

**Method for Computing and Reducing Frequency “Lock-In”
for a Vibrating Beam Accelerometer and Optional Linear Enhancement**

FIELD OF THE INVENTION

This invention relates to Vibrating Beam Accelerometers and more particularly to reduction of the “blind zone” where the sensor does not produce a useful output in the presence of an external acceleration.

5 **PRIOR ART**

Prior art references considered to be relevant as a background to the invention are listed below and their contents are incorporated herein by reference. Acknowledgement of the references herein is not to be inferred as meaning that these are in any way relevant to the patentability of the invention disclosed herein. Each reference is identified by a number enclosed in square brackets and accordingly the prior art will be referred to throughout the specification by numbers enclosed in square brackets.

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BACKGROUND OF THE INVENTION

Vibrating Beam Accelerometers are based on the fact that the resonant frequency of a vibrating beam attached to a seismic-mass changes when subjected to an external acceleration. The beam that is pulled vibrates at a higher frequency while the beam that is pushed vibrates at a lower frequency. This action is similar to that of a violin string, which increases pitch with an increase in tension, and decreases pitch with a decrease in tension (or relative compression). Typically, two vibrating beams are used in a differential arrangement in order to reduce common parasitic sensitivities e.g. temperature, pressure or aging. The two beams operate in a push-pull mode whereby when one beam is pushed and is subjected to compression, the other beam is pulled and subjected to tension. During operation, the two beams constantly change roles and, as is described in the literature, for each beam there is a measured vibrating frequency and it can be shown that they vibrate at two frequencies: one slightly higher corresponding to pull-mode and the other slightly lower corresponding to push-mode.

However, the use of two beams coupled by a seismic mass may lead to the presence of a “blind zone” where the sensor does not produce a useful output in the presence of an external acceleration. It is widely believed that this blind zone is caused by the “lock-in” phenomenon whereby the resonant frequency of both beams becomes “locked” (i.e. both beams vibrate with the same frequency) and therefore the output acceleration (which is proportional to the frequency difference) becomes zero even in the presence of an external acceleration. This effect limits the range of accelerations for which the sensor can be used.

SUMMARY OF THE INVENTION

The invention is based on the unexpected finding that, contrary to widespread belief, the lock-in of frequencies does not exist in the rigorous sense. This is because both beams vibrate at their two Eigen-frequencies for the whole range of external accelerations, excepting for zero acceleration. The presence of two vibration frequencies is the consequence of the split of the resonant frequency caused by the coupling of both resonators through the seismic mass.

Specifically, for each of the beams there are corresponding first and second frequencies (f^- , f^+) at which the respective amplitude is dominant. The lock-in zone is bound

by respective crossover points for each beam where the respective amplitudes (A_- , A_+) at both the first and second frequencies (f_- , f_+) are the same and which define a crossover region within which the respective dominant amplitudes of the first and second frequencies cross over such that the frequency for which amplitude is higher outside the crossover region has a smaller amplitude within the crossover region and *vice versa*.

This is distinguished over conventional approaches that measure only a single frequency, corresponding to the frequency whose amplitude is dominant.

The present inventor has found that a principal cause of lock-in is that conventional approaches to measuring the vibration frequency of the beams measure only the dominant frequency, i.e. the frequency whose amplitude is higher and, as a result, ignore the less dominant frequency whose amplitude is lower. Throughout the present disclosure, we will refer to these frequencies as the dominant and sub-dominant frequencies. In contrast, the present invention detects both the dominant and sub-dominant frequencies, which when properly taken into account can determine the lock-in boundary within which the amplitudes of the dominant and sub-dominant frequencies cross over. The frequencies within the lock-in boundary can then be selected to avoid the effect of lock-in.

It may appear to be a contradiction to suggest that, on the one hand, the present invention is directed to the reduction or elimination of lock-in while, on the other hand, to state that the frequency lock-in does not exist in the rigorous sense. Therefore, for the sake of abundant clarity and to avoid doubt, lock-in will occur if only the dominant frequencies of both beams are recorded across the complete range of accelerations to which the accelerometer is subjected. This is because, as will be shown analytically below, the dominant frequencies for both beams are identical in the lock-in range and therefore in an accelerometer whose output is a function of differential frequency, no signal will be output for any acceleration in the lock-in zone. The invention effectively circumvents this problem by taking into account the non-dominant frequencies in the lock-in zone.

