[0063] At step 406, process 400 then formulates index symbols based on the processed sub-sequences, whereby a first bit of each of the index symbols is obtained from a first processed sub-sequence. At the final step 408, process 400 then provides the index symbols to an OAM mode selector module for selecting corresponding OAM mode combinations for transmission by the transmitter, whereby optimal OAM mode combinations of the transmitter are associated with index symbols that comprise primarily of the predetermined binary value.

[0064] A flowchart which illustrates a process for processing the sub-sequences using a shaping function in accordance with embodiments of the present disclosure is shown in Figure 5. In embodiments of the disclosure, process 500 as illustrated in Figure 5 may be performed by shaping encoder module 202 and/or OAM-based index module 110.

[0065] Process 500 begins at step 502 by generating a codebook comprising an initial set of distinct binary sequences that are mapped to a final set of distinct binary sequences that comprise primarily of the predetermined binary value, whereby a length of each binary sequence in the initial set of distinct binary sequences is equal to the first length and a length of each binary sequence in the final set of distinct binary sequences is equal to the second length. Process 500 then proceeds to convert the first sub-sequence to a shaped first sub-sequence based on the mappings in the codebook at step 504.

[0066] In embodiments of the disclosure, the step of formulating the index symbols based on the processed sub-sequences at step 406 may comprise the steps of, for the formulation of each index symbol, sequentially extracting, using the shaping encoder module, a predefined number of distinct bits from each of the processed sub-sequences and concatenate the extracted bits to form the index symbol.

[0067] In embodiments of the disclosure, the step of selecting the optimal mode combinations using a MI-based theoretical-analysis method comprises the steps of performing one or more rounds of mode-combination selection, using a MI-based theoretical-analysis module that is communicatively coupled to the OAM mode selector, each round comprising the steps of: generating a modulated symbol sequence for a distinct signal-to-noise ratio (SNR); sequentially dividing the modulated symbol sequence into a N number of sub-sequences, whereby the modulated symbols within a k-th sub-sequence are to be transmitted by a k-th mode, where k is defined as a number from a set {1, 2, ..., N} and N is defined as the total number of different OAM modes; computing, using an MI calculator, extrinsic MI values for each mode based on the sub-sequence associated with the mode; and determining mode combinations to be used as the optimal mode combinations based on mode combinations that have the largest MI values.

[0068] In embodiments of the disclosure, the MI calculator may be configured to compute extrinsic MI values based on a customized Box-Cox transformation (CBCT)-based protograph extrinsic information transfer (PEXIT) algorithm and the binary coded-bit sequence may comprise a bit sequence coded based on a protograph low-density parity-check (PLDPC) code,

an improved unpunctured (IUP) PLDPC code, or one of two improved punctured (IP1 and IP2) PLDPC codes.

[0069] **Customized Box-Cox Transformation**

[0070] In certain transmission environments, it is useful to consider channel characteristics so that coding schemes that approach channel capacity may be developed. To achieve this, Monte-Carlo simulations were performed and the extrinsic LLR values of coded bits output from the channel detector (as shown in Figure 1) are analyzed. The simulation parameters for the simulated OAM-IM systems are set out in Table 4 below.

OAM simulation system parameters	
UCA configurations	4 x 4
Transmission distance (D)	500 λ m
Transmitting UCA radius (R_{tx})	0.6 m
Receiving UCA radius (R_{rx})	0.6 m
Total transmission power (P_t)	1 W
Antenna gain (β)	1
Wavelength (λ)	0.03 m

TABLE 4

[0071] For this simulation, a regular-(3, 6) PLDPC code with a code rate $R_p = 0.5$ and a codeword length $n_e = 2400$ was adopted, while the modulation scheme comprised a quadrature PSK (QPSK) modulation scheme. Although the conditional distribution analysis for LLRs obtained from the detector does not require channel coding, a regular-(3, 6) PLDPC code was used to provide a more practical analysis for PLDPC-coded OAM-IM systems. b_i was denoted as the i -th coded bit in a frame (codeword) and l_i was denoted as the corresponding LLR value obtained from the channel detector.

[0072] After the LLRs were generated from the simulations, the range of LLR values are sequentially divided into several uniformed intervals, each with a length of $l_{int} = 0.1$, and the occurrence frequencies of these LLRs were computed for different intervals. These occurrence frequencies are then scaled by a factor of $1/l_{int}$ to generate the LLR histograms $h(l_i|b_i = 0)$ and $h(l_i|b_i = 1)$, which correspond to the Probability Density Functions (PDFs) of an appropriately quantized version of LLRs for the values of $b_i = 0$ and $b_i = 1$, respectively.

[0073] Further, symmetric Gaussian distributions $\mathcal{N}(u, 2u)$ (i.e., $\mathcal{N}(u_0, 2u_0)$ and $\mathcal{N}(u_1, 2u_1)$) were considered for comparison, where u is the mean, $2u$ is the variance, and u_0 (u_1) denotes the average *extrinsic* LLR values for $b_i = 0$ ($b_i = 1$). With reference to Figure 6a, when an OAM-IM with $N_A = 1$ and a SNR = 80 dB is considered for $\theta = 0$ (i.e., there is perfect alignment between the transmitting and receiving UCAs), it can be seen that the *extrinsic* LLR histograms $h(l_i|b_i = 0)$ and $h(l_i|b_i = 1)$ (i.e., the scattered points) are consistent with the symmetric Gaussian distributions $\mathcal{N}(u_0, 2u_0)$ – plot 604 and $\mathcal{N}(u_1, 2u_1)$ – plot 602 (i.e., solid lines), respectively.

[0074] When two UCAs are misaligned with each other (i.e., $\theta \neq 0$), the *extrinsic* LLR histograms $h(l_i|b_i = 0)$ and $h(l_i|b_i = 1)$ may not exhibit such PDF characteristics. An example when an OAM-IM with $N_A = 1$ and a SNR = 80 dB is considered for $\theta = 0.8$ is illustrated in Figure 6b, where $h(l_i|b_i = 0)$ and $h(l_i|b_i = 1)$ tend to follow skewed distributions (i.e., the scattered points), but not the symmetric Gaussian distributions $\mathcal{N}(u_0, 2u_0)$ – plot 608 and $\mathcal{N}(u_1, 2u_1)$ – plot 606 (i.e., solid lines), respectively.

[0075] A similar phenomenon is illustrated in Figures 7a and 7b when an OAM-IM with $N_A = 2$ and a SNR = 82 dB is considered for $\theta = 0$ (Figure 7a) and for $\theta = 0.8$ (Figure 7b). In particular, it can be seen from Figure 7a that the *extrinsic* LLR histograms $h(l_i|b_i = 0)$ and $h(l_i|b_i = 1)$ (i.e., the scattered points) are consistent with the symmetric Gaussian distributions $\mathcal{N}(u_0, 2u_0)$ – plot 704 and $\mathcal{N}(u_1, 2u_1)$ – plot 702 (i.e., solid lines), respectively. However, it can be seen from Figure 7b that the *extrinsic* LLR histograms $h(l_i|b_i = 0)$ and $h(l_i|b_i = 1)$ (i.e., the scattered points) are not consistent with the symmetric Gaussian distributions $\mathcal{N}(u_0, 2u_0)$ – plot 708 and $\mathcal{N}(u_1, 2u_1)$ – plot 706 (i.e., solid lines), respectively.