BRIEF DESCRIPTION OF THE DRAWINGS

In order to understand the invention and to see how it may be carried out in practice, embodiments will now be described, by way of non-limiting example only, with reference to the accompanying drawings, in which:

- 5 **Fig. 1** shows schematically a simple differential vibrating beam accelerometer;
 Fig. 2 is the equivalent spring-mass model of the device shown in Fig. 1;
 Fig. 3 is a graphical representation of the amplitude-acceleration characteristic for the device as derived using the model of Fig. 2;
 Fig. 4 is a graphical representation of the frequency-acceleration characteristic for
10 conventional accelerometers as derived using the model of Fig. 2;
 Figs. 5 and 6 show the dominant frequencies of Beam 1 and Beam 2, respectively which are measured by conventional accelerometers;
 Fig. 7 is a graphical representation showing frequency-difference ($\Delta f(t)$) versus acceleration for the device as derived using the model of Fig. 2 when measuring frequency
15 according to conventional methods;
 Figs. 8 and 9 show the dominant and sub-dominant frequencies of Beam 1 and Beam 2, respectively both of which may be measured according to a first embodiment of the invention;
 Fig. 10 is a graphical representation of the amplitude-acceleration characteristic
20 for a device having beams of different masses as derived using a suitably modified model to the model of Fig. 2;
 Fig. 11 shows graphically the relationship between the vibration amplitudes of both beams inside and outside of the lock-in zone;
 Figs. 12 and 13 are graphical representations showing corrected frequency-
25 difference ($\Delta f(t)$) versus acceleration over different ranges of acceleration according to a first embodiment of the invention;
 Fig. 14 shows graphically dominant and sub-dominant frequency versus acceleration for a single beam showing that the curves veer toward each at zero acceleration;
 Figs. 15 and 16 show graphically dominant frequency versus acceleration for both
30 beams when vibrated in-phase and anti-phase, respectively;

Fig. 17 shows frequency-difference versus acceleration using the frequency-difference, $\Delta f(\Gamma)$, of Fig. 12 as compared with an enhanced linear relationship using a corrected frequency-difference, $\hat{\Delta f}(\Gamma)$, according to a second embodiment;

Fig. 18 shows the same comparison of Fig. 17 drawn to a larger scale;

5 **Fig. 19** shows deviation from linearity for the first and second embodiments of the invention; and

Figs 20 and 21 are flow charts showing alternative methods for determining the boundary of the lock-in zone during calibration of the accelerometer.

DETAILED DESCRIPTION OF EMBODIMENTS

10 A proper understanding of the invention requires a detailed analysis of the vibrating beam accelerometer, much of which will be familiar to those skilled in the art. For the sake of completeness, we present a model of a vibrating beam accelerometer and derive equations that define the boundaries of the lock-in zone.

Fig. 1 shows schematically a simple vibrating beam accelerometer that can be
15 analyzed mathematically using the equivalent spring-mass model shown in Fig. 2. For illustrative purposes and sake of simplicity, in the following explanation the model shown in Fig 2 considers the case of equal beams, it being understood that the invention defined by the appended claims encompasses any practical implementation of the vibrating beam accelerometer that can be reduced to the same or similar mathematical model including the
20 case where the beams are not equal. Thus, Fig. 1 shows an accelerometer 10 comprising a pair of collinear or parallel beams 11, 11' each anchored at one end to a respective support 12, 12' and commonly coupled at their opposite ends to a seismic mass 13. The beams 11, 11' may be formed of piezoelectric material such as quartz that generates an electric charge in response to mechanical stress. When the beams are compressed or stretched, they produce
25 an electric voltage across their surfaces. Conversely, an electric field applied across the beams will induce vibration. To this end, an AC voltage 14 is commonly applied across a respective pair of electrodes 15, 15' deposited on opposing surfaces of each beam, thereby inducing a steady-state vibration of the beams whose amplitudes are measured as analog signals across respective electrodes 16, 16' deposited on opposing surfaces of both beams.

30 When the structure is subjected to an external acceleration, a force component parallel to the beams' longitudinal axes shifts the seismic mass axially to and fro, thereby

pushing one of the beams while pulling the other, thereby giving rise to slight differences in their respective resonant frequencies. Specifically, when using conventional methods for measuring the frequency, the shorter compressed beam will vibrate at a lower frequency, which we shall denote as f^- compared to the longer stretched beam, whose resonant frequency f^+ is higher. In a practical implementation, the beams are used in flexural mode as sensitive elements, and vibration is maintained by means of an oscillator circuit denoted schematically by the AC voltage 14. For so long as the beams are subjected to axial vibration, they will continually elongate and foreshorten alternately, and their resonant frequencies will likewise decrease and increase alternately.

10 The applied acceleration induces tensile or compressive stresses into each beam which modify the resonant frequency, measurement of which is thus indicative of the applied acceleration.

The equations of motion that describe the dynamics of the model of Fig. 2 are:

$$\begin{aligned} m_0 \ddot{x}_1 + k^+(x_1 - X) &= m_0 \Gamma \\ M \ddot{X} + k^+(X - x_1) + k^-(X - x_2) &= M \Gamma \\ m_0 \ddot{x}_2 + k^-(x_2 - X) &= m_0 \Gamma \end{aligned} \quad (1)$$

where Γ is the external applied acceleration.

$$\begin{aligned} k^+ &= k_0(1 + \lambda \Gamma) \\ k^- &= k_0(1 - \lambda \Gamma) \end{aligned} \quad (2)$$

15 It is clear from Eq. (2) that k^+ and k^- are functions of Γ . However, in all the following equations, we shall omit the dependency $k^+(\Gamma)$ and $k^-(\Gamma)$ on Γ in order to avoid the equations becoming unwieldy and refer simply to k^+ and k^- .

Dividing by m_0 and M , respectively, leads to:

$$\begin{aligned} \ddot{x}_1 + \frac{k^+}{m_0}(x_1 - X) &= \Gamma \\ \ddot{X} + \frac{k^+}{M}(X - x_1) + \frac{k^-}{M}(X - x_2) &= \Gamma \\ \ddot{x}_2 + \frac{k^-}{m_0}(x_2 - X) &= \Gamma \end{aligned} \quad (3)$$

Subtracting the first and third equations from the second leads to:

$$\begin{aligned}
\ddot{X} - \ddot{x}_1 + \left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)(X - x_1) + \frac{k^-}{M}(X - x_2) &= 0 \\
\ddot{X} - \ddot{x}_2 + \frac{k^+}{M}(X - x_1) + \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)(X - x_2) &= 0
\end{aligned} \tag{4}$$

We define new variables y_1 and y_2 as follows:

$$\begin{aligned}
y_1 &\equiv X - x_1 \\
y_2 &\equiv X - x_2
\end{aligned}$$

Substituting in Eq. (4) yields:

$$\begin{aligned}
\ddot{y}_1 + \left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)y_1 + \frac{k^-}{M}y_2 &= 0 \\
\ddot{y}_2 + \frac{k^+}{M}y_1 + \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)y_2 &= 0
\end{aligned} \tag{5}$$

5 We define new variables y_3 and y_4 as follows:

$$\begin{aligned}
y_3 &\equiv \dot{y}_1 \\
y_4 &\equiv \dot{y}_2
\end{aligned} \tag{6}$$

Substituting in Eq. (5) gives:

$$\begin{aligned}
\dot{y}_1 &= y_3 \\
\dot{y}_2 &= y_4 \\
\dot{y}_3 &= -\left(\frac{k^+}{M} + \frac{k^+}{m_0}\right)y_1 - \frac{k^-}{M}y_2 \\
\dot{y}_4 &= -\frac{k^+}{M}y_1 - \left(\frac{k^-}{M} + \frac{k^-}{m_0}\right)y_2
\end{aligned} \tag{7}$$

Equation (7) can be more conveniently written in matrix form:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\left(\frac{k^+}{M} + \frac{k^+}{m_0}\right) & -\frac{k^-}{M} & 0 & 0 \\ -\frac{k^+}{M} & -\left(\frac{k^-}{M} + \frac{k^-}{m_0}\right) & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \tag{8}$$

We define:

$$M_0 \equiv \frac{m_0 M}{M + m_0}$$

We can re-write Eq. (8) as follows:

$$\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k^+}{M_0} & -\frac{k^-}{M} & 0 & 0 \\ -\frac{k^+}{M} & -\frac{k^-}{M_0} & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} \quad (9)$$