[0076] It should be noted that these numerical results shown in Figures 6a and 6b, Figures 7a and 7b are all calculated based on *extrinsic* LLRs obtained from simulations. Further, discrete point representations were adopted in these plots as compared to the traditional histograms in order to make the observations clearer. Thus, it can be seen that the conventional theoretical-analysis tools used for the density evolution analysis of LDPC codes (i.e., *extrinsic* information transfer (EXIT) chart and protograph *extrinsic* information transfer (PEXIT) algorithm) and coding schemes tailored for channels with symmetric Gaussian characteristics cannot be directly applied to OAM-IM systems with inter-mode interference.

[0077] From a mathematical perspective, given the original data has a non-Gaussian distribution (especially skewed distribution), a parametric transformation technique, named Box-Cox transformation (BCT) may be utilized to convert this data into a new data set (i.e., transformed data) that is able to approximate a Gaussian distribution. The BCT may be defined as:

$$\varsigma(\lambda_t) = \begin{cases} \frac{\varsigma^{\lambda_t-1}}{\lambda_t}, & \lambda_t \neq 0, \\ \log \varsigma, & \lambda_t = 0, \end{cases} \quad \dots(15)$$

where $\varsigma > 0$ denotes the original data, and λ_t is the transformation parameter which is used to make the transformed data $\varsigma(\lambda_t)$ approximate a Gaussian distribution.

[0078] However, in certain scenarios, some LLR values l_i (i.e., original data) obtained from the channel detector may comprise a negative value, especially for the set of coded bits $b_i = 1$ (see the scattered points in Figures 6a and 6b, and Figures 7a and 7b). As a result, the BCT technique as disclosed in equation 15 may not be used. Instead, a variant referred to as the Customized Box-Cox transformation (CBCT) may be utilized and the CBCT is defined as:

$$\begin{cases} l^{CBCT} = \pm \frac{|l|^{\lambda_t-1}}{\lambda_t}, & \lambda_t \neq 0, \\ \text{subject to: } l^{CBCT} \times l > 0, l^{CBCT} \neq 0 \text{ \& } l \neq 0 \end{cases} \quad \dots(16)$$

where $|\cdot|$ denotes an absolute value operator. The transformed data can then be attained through the use of equation (16), whereby histograms associated with these transformed data may approximate symmetric Gaussian distributions. To verify this, the histograms of transformed *extrinsic* LLR values when $b_i = 0$ with $\lambda_t = 5/8$ and $b_i = 1$ with $\lambda_t = 3/8$ are plotted in Figure 6b (i.e., the dashed line and the line with circle in Figure 6b). Based on these plots, it can be seen that histograms associated with these transformed *extrinsic* LLR values are able to match with the symmetric Gaussian distributions having the values of $u_0 = 1.5$ and $u_1 = -1.5$. A similar occurrence may be observed in Figure 7b, where the histograms of transformed *extrinsic* LLR values are attained using $\lambda_t = 3/8$ for both $b_i = 0$ and 1, and the mean values of the corresponding symmetric Gaussian distributions are $u_0 = 1.5$ and $u_1 = -1.5$.

[0079] In order to determine the optimal λ_t to be used in the CBCT procedure, a Kullback-Leibler divergence (KLD) analysis is utilized together with a wave-crest-based constraint.

When these two approaches are combined, this ensures that a small KLD (i.e., $d_{kl} < 0.1$) exists between the transformed distribution and its corresponding Gaussian distribution while ensuring that such two distributions have similar wave crests. For other angles $\theta \neq 0$ (e.g., $\theta = 1.0$), the original LLRs obtained from the channel detector of OAM-IM systems may also follow a skewed distribution. As such, the CBCT technique may be utilized to attain the corresponding transformed LLRs, whose histograms approximate a symmetric Gaussian distribution. It is useful to note that the plots illustrated in Figures 6 and 7 may be made smoother by increasing the number of samples (i.e., the number of frames).

[0080] As observed from Figures 6b and 7b, the proposed CBCT technique primarily shapes the long left/right tails of the skewed distributions, which make them approach the symmetric Gaussian distributions. In particular, as can be seen from equation (16), both the original and transformed LLRs have the same positive/negative sign. This implies that when a coded bit has a high probability of being decoded as 0/1 in the original model (i.e., the original LLR has a positive/negative sign), it is also likely to be decoded as 0/1 in the transformation model (i.e., a CBCT-aided OAM-IM system over an LoS channel that utilizes the CBCT technique) and vice versa. Therefore, the transformation model does not deteriorate any accuracy of the BP decoding, but only simplifies the EXIT analysis and code design. For the above reason, the transformation model is able to capture the performance of the PLDPC codes in the original model. Furthermore, through the use of the CBCT, a feasible low-complexity theoretical-analysis tool may be devised to guide the design and optimization of PLDPC codes and index schemes for improving the performance of OAM-IM systems over LoS channels. However, the CBCT itself cannot directly improve the performance of the system since it is not used in the practical data transmission process.

[0081] **Protograph-Based Low-Density Parity-Check (PLDPC) Codes**

[0082] PLDPC codes comprise codes which are generated by expanding a small Tanner graph, referred to as protograph. Specifically, a protograph can be represented by $\mathcal{G}_p = (\mathcal{V}_p, \mathcal{C}_p, \mathcal{E}_p)$, where $\mathcal{V}_p = \{v_1, v_2, \dots, v_{n_v}\}$ is a variable-node (VN) set, $\mathcal{C}_p = \{c_1, c_2, \dots, c_{n_c}\}$ is a check-node (CN) set, $\mathcal{E}_p = \{e_{i,j}\}$ is an edge set and $e_{i,j}$ denotes the edge connecting the i -th CN c_i ($i = 1, 2, \dots, n_c$) to the j -th VN v_j ($j = 1, 2, \dots, n_v$). The code rate of a protograph is $R_p = (n_v - n_c)/n_v$. A protograph can be expressed as a base matrix $\mathbf{B}_p = (b_{i,j})$, whose rows

and columns correspond to the CNs and VNs, respectively, where $b_{i,j}$ denotes the number of edges connecting c_i to v_j . Given a protograph (i.e., base matrix), a lifting operation may be performed on it and the corresponding derived graph (i.e., parity-check matrix) may be attained, which in turn may be used to generate the PLDPC code. In particular, unlike the conventional LDPC codes, multiple edges connecting a CN and a VN (i.e., parallel edges) are permitted in a protograph, but they must be eliminated in a lifting process. A length- n_e PLDPC code can be generated from a protograph with n_v VNs, where $n_e = n_v \times f$ and f is a lifting factor. In embodiments of this disclosure, the lifting operation is implemented using a modified progressive-edge-growth (PEG) algorithm.

[0083] **A CBCT-PEXIT Algorithm**

[0084] A Protograph Extrinsic Information Transfer (PEXIT) algorithm is an extension of the Extrinsic Information Transfer (EXIT) analysis used specifically for protograph-based Low-Density Parity-Check (LDPC) codes. Protograph-based LDPC codes are a class of structured LDPC codes that are designed based on a small bipartite graph, called a protograph, which is then lifted to create a larger LDPC code. The PEXIT algorithm is an asymptotic-performance analysis tool, which can trace the mutual information (MI) evolution of a PLDPC code to predict its decoding threshold. The decoding threshold is regarded as the minimum SNR that allows the PLDPC code to achieve error-free transmission, and thus can be used to evaluate the convergence performance of PLDPC-coded OAMIM systems. For the PEXIT algorithm, the MI evolution process includes two different parts. The first part is the calculation of MI output from the channel detector, while the second part is the MI update between the VN decoder and CN decoder. Note that the computational complexity of the PEXIT algorithm is mainly dominated by the first part.