Eq. (9) has a solution of the form:

$$\begin{aligned} y_1(t) &= 2c_1 \cos \omega_+ t - 2e_1 \sin \omega_+ t + 2c_2 \cos \omega_- t - 2e_2 \sin \omega_- t \\ y_2(t) &= -(2c_1 \cos \omega_+ t - 2e_1 \sin \omega_+ t) \frac{M}{k^-} \left(\frac{k^+}{M_0} - \omega_+^2 \right) \\ &\quad - (2c_2 \cos \omega_- t - 2e_2 \sin \omega_- t) \frac{M}{k^-} \left(\frac{k^+}{M_0} - \omega_-^2 \right) \\ y_3(t) &= -(2c_1 \sin \omega_+ t + 2e_1 \cos \omega_+ t) \omega_+ \\ &\quad - (2c_2 \sin \omega_- t + 2e_2 \cos \omega_- t) \omega_- \\ y_4(t) &= (2c_1 \sin \omega_+ t + 2e_1 \cos \omega_+ t) \frac{\omega_+ M}{k^-} \left(\frac{k^+}{M_0} - \omega_+^2 \right) \\ &\quad + (2c_2 \sin \omega_- t + 2e_2 \cos \omega_- t) \frac{\omega_- M}{k^-} \left(\frac{k^+}{M_0} - \omega_-^2 \right) \end{aligned} \quad (10)$$

and

$$\begin{aligned} c_1 &= \frac{y_1(0)}{2} - \frac{y_1(0)}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) - \frac{y_2(0)k^-}{2M(\omega_-^2 - \omega_+^2)} \\ e_1 &= -\frac{y_3(0)}{2\omega_+} + \frac{y_3(0)}{2\omega_+} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) + \frac{y_4(0)k^-}{2M\omega_+(\omega_-^2 - \omega_+^2)} \\ c_2 &= \frac{y_1(0)}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) + \frac{y_2(0)k^-}{2M(\omega_-^2 - \omega_+^2)} \\ e_2 &= -\frac{y_3(0)}{2\omega_-} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) - \frac{y_4(0)k^-}{2M\omega_-(\omega_-^2 - \omega_+^2)} \end{aligned} \quad (11)$$

where:

$$\begin{aligned}\omega_- &\equiv \sqrt{k_0 \left[\frac{1}{M_0} - \sqrt{\frac{1}{M^2} + \lambda^2 \Gamma^2 \left(\frac{1}{M_0^2} - \frac{1}{M^2} \right)} \right]} \\ \omega_+ &\equiv \sqrt{k_0 \left[\frac{1}{M_0} + \sqrt{\frac{1}{M^2} + \lambda^2 \Gamma^2 \left(\frac{1}{M_0^2} - \frac{1}{M^2} \right)} \right]}\end{aligned}\quad (12)$$

It is reiterated that the dependency of ω_- and ω_+ on Γ is omitted for the sake of simplicity, The differential frequency $\Delta\omega = \omega_+ - \omega_-$ is given by:

$$\Delta\omega = \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - \sqrt{(1 - \lambda^2 \Gamma^2) \left(1 + 2 \frac{m_0}{M} \right)}} \quad (13)$$

For the case that $m_0 \ll M$ and $\lambda\Gamma \ll 1$, we can neglect $2 \frac{m_0}{M} \lambda^2 \Gamma^2$ and Eq. (13)

5 can be approximated to:

$$\Delta\omega \approx \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - \sqrt{1 + 2 \frac{m_0}{M} - \lambda^2 \Gamma^2}} \quad (14)$$

Using the partial Taylor expansion for $\sqrt{1+x} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ where $x = 2 \frac{m_0}{M} - \lambda^2 \Gamma^2$ and neglecting terms in $4 \frac{m_0}{M} \lambda^2 \Gamma^2$ and $\lambda^4 \Gamma^4$:

$$\begin{aligned}\Delta\omega &\approx \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - 1 - \frac{1}{2} \left(2 \frac{m_0}{M} - \lambda^2 \Gamma^2 \right) + \frac{1}{8} \cdot 4 \left(\frac{m_0}{M} \right)^2} \\ &\approx \sqrt{2 \frac{k_0}{m_0}} \sqrt{1 + \frac{m_0}{M} - 1 - \frac{m_0}{M} + \frac{1}{2} \lambda^2 \Gamma^2 + \frac{1}{2} \left(\frac{m_0}{M} \right)^2}\end{aligned}\quad (15)$$