[0085] To be specific, for a given channel environment, the *extrinsic* MI I_E^{Det} obtained from the detector can be calculated as:

$$I_E^{Det} = \frac{1}{2} \sum_{\mu \in \{+1, -1\}} \int_{-\infty}^{\infty} f(l|x = \mu) \times \log_2 \frac{2^{f(l|x=\mu)}}{f(l|x=+1) + f(l|x=-1)} dl \quad \dots (17)$$

where $\mu = (-1)^b \in \{+1, -1\}$ is a converted binary symbol and $b \in \{0, 1\}$ is a coded bit. Under the assumption that the conditional PDF $f(l|x = \mu)$ in equation (17) is approximated by a symmetric Gaussian distribution (i.e., $\mathcal{N}(\sigma_n^2/2, \sigma_n^2)$), the conditional PDF may be defined as:

$$f(l|x = \mu) = \frac{1}{\sqrt{2\pi}\sigma_n} \times \exp\left(-\frac{(l - \sigma_n^2\mu/2)^2}{2\sigma_n^2}\right) \quad \dots(18)$$

As a result, equation (17) may be simplified as:

$$\begin{aligned} I_E^{Det} &= 1 - \int_{-\infty}^{\infty} f(l|x = \mu) \times \log_2(1 + \exp(-\mu l)) dl \\ &= 1 - \mathbb{E}[\log_2(1 + \exp(-\mu l))] \\ &\approx 1 - \frac{1}{n_e} \sum_{k=1}^{n_e} \log_2(1 + \exp(-\mu_k l_k)) \end{aligned} \quad \dots(19)$$

where $\mathbb{E}[\cdot]$ is the expectation function and l is the LLR corresponding to μ .

[0086] For existing OAM-IM systems (i.e., the original model), equation (17) has to be utilized to calculate the *extrinsic* MI I_E^{Det} as the original extrinsic LLRs tend to follow a skewed distribution. Based on equation (17), the discrete numerical values have to be computed, the corresponding conditional PDFs must be estimated, while this involves an integration operation. In particular, for a non-Gaussian distribution (e.g., a skewed distribution), it is challenging to obtain a close-form solution for the integration, as such the numerical method (i.e., histogram) is usually adopted to calculate the *extrinsic* MI I_E^{Det} and this leads to a highly complex calculation.

[0087] This issue may be addressed through the use of the CBCT, in which the transformed extrinsic LLRs have a close correlation with a symmetric Gaussian distribution. A transformed *extrinsic* MI I_E^{CBCT} may then be obtained for the CBCT-aided OAM-IM systems (i.e., the transformation model) based on equation (19), which involves much simpler steps as compared to equation (17) as it only involves discrete numerical calculations and an expectation operation. In view of the above, a CBCT-PEXIT algorithm is defined below.

[0088] Figure 8 illustrates a principle of this CBCT-PEXIT algorithm, in which the mutual information (MI) is defined as follows.

- $I_E^{CBCT}(j)$ represents the transformed extrinsic MI of v_j as obtained from the MI calculator;
- $I_{Av}^{Dec}(i, j)$ and $I_{Ac}^{Dec}(i, j)$ indicate the *a-priori* MIs sent from c_i to v_j and v_j to c_i respectively;
- $I_{Ev}^{Dec}(i, j)$ and $I_{Ec}^{Dec}(i, j)$ indicate the extrinsic MIs sent from v_j to c_i and c_i to v_j respectively;
- $I_{AP}^{Dec}(j)$ indicates the a-posteriori MI of v_j .

[0089] In particular, $I_{Av}^{Dec}(i, j)$ and $I_{Ac}^{Dec}(i, j)$ are equal to $I_{Ec}^{Dec}(i, j)$ and $I_{Ev}^{Dec}(i, j)$, respectively, in each inner iteration between the VN decoder and the CN decoder. T_m is set as the maximum number of inner iterations and the CBCT-PEXIT algorithm is defined as follows.

[0090] During an initialization step, an initial SNR is set, and a Monte Carlo simulation is implemented. The receiving UCA is then configured to obtain a sequence of channel outputs 801 $\{y_1, y_2, \dots, y_{(n_e/\eta)}\}$. This sequence 801 will be demultiplexed to be $\{\tilde{y}_1, \tilde{y}_2, \dots, \tilde{y}_{(n_e/\eta)}\}$ and then processed by channel detector 802.

[0091] During the step of computing the transformed *extrinsic* MIs output from MI calculator 806, channel detector 802 outputs a sequence of extrinsic LLRs $\{l_1, l_1, \dots, l_{n_e}\}$ to CBCT module 804. Through the use of equation (16), these extrinsic LLRs are converted to a new sequence $\{l_1^{CBCT}, l_2^{CBCT}, \dots, l_{n_e}^{CBCT}\}$. Based on the transformed extrinsic LLRs and their corresponding coded bits, the transformed *extrinsic* MI of v_j $I_E^{CBCT}(j)$ where $(j = 1, 2, \dots, n_v)$ is calculated as:

$$I_E^{CBCT}(j) = 1 - \frac{1}{f} \sum_k \log_2(1 + \exp(-\mu_k l_k^{CBCT})), \quad \dots (20)$$

$$k = \{(j-1)f + 1, (j-1)f + 2, \dots, (j-1)f + f\}$$

where $\mu_k = (-1)^{b_k} \in \{+1, -1\}$ and $b_k \in \{0, 1\}$ are the k -th converted binary symbol and coded bit, respectively. Subsequently, $I_E^{CBCT}(j)$ is sent to VN decoder 808 and used as initial MI (i.e., $I_{init}(j) = I_E^{CBCT}(j)$). When the j -th VN v_j is punctured in a protograph, $I_{init}(j) = 0$. In embodiments of the disclosure, MI calculator 806 may comprise a module that is configured to calculate the mutual information between the input and output of a channel in order to assess the channel capacity and the efficiency of the channel's transmission. MI calculators typically

apply a formula for mutual information, as known to one skilled in the art, to probability distributions of the random variables and then compute the entropy of the individual variables. The output produced by the MI calculator comprises the mutual information value, which is a non-negative number measured in bits (for binary variables).

[0092] During the step of determining the extrinsic MIs obtained from VN decoder 808, i.e., the initial MIs $I_{init}(j)$ and the *a-priori* MIs $I_{Av}^{Dec}(i, j)$ are exploited. The extrinsic MIs $I_{Ev}^{Dec}(i, j)$ are updated and considered as $I_{Ac}^{Dec}(i, j)$ if $b_{i,j} \neq 0$. The detailed process can be found in the publication by Z. Yang *et. al.* titled “Analysis and optimization of tail-biting spatially coupled protograph LDPC codes for BICM-ID systems,” in the *IEEE Trans. Veh. Technol.*, vol. 69, no. 1, at pp. 390–404, Jan. 2020; and in the publication by G. Liva and M. Chiani titled “Protograph LDPC codes design based on EXIT analysis,” in the *Proc. IEEE Glob. Commun. Conf. (GLOBECOM)*, Nov. 2007, at pp. 3250–3254.

[0093] During the step of determining the extrinsic MIs output from CN decoder 810, given the *a-priori* MIs $I_{Ac}^{Dec}(i, j)$, the *extrinsic* MIs $I_{Ec}^{Dec}(i, j)$ can be updated if $b_{i,j} \neq 0$ and considered as $I_{Av}^{Dec}(i, j)$ in next inner iteration. The detailed process of this step is also set out in the two publications mentioned above.

[0094] During the step of computing the *a-posteriori* MIs for VNs, based on the initial MIs $I_{init}(j)$ and the *a-priori* MIs $I_{Av}^{Dec}(i, j)$, the *a-posteriori* MIs $I_{AP}^{Dec}(j)$ (where $j = 1, 2, \dots, n_v$) can be computed. More detailed calculations concerning $I_{AP}^{Dec}(j)$ can be found in the two publications mentioned above.

[0095] At the final step, the CBCT-PEXIT algorithm is terminated, and the current SNR is used as a decoding threshold if $I_{AP}^{Dec}(j) = 1$ for all VNs. Otherwise, i) this algorithm will restart with a larger SNR if T_{in} is reached; ii) return to Step 3) and continue to trace MI convergence if T_{in} is not reached.

[0096] A traditional EXIT chart is not applicable to PLDPC codes because the protographs allow the presence of degree-1 VNs and punctured VNs. Also, different protographs may have the same degree distribution. In the traditional PEXIT algorithm, the initial MI calculation is based on the assumptions of infinite-length codeword, binary modulation, and an AWGN

channel. For the OAM-IM systems with finite-length codeword, high-order modulation, and non-Gaussian channel, the initial MI calculation (i.e., equation (20)) is different from that in the PEXIT algorithm.

[0097] In summary, for the CBCT-PEXIT algorithm, the CBCT is an essential link between the channel detector and VN decoder, which allows equation (19) to be used when the transformed extrinsic MIs for OAM-IM systems with inter-mode interference are calculated. The analysis based on the Kullback-Leibler Divergence (KLD) has revealed that a short codeword length does not provide sufficient extrinsic LLRs for accurate approximation. Consequently, the conditional histogram of the transformed extrinsic LLRs obtained through CBCT cannot accurately approximate a symmetric Gaussian distribution. Additionally, the standard PEXIT analysis assumes an infinite-length codeword to calculate the decoding threshold. Therefore, to make an accurate prediction of the convergence performance limit of the PLDPC code in this algorithm, it is recommended to set a large codeword length (e.g., $n_e = 9000$). While the transformed extrinsic LLRs exhibit symmetric and Gaussian characteristics, it is important to note that they are the output of a transformation model (i.e., the CBCT-aided OAM-IM system with inter-mode interference) rather than Additive White Gaussian Noise (AWGN) channels.

[0098] **Proposed PLDPC Codes :- IUP PLDPC / IP1 PLDPC / IP2 PLDPC codes**

[0099] A PLDPC code that performs well in a specific transmission environment may not necessarily be a promising candidate for other transmission environments due to the difference in channel characteristics. Moreover, even if the channel's characteristics are given, a PLDPC code is usually unable to achieve desirable performance in both low and high-SNR regions simultaneously. For AWGN channels, the accumulate-repeat-by-4-jagged-accumulate (AR4JA) code is viewed as a competitive PLDPC code since it can achieve a good performance tradeoff between the waterfall and error-floor regions.

[00100] Figure 9 illustrates protographs for a family of AR4JA codes with a wide range of rates from $1/2$ to $(e+1)/(e+2)$, where e denotes an arbitrarily large positive value; the dark circles, blank circle, and circles with a plus sign represent the transmitted VNs, punctured VN, and CNs, respectively. The rate $\frac{e+1}{e+2}$ AR4JA code is constructed by repeatedly adding $2e$

degree-4 VNs that connect the two highest-degree CNs into a rate-1/2 AR4JA code. In OAM-IM systems over LoS channels, the transmitted signals will suffer from inter-mode interference and their extrinsic LLRs output from channel detector may follow a non-Gaussian distribution. Consequently, the PLDPC-code design for OAM-IM systems has to be addressed.

[00101] In the following section, a CBCT-PEXIT-aided computer-search method is employed to construct two types of rate-1/2 improved PLDPC codes tailored for the proposed transformation model. These two types of rate-1/2 improved PLDPC codes are also the promising alternatives for use in original OAM-IM systems (i.e., without using CBCT).

[00102] Firstly, a small-size (i.e., $n_c \times n_v$) all-zero-base matrix $\mathbf{B}_0$ is defined. The search space then equals $(1 + b_{max})^{n_c \times n_v} - 1$ if an exhaustive computer-search method is used to find the resultant base matrix $\mathbf{B}_{reul}$ that corresponds to a PLDPC code having an optimal decoding threshold. In particular, b_{max} is the maximum integer element in $\mathbf{B}_{reul}$. With an aim of reducing the search space and lowering the encoding/decoding complexity, it is assumed that b_{ij} within $\mathbf{B}_{reul}$ satisfies the condition that $0 \leq b_{ij} \leq 3$ ($i \in \{1, 2, \dots, n_c\}$ and $j \in \{1, 2, \dots, n_v\}$). Meanwhile, two fundamental characteristics of a good PLDPC code should be taken into account which is that: (1) an appropriate proportion of degree-1 and degree-2 VNs should be added into the protograph to ensure that the resultant PLDPC code attains a relatively low decoding threshold; and (2) a CN should be used to connect a degree-1 unpunctured VN and a highest-degree punctured VN to formulate a precoding structure that can further improve the threshold performance for a punctured PLDPC code.

[00103] Based on the above, the design for two different types of improved PLDPC codes are described below when simple constraints are imposed on $\mathbf{B}_0$ to formulate an initial base matrix $\mathbf{B}_{init}$ with the same size, and when the corresponding $\mathbf{B}_{reul}$ is attained with the optimal decoding threshold using the CBCT-PEXIT-aided computer-search method depicted in Algorithm 3 below.

Algorithm 3 CBCT-PEXIT-aided computer-search method

Input: $\mathbf{B}_{init}$, constraints related to $\mathbf{B}_{reul}$, and a pre-set decoding threshold (e.g., $T_{pre} = 100$);

Output: $\mathbf{B}_{reul}$;

1: **for** $i = 1, 2, \dots, n$ **do**

```

2:   Generate a tentative base matrix  $\mathbf{B}_{\text{tent}}$  based on  $\mathbf{B}_{\text{init}}$  via a
   constraint-based enumeration;
3:   Calculate the decoding threshold of  $\mathbf{B}_{\text{tent}}$  (i.e.,  $\tau_{\text{tent}}$ ) using
the CBCT-PEXIT algorithm;
4:   if  $\tau_{\text{tent}} < \tau_{\text{pre}}$  then
5:      $\tau_{\text{pre}} \leftarrow \tau_{\text{tent}}$ , and  $\mathbf{B}_{\text{reul}} \leftarrow \mathbf{B}_{\text{tent}}$ ;
6:   end if
7: end for

```

[00104] **Improved unpunctured PLDPC codes:**

[00105] For a base matrix, the column swapping operation does not change its decoding threshold. Under the assumption that an initial 4×8 base matrix $\mathbf{B}_0$ with a degree-1 VN and a degree-2 VN is given, the corresponding $\mathbf{B}_{\text{init}}$ may then be defined as:

$$\mathbf{B}_{\text{init}} = \begin{bmatrix} 1 & 0 & b_{1,3} & b_{1,4} & b_{1,5} & b_{1,6} & b_{1,7} & b_{1,8} \\ 0 & 0 & b_{2,3} & b_{2,4} & b_{2,5} & b_{2,6} & b_{2,7} & b_{2,8} \\ 0 & 1 & b_{3,3} & b_{3,4} & b_{3,5} & b_{3,6} & b_{3,7} & b_{3,8} \\ 0 & 1 & b_{4,3} & b_{4,4} & b_{4,5} & b_{4,6} & b_{4,7} & b_{4,8} \end{bmatrix} \quad \dots(21)$$

[00106] When three different constrains (i.e., the CN-degree, VN-degree, and element constrains) are considered: i) $\sum_{j=1}^8 b_{i,j} \leq 8$ ($i = 1, 2, 3, 4$); ii) $\sum_{i=1}^4 b_{i,j} \leq 3$ ($j = 3, 4, 5, 6$); iii) $0 \leq b_{i,j} \leq 3$ for $i = 1, 2, 3, 4$ and $j = 3, 4, \dots, 8$, with the aid of a computer-search-based procedure, a rate-1/2 improved unpunctured PLDPC code, called IUP PLDPC code may be obtained and its base matrix $\mathbf{B}_{\text{reul}}^{\text{IUP}}$ may be defined by:

$$\mathbf{B}_{\text{reul}}^{\text{IUP}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 3 & 3 \\ 0 & 1 & 2 & 2 & 2 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 & 2 & 1 & 2 \end{bmatrix} \quad \dots(22)$$

[00107] **Improved punctured PLDPC codes:**

[00108] When a 3×5 base matrix $\mathbf{B}_0$ is initialized and added with a degree-1 VN and a degree-2 VN, and under the assumption that the second column of the initial base matrix $\mathbf{B}_{\text{init}}$ corresponds to a punctured VN, the corresponding $\mathbf{B}_{\text{init}}$ may be defined as:

$$\mathbf{B}_{\text{init}} = \begin{bmatrix} 1 & b_{1,2} & b_{1,3} & b_{1,4} & 0 \\ 0 & b_{2,2} & b_{2,3} & b_{2,4} & 1 \\ 0 & b_{3,2} & b_{3,3} & b_{3,4} & 1 \end{bmatrix} \quad \dots(23)$$

[00109] When three different constrains (i.e., the CN-degree, VN-degree, and element constrains) are considered: i) $\sum_{j=1}^5 b_{i,j} \leq 4$ and $\sum_{j=1}^5 b_{i,j} \leq 6$ ($i = 2, 3$); ii) $\sum_{i=1}^3 b_{i,2} > \sum_{i=1}^3 b_{i,j}$ ($j = 3, 4$); iii) $0 \leq b_{i,j} \leq 3$ for $i = 1, 2, 3$ and $j = 2, 3, 4$. By further adjusting the number of degree-2 VNs (i.e., $\sum_{i=1}^3 b_{i,4} > 2$ or $= 2$) and using the computer-search method, two different rate-1/2 improved punctured PLDPC codes denoted as IP1 and IP2 PLDPC codes are obtained. Their base matrices (i.e., $\mathbf{B}_{\text{reul}}^{\text{IP1}}$ and $\mathbf{B}_{\text{reul}}^{\text{IP2}}$) are given by:

$$\mathbf{B}_{\text{reul}}^{\text{IP1}} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 & 1 \end{bmatrix}, \quad \mathbf{B}_{\text{reul}}^{\text{IP2}} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 & 1 \end{bmatrix} \quad \dots(24)$$

[00110] In particular, the IP1 PLDPC code was found to achieve a desirable tradeoff between the waterfall and error-floor performance; while the IP2 PLDPC code was found to have the optimal waterfall performance but suffers from error-floor behavior in high-SNR region. For most wireless transmission environments, high-rate PLDPC codes are preferred over low-rate counterparts, which inspires the use of a simple rate-extension scheme to construct high-rate IP1 and IP2 PLDPC codes. As it has been determined that when VNs with degrees no less than 3 are appended to a protograph, it results in PLDPC code that is able to achieve good waterfall performance without deterioration in the error-floor performance, the same VN pattern composed of two different degree-3 VNs is thus repeatedly added into the protographs of rate-1/2 IP1 and IP2 PLDPC codes. Under the assumption that the number of newly added VN patterns is $e \geq 1$, the code rate of an extended IP1/IP2 PLDPC code is $R_p = \frac{e+1}{e+2}$ where $e = 1, 2, \dots$. The protographs corresponding to such two families of improved punctured PLDPC codes are presented in Figures 10a in relation to the family of IP1 PLDPC codes and Figure 10b in relation to the family of IP2 PLDPC codes, while their base matrices are expressed as:

$$\mathbf{B}_{\text{real}}^{\text{IP1}} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & \overbrace{0 & 0 & \dots & 0 & 0}^{2e} \\ 0 & 2 & 0 & 1 & 1 & 1 & 2 & \dots & 1 & 2 \\ 0 & 1 & 2 & 2 & 1 & 2 & 1 & \dots & 2 & 1 \end{bmatrix} \quad \dots(25)$$

$$\mathbf{B}_{\text{real}}^{\text{IP2}} = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 & \overbrace{0 & 0 & \dots & 0 & 0}^{2e} \\ 0 & 2 & 0 & 1 & 1 & 1 & 2 & \dots & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 & 2 & 1 & \dots & 2 & 1 \end{bmatrix} \quad \dots(26)$$

[00111] It is noted that better high-rate PLDPC codes may be constructed if the CBCT-PEXIT-aided computer-search method is used to search the large-size base matrices without considering any constraints. However, to reduce the search space (i.e., lower the PLDPC-code construction complexity), a rate-1/2 punctured PLDPC code is first constructed with a small size base matrix based on some given constraints, and then the same VN pattern is repeatedly introduced into the protograph of this code to formulate the high-rate counterparts.

[00112] Also, as OAM systems are mostly used for LoS backhaul links between base stations, the misaligned condition usually remains constant for a long time. As a result, the channel coefficient, which is closely related to the OAM system setting, also remains constant during each transmission period. The OAM-based LoS channel then can be viewed as a wireless channel with inter-mode interference. Hence, the natural step would be to design PLDPC codes for such a particular channel configuration since the codes designed for conventional fading MIMO channels may not be able to preserve their performance advantage for OAM applications.

[00113] Mutual Information (MI)-based Index Scheme

[00114] For an OAM-IM system with N array elements in both transmitting and receiving UCAs and N_A active modes, the number of effective OAM mode combinations, B_e , may be defined as $B_e = 2^{\lfloor \log_2 \binom{N}{N_A} \rfloor}$. For example, when N and N_A are respectively set to be 4 and 2, the number of effective OAM mode combinations, B_e equals 4. Channel gains of a 4×4 OAM-IM system without inter-mode interference (i.e., $\theta = 0$) are illustrated in Figure 11a; and with inter-mode interference (i.e., $\theta = 0.8$) are illustrated in Figure 11b. As shown, it can be seen

that in the case with inter-mode interference (see Figure 11b), the signals emitted by different modes will suffer from different channel gains. In the following section, a method for determining an optimal number of effective OAM mode combinations, B_e will be disclosed.

[00115] From the perspective of Monte-Carlo simulations, an $N \times N$ OAM-IM system can be viewed as N different $1 \times N$ OAM-IM systems. Moreover, the CBCT-PEXIT algorithm described above can be used to evaluate the reliability of signals transmitted by N different OAM modes. Specifically, for a given SNR, a sufficiently large, modulated symbol sequence is first generated, and then divided into N subsequences in a sequential order. The modulated symbols within the k -th sub-sequence will be transmitted by the k -th mode, where $k = 1, 2, \dots, N$. Subsequently, the previously described CBCT-PEXIT algorithm is utilized to calculate N different extrinsic MIs as obtained from the MI calculator. By repeatedly executing the above procedure for different SNRs, a N number of different extrinsic MI convergence curves corresponding to N different modes may be obtained. Based on the extrinsic MI analysis, B_e mode combinations having the largest MIs may then be determined, and these mode combinations may then be arranged in a descending order such that the reliability of the k -th mode combination is higher than that of the $(k - 1)$ -th mode combination, where $k = 1, 2, \dots, B_e$.

[00116] As an example, the extrinsic MI curves in a 4×4 OAM-IM system when $\theta = 0.8$, and where the regular-(3, 6) PLDPC code and Gray-labeled QPSK modulation are used is illustrated in Figure 12. In this figure, the extrinsic MIs obtained from the detector using CBCT are analyzed, and it can be seen that these values correspond to coded bits transmitted by 4 different modes (see dashed lines). The extrinsic MIs obtained from the detector using conventional PEXIT algorithms (see solid lines) are also analyzed, and it can be seen that their trends are consistent with the dashed lines. This implies that if a code is able to achieve a good convergence performance in the transformation model, it can also achieve a good convergence performance in the original model. In this embodiment of the disclosure, r_k is defined as the reliability of the k -th mode. Hence, the greater the extrinsic MI of coded bits transmitted by the k -th mode, the higher the corresponding r_k will be. As shown in Figure 12, 4 different modes satisfy this condition, i.e., $r_4 > r_2 > r_1 > r_3$. Under the assumption that the number of active modes is $N_A = 2$, this implies that there will be 4 different mode combinations, i.e., $B_e = 4$, with largest MIs, and this hereinafter is defined as an MI-based index scheme. Table 5 shows the

MI-based index scheme, the natural index scheme, and the mapping relationship between them and coded bits.

Coded Bits (Index Symbols)		Natural Index					MI-based Index				
b_1	b_2	Φ :	m_1	m_2	m_3	m_4	Φ :	m_1	m_2	m_3	m_4
0	0	ϕ_1 :	✓	✓			ϕ_1 :		✓		✓
0	1	ϕ_2 :	✓		✓		ϕ_2 :	✓			✓
1	0	ϕ_3 :	✓			✓	ϕ_3 :			✓	✓
1	1	ϕ_4 :		✓	✓		ϕ_4 :	✓	✓		

TABLE 5

[00117] To verify the superiority of the mode-combination selection over the natural index scheme, the constellation-constrained capacities are computed for the natural index and MI-based index schemes and the results are plotted in Figure 13. For both QPSK and 8PSK modulations, it can be seen that the MI-based index scheme has larger capacities than the natural index scheme over a wide range of SNRs (i.e., 65 ~ 85 dB). In particular, it should be noted that the performance of the MI-based index scheme may be further improved using a shaped Index Modulation scheme (as previously described in Embodiment 2 of the transmitter).

[00118] Decoding-Threshold Analysis

[00119] In order to demonstrate the superiorities of the improved PLDPC codes (i.e., IUP, IP1, and IP2 PLDPC codes) and the shaped index scheme, the convergence performance of these codes and scheme are assessed for 4×4 OAM-IM systems with $\theta = 0.8$ over LoS channels. Additionally, two different PLDPC codes (i.e., regular-(3, 6) and AR4JA PLDPC codes) and two different index schemes (i.e., natural and ED-based index schemes) are considered as benchmarks for the proposed code construction and shaping index design, respectively.

[00120] Table 6 below illustrates the decoding thresholds of the above five PLDPC codes with QPSK modulation and the natural index are calculated by exploiting the above described CBCT-PEXIT algorithm.

Parameter	$N_A = 1$					$N_A = 2$				
	Unpunctured Codes		Punctured Codes			Unpunctured Codes		Punctured Codes		
	regular-(3,6)	IUP	AR4JA	IP1	IP2	regular-(3,6)	IUP	AR4JA	IP1	IP2
Natural Index	79.82	79.35	79.51	79.37	79.23	81.86	81.35	81.57	81.38	81.29

TABLE 6

[00121] When $N_A = 1$, the proposed IUP PLDPC code achieves a smaller decoding threshold over regular-(3, 6) PLDPC code. The proposed IP1 and IP2 PLDPC codes also exhibit smaller decoding thresholds over AR4JA PLDPC code. Furthermore, a similar phenomenon can be observed in the case of $N_A = 2$. This observation demonstrates that the proposed improved PLDPC codes are promising alternatives for use in OAM-IM systems.

[00122] The decoding thresholds of these five PLDPC codes in the case of using original LLRs (i.e., without CBCT) are analyzed in Table 7 below.

Modulation	QPSK					8PSK				
	Unpunctured Codes		Punctured Codes			Unpunctured Codes		Punctured Codes		
	regular-(3,6)	IUP	AR4JA	IP1	IP2	regular-(3,6)	IUP	AR4JA	IP1	IP2
Natural Index	78.77	78.38	78.57	78.41	78.19	78.82	78.47	78.67	78.52	78.33

TABLE 7

[00123] When $N_A = 1$, one may notice that the proposed IUP PLDPC code and IP1/IP2 PLDPC code significantly outperforms the regular-(3, 6) PLDPC code and AR4JA PLDPC code for both QPSK and 8PSK modulations, respectively. The convergence performance trends of these PLDPC codes are also consistent with those in Table 6 above, which further implies the effectiveness of the CBCT-PEXIT algorithm.

[00124] In Table 8, the decoding thresholds of these PLDPC codes are compared with three different index schemes for $N_A = 2$.

Modulation	QPSK					8PSK				
	Unpunctured Codes		Punctured Codes			Unpunctured Codes		Punctured Codes		
	regular-(3,6)	IUP	AR4JA	IP1	IP2	regular-(3,6)	IUP	AR4JA	IP1	IP2
Natural Index	80.70	80.30	80.47	80.33	80.11	81.23	80.71	81.02	80.75	80.54
ED-based Index	79.10	78.52	78.77	78.55	78.31	79.47	78.77	79.11	78.89	78.68
Shaped Index	78.51	77.96	78.25	77.90	77.83	79.13	78.53	78.76	78.45	78.42

TABLE 8

[00125] Specifically, for a given QPSK/8PSK modulation, the PLDPC codes with the proposed shaped index scheme are capable of achieving better decoding thresholds than those with existing index schemes (i.e., non-shaped index schemes). Therefore, the performance of PLDPC-coded OAM-IM systems can be significantly enhanced with the use of shaping index described in the previous sections.

[00126] **Simulation Results**

[00127] In these simulations, the simulated BERs of two known LDPC codes (i.e., Code I and Code II) designed for MIMO channels, the regular-(3, 6), AR4JA, and the IUP, IP1, and IP2 PLDPC codes in 4×4 OAM systems over LoS channels are compared. In particular, QPSK and 8PSK modulations as well as three different index schemes (i.e., natural index, ED-based index, and shaped index schemes) are considered. Unless otherwise stated, it is assumed that the parameters are $\theta = 0.8$, and $N_A = 1$ and 2. Furthermore, all PLDPC codes are configured to have the same code rate $R_p = 1/2$, a transmitted codeword length $n'_e = 2400$ and the maximum number of BP decoding iterations is set to be equal to 100.

[00128] *A. PLDPC Codes with $R_p = 1/2$ (Natural Index Scheme)*

[00129] With reference to Figure 14a, when $N_A = 1$ and QPSK modulation are considered, it is shown from the plots that the IUP, IP1, and IP2 PLDPC codes (as described in the sections above) significantly outperform the other four benchmark codes. Specifically, at a BER of 10^{-5} , the proposed IUP PLDPC code achieves performance gains around 0.3, 0.3, and 0.1 dB over the regular-(3, 6) PLDPC code, Code I, and Code II, respectively. Also, the proposed IP1 and IP2 PLDPC codes achieve performance gains around 0.2 and 0.25 dB over the AR4JA PLDPC code, respectively.

[00130] A similar observation can be seen in Figure 14b, when $N_A = 1$ and 8PSK modulation are used, the proposed IP2 PLDPC code exhibits the best waterfall performance among these seven different codes but suffers from an error-floor behavior in the high-SNR region. This is because the IP2 PLDPC code includes a high proportion of degree-2 VNs (i.e., 40% VNs in each codeword), which forms a girth-4 cycle (see equation (24)). By increasing additional VNs with degree at least 3 in the protograph of the IP2 PLDPC code, its error-floor performance can be improved significantly. It is useful to note that the large value of E_b/N_0 in simulations is caused by the system parameter settings (i.e., $P_t = 1$ W and $\beta = 1$), which results in a small channel gain. For practical OAM applications, there exist some OAM antennas which can generate a high-gain OAM vortex beam (e.g., the antenna gain $\beta > 100$), and then one only needs a relatively small SNR to lower the BER.

[00131] As shown in 15a and 15b, when $N_A = 2$ is considered, it can be observed that the performance of the IUP, IP1, and IP2 PLDPC codes are still better than that of other four benchmark codes for both QPSK (see Figure 15a) and 8PSK (see Figure 15b) modulations. Thus, it may be concluded that for different modulations and numbers of active OAM modes (i.e., N_A), the proposed improved PLDPC codes are promising alternatives for use in OAM-IM systems over LoS channels.

[00132] It is useful to note at this stage that both the IUP and IP1 PLDPC codes have a low proportion of degree-2 VNs and no girth-4 cycle. For this reason, these codes are able to avoid the error floor and achieve better performance in the high SNR region with respect to the IP2 PLDPC code.

[00133] With reference to equations (22) and (24), it can be observed that the base matrix of the IUP PLDPC code has (i) higher edge density (i.e., the average degree for one VN is 3.625) and (ii) higher proportion of parallel edges (i.e., the average number of parallel edges for one VN is 1.125) than the base matrix of the IP1 PLDPC code, whose average degree and average number of parallel edges for one VN are 2.8 and 0.8, respectively. Then, the IUP PLDPC code is more likely to have short cycles in the resultant parity-check matrix and thus perform worse in the high SNR region, compared with the IP1 PLDPC code.

[00134] Figure 16 further compares the performance of the 5G LDPC code, three irregular LDPC codes (i.e., irregular LDPC codes with maximum VN degrees $d_v = 4, 5$, and 50), (3, 6) spatially coupled (SC)-LDPC code, and the proposed improved PLDPC codes. It is obvious that the proposed improved PLDPC codes are the best-performing codes, especially in the high-SNR region. The proposed IUP, IP1, and IP2 PLDPC codes exhibit gaps of about 0.59, 0.62, and 0.40 dB, respectively, to the Shannon limit (i.e., 79.71 dB). The gaps of the three proposed PLDPC codes to the Shannon limit are very desirable for the code design over OAM-based LoS channels.

[00135] *B. PLDPC Codes with $R_p > 1/2$ (Natural Index Scheme)*

[00136] Figure 17 illustrates the BER results of the AR4JA, IP1, and IP2 PLDPC codes with code rates $R_p = 2/3, 3/4, 4/5, 5/6$. Based on this, it was found that the IP1 and IP2 PLDPC codes are superior to the AR4JA PLDPC code. Specifically, the IP1 and IP2 PLDPC codes respectively possess additional performance gains of about 0.2 and 0.2 ~ 0.3 dB over the AR4JA PLDPC code in the whole range of code rates. Additionally, since the degree of newly added VNs is greater than 2, the high-rate IP2 PLDPC codes have no error-floor behavior even when the BER equals 10^{-6} . Based on the above, it can be seen that the IUP, IP1, and IP2 PLDPC codes with rate-compatible structures are good candidates for use in OAM-IM systems with a high-transmission-rate demand.

[00137] *C. PLDPC Codes with Shaped Index Schemes*

[00138] Figures 18a and 18b compare the BER performance of the regular-(3, 6) PLDPC code, AR4JA PLDPC code, and three improved PLDPC codes with the natural index and proposed shaped index schemes in the case of $N_A = 2$. It can be seen that all PLDPC codes are capable of achieving significant performance gains with the aid of proposed shaped index scheme. Specifically, for QPSK modulation (see Figure 18a), the regular-(3, 6), AR4JA, and proposed IUP, IP1, and IP2 PLDPC codes achieve performance gains of about 2.2, 2.2, 2.3, 2.4, and 2.5 dB, respectively, at a BER of 2×10^{-5} by using the proposed shaped index scheme. On the other hand, this technique suffers from a spectral-efficiency loss of only 0.5 bpcu (i.e., 8.33% of the original bpcu). Additionally, a similar phenomenon can be noticed in Figure 18b where 8PSK modulation is considered, but the spectral-efficiency loss is only 6.25% of the original bpcu.

[00139] Figures 19a, 19b and 19c investigate the performance of three improved PLDPC codes with the ED-based index and proposed shaped index schemes. Specifically, for IUP PLDPC code (see Figure 19a) the shaped index scheme achieves a significant performance gain over the ED-based index scheme. Also, a similar phenomenon can be observed for IP1 and IP2 PLDPC codes in Figures 19b & 19c, respectively. In view of the above, it can be said that the proposed shaped index scheme is a good alternative for conventional PLDPC codes (e.g., regular-(3, 6) and AR4JA PLDPC codes), and thus can be treated as an appealing technique for boosting system performance with a slight spectral-efficiency loss.

[00140] *Simulated Implementation in a multiple-input multiple-output (MIMO) communications for 5G*

[00141] Figure 20 illustrates the performance of the (3, 6) PLDPC code in 4×4 MIMO visible light communication (VLC) systems with $N_t = 1, 2$, and four different index schemes, including a conventional spatial modulation (CONSM) scheme, an ED scheme and their shaped counterparts. From this plot, it can be seen that the MIMO-VLC systems can achieve significant performance gains with the aid of the shaped index scheme (i.e., shaped ED (proposed) and shaped CONSM (proposed)).

[00142] *Simulated Implementation in multi-cell network systems*

[00143] Figures 21a, 21b and 21c illustrate the performance of (3, 6) PLDPC code in multi-cell network systems with two different index schemes, including conventional spatial modulation (CONSM) scheme and its shaped counterpart. It can be seen from these plots that cell-edge users achieve significant performance gains with the aid of the shaped index scheme. Specifically, the larger the multi-user interference (i.e., superimposition of user interferences in a cell) is, the larger the performance gain can be obtained.

[00144] Numerous other changes, substitutions, variations, and modifications may be ascertained by the skilled in the art and it is intended that the present application encompass all such changes, substitutions, variations, and modifications as falling within the scope of the appended claims.

CLAIMS:

1. A shape index modulation transmitter for a multi-mode orbital angular momentum (OAM) system comprising:
a processing unit; and
a non-transitory media readable by the processing unit, the media storing instructions that when executed by the processing unit causes the processing unit to:
 divide, using a shaping encoder module, a binary coded-bit sequence received from a bit-splitter module into a K -number of sub-sequences, whereby K is an integer that is determined based on a total number of OAM modes and a number of active OAM modes;
 process, using a shaping function provided within the shaping encoder module, the sub-sequences to increase a probability that bits in the sub-sequences comprise a predetermined binary value;
 formulate, using the shaping encoder module, index symbols based on the processed sub-sequences, whereby a first bit of each of the index symbols is obtained from a first processed sub-sequence; and
 provide, using the shaping encoder module, the index symbols to an OAM mode selector module for selecting corresponding OAM mode combinations for transmission by the transmitter, whereby optimal OAM mode combinations of the transmitter are associated with index symbols that comprise primarily of the predetermined binary value.
2. The transmitter according to claim 1 whereby a length of the first sub-sequence of the series of sub-sequences comprises a first length and lengths of remaining sub-sequences of the series of sub-sequences each comprise a second length, wherein the first length differs from the second length.
3. The transmitter according to claim 2 whereby the instructions to process the series of sub-sequences using the shaping function comprises instructions for directing the processing unit to:
 generate, using the shaping encoder module, a codebook comprising an initial set of distinct binary sequences that are mapped to a final set of distinct binary sequences that comprise primarily of the predetermined binary value, whereby a length of each binary sequence in the initial set of distinct binary sequences is equal to the first length and a

length of each binary sequence in the final set of distinct binary sequences is equal to the second length; and

convert, using the shaping encoder module, the first sub-sequence to a shaped first sub-sequence based on the mappings in the codebook.

4. The transmitter according to any one of claims 1 to 3 whereby the instructions to formulate the index symbols based on the processed sub-sequences comprises instructions for directing the processing unit to:
 - for the formulation of each index symbol,
 - sequentially extract, using the shaping encoder module, a predefined number of distinct bits from each of the processed sub-sequences and concatenate the extracted bits to form the index symbol.
5. The transmitter according to claim 4 wherein the predefined number of distinct bits extracted from each of the processed sub-sequences is defined as the number K of the sub-sequences.
6. The transmitter according to any one of claims 1 to 5, whereby the optimal OAM mode combinations of the transmitter comprise $2^{\lfloor \log_2(N_A) \rfloor}$ optimal mode combinations that were selected by channel estimation of a $N \times N$ orbital angular momentum-index modulation system having N_A active OAM modes where N is defined as a number of array elements and N_A is defined as a number of active modes, whereby $1 \leq N_A \leq N$.
7. The transmitter according to any one of claims 1 to 5, whereby the optimal OAM mode combinations of the transmitter comprise optimal mode combinations that were selected using a mutual information (MI)-based index scheme.
8. The transmitter according to claim 7, whereby the selection of the optimal mode combinations using the MI-based index scheme comprises instructions for directing the processing unit to:
 - perform one or more rounds of mode-combination selection, using a MI-based index scheme module that is communicatively coupled to the OAM mode selector, each round comprising:
 - generating a modulated symbol sequence for a distinct signal-to-noise ratio (SNR);

sequentially dividing the modulated symbol sequence into a N number of sub-sequences, whereby the modulated symbols within a k-th sub-sequence are to be transmitted by a k-th mode, where k is defined as a number from a set {1, 2, ..., N} and N is defined as the total number of different OAM modes;

computing, using an MI calculator, extrinsic MI values for each mode based on the sub-sequence associated with the mode; and

determining mode combinations to be used as the optimal mode combinations based on mode combinations that have the largest MI values.

9. The transmitter according to claim 8, whereby the MI calculator computes extrinsic MI values based on a customized Box-Cox transformation (CBCT)-based protograph extrinsic information transfer (PEXIT) algorithm.
10. The transmitter according to any one of claims 1-9 wherein the binary coded-bit sequence comprises a bit sequence coded based on protograph low-density parity-check (PLDPC) codes, an improved unpunctured (IUP) PLDPC code, or one of two improved punctured PLDPC codes.
11. A method for shaping index modulation in a multi-mode orbital angular momentum (OAM) system comprising a shape index modulation transmitter having a shaping encoder module communicatively coupled to an OAM mode selector module, the method comprising the steps of:
 - dividing, using the shaping encoder module, a binary coded-bit sequence received from a bit-splitter module into a K-number of sub-sequences, whereby K is an integer that is determined based on a total number of OAM modes and a number of active OAM modes;
 - processing, using a shaping function provided within the shaping encoder module, the sub-sequences to increase a probability that bits in the sub-sequences comprise a predetermined binary value;
 - formulating, using the shaping encoder module, index symbols based on the processed sub-sequences, whereby a first bit of each of the index symbols is obtained from a first processed sub-sequence; and
 - providing, using the shaping encoder module, the index symbols to an OAM mode selector module for selecting corresponding OAM mode combinations for transmission by the

transmitter, whereby optimal OAM mode combinations of the transmitter are associated with index symbols that comprise primarily of the predetermined binary value.

12. The method according to claim 11 whereby a length of the first sub-sequence of the series of sub-sequences comprises a first length and lengths of remaining sub-sequences of the series of sub-sequences each comprise a second length, wherein the first length differs from the second length.

13. The method according to claim 12 whereby the step of processing the series of sub-sequences using the shaping function comprises:

generating, using the shaping encoder module, a codebook comprising an initial set of distinct binary sequences that are mapped to a final set of distinct binary sequences that comprise primarily of the predetermined binary value, whereby a length of each binary sequence in the initial set of distinct binary sequences is equal to the first length and a length of each binary sequence in the final set of distinct binary sequences is equal to the second length; and

converting, using the shaping encoder module, the first sub-sequence to a shaped first sub-sequence based on the mappings in the codebook.

14. The method according to any one of claims 11 to 13 whereby the step of formulating the index symbols based on the processed sub-sequences comprises:

for the formulation of each index symbol,

sequentially extracting, using the shaping encoder module, a predefined number of distinct bits from each of the processed sub-sequences and concatenate the extracted bits to form the index symbol.

15. The method according to claim 14 wherein the predefined number of distinct bits extracted from each of the processed sub-sequences is defined as the number K of the sub-sequences.

16. The method according to any one of claims 11 to 15, whereby the optimal OAM mode combinations of the transmitter comprise $2^{\lfloor \log_2 \binom{N}{N_A} \rfloor}$ optimal mode combinations that were selected by channel estimation of a $N \times N$ orbital angular momentum-index modulation

system having N_A active OAM modes where N is defined as a number of array elements and N_A is defined as a number of active modes, whereby $1 \leq N_A \leq N$.

17. The method according to any one of claims 11 to 15, whereby the optimal OAM mode combinations of the transmitter comprise optimal mode combinations that were selected using a mutual information (MI)-based index scheme.

18. The method according to claim 17, whereby the step of selecting the optimal mode combinations using the MI-based index scheme comprises:

performing one or more rounds of mode-combination selection, using a MI-based index scheme module that is communicatively coupled to the OAM mode selector, each round comprising the steps of:

generating a modulated symbol sequence for a distinct signal-to-noise ratio (SNR);

sequentially dividing the modulated symbol sequence into a N number of sub-sequences, whereby the modulated symbols within a k -th sub-sequence are to be transmitted by a k -th mode, where k is defined as a number from a set $\{1, 2, \dots, N\}$ and N is defined as the total number of different OAM modes;

computing, using an MI calculator, extrinsic MI values for each mode based on the sub-sequence associated with the mode; and

determining mode combinations to be used as the optimal mode combinations based on mode combinations that have the largest MI values.

19. The method according to claim 18, whereby the MI calculator computes extrinsic MI values based on a customized Box-Cox transformation (CBCT)-based protograph extrinsic information transfer (PEXIT) algorithm.

20. The method according to any one of claims 11-19 wherein the binary coded-bit sequence comprises a bit sequence coded based on protograph low-density parity-check (PLDPC) codes, an improved unpunctured (IUP) PLDPC code, or one of two improved punctured PLDPC codes.