This leads to:

$$\Delta\omega \approx \sqrt{\frac{k_0}{m_0}} \sqrt{\lambda^2 \Gamma^2 + \frac{m_0^2}{M^2}} \quad (16)$$

and for $\Gamma = 0$:

$$\Delta\omega(0) \approx \sqrt{\frac{k_0}{m_0} \frac{m_0}{M}} \quad (17)$$

10 and for $\Gamma \gg 0$:

$$\Delta\omega(\Gamma) \approx \lambda \sqrt{\frac{k_0}{m_0}} \Gamma = \eta \Gamma \quad (18)$$

where $\eta \equiv \lambda \sqrt{\frac{k_0}{m_0}} = \lambda \omega_0$ is the scale factor.

Example I:

Accelerations are commonly represented in units of the local value of the gravity constant g , which was approximated to 10 ms^{-2} for the purpose of simulation. By way of
 5 example, consider a concrete example having a seismic mass $M = 2 \times 10^{-5} \text{ kg}$, beams of mass $m_0 = 2 \times 10^{-8} \text{ kg}$, spring constant $k_0 = 3850 \text{ kg s}^{-2}$, and $\lambda = \frac{0.0005}{g}$. The initial conditions are $y_1(0) = y_2(0) = 1 \mu\text{m}$ and $y_3(0) = y_4(0) = 0$. We use Eq. (10) to calculate the output signals $y_1(t)$ and $y_2(t)$.

A frequency counter based on signal zero-cross in a given time interval is used in
 10 order to measure the vibrating frequency of each beam. Note that a frequency counter gives a single output frequency. The frequency difference Δf is proportional to the applied external acceleration. The vibration amplitudes of each beam can be calculated from Eq. (10) as:

$$\begin{aligned} A_{1+} &= 2\sqrt{c_1^2 + e_1^2} \\ A_{1-} &= 2\sqrt{c_1^2 + e_2^2} \\ A_{2+} &= 2\sqrt{(c_1^2 + e_1^2) \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_+^2\right)^2} \\ A_{2-} &= 2\sqrt{(c_1^2 + e_2^2) \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_-^2\right)^2} \end{aligned} \quad (19)$$

Again, for the sake of abundant clarity, we reiterate that in Eq. (19) the
 15 dependency of k^- and k^+ as well ω_- and ω_+ on Γ is omitted for the sake of simplicity.

Fig. 3 depicts the vibration amplitudes for each beam. The numbers 1 and 2 refer to the beams 1 and 2 also denoted 11 and 11', respectively in Fig. 1. In order to distinguish between the four frequencies (two for each of the two beams), the amplitude-acceleration curves are shown using different line types as follows:

Beam	Frequency	Line Type	Amplitude
Beam 1	f^+		A_{1+}
	f^-		A_{1-}
Beam 2	f^+		A_{2+}
	f^-		A_{2-}

Table I: Line type key for frequency curves

From the figure we see that the vibration amplitudes of both beams change in a continuous way.

When the external acceleration is zero, both beams get “locked” and vibrate at the same frequency $f_+(0) = \frac{\omega_+(0)}{2\pi} = \frac{1}{2\pi} \sqrt{k_0 \frac{M+2m_0}{m_0 M}}$ or $f_-(0) = \frac{\omega_-(0)}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}}$ depending on the initial conditions. On one hand, we obtain that if $y_1(0) = y_2(0)$ both beams vibrate at $f_+(0) = \frac{\omega_+(0)}{2\pi} = \frac{1}{2\pi} \sqrt{k_0 \frac{M+2m_0}{m_0 M}}$ and for $m_0 \ll M$ then the lock-in frequency is $f_+(0) = \frac{\omega_+(0)}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}}$. This result shows that, for this case, the seismic mass acts as an anchor for both beams without any transfer of energy between them.

On the other hand, if the initial conditions are such that $y_1(0) = -y_2(0)$, then both beams vibrate at their uncoupled resonant frequency $f_-(0) = \frac{\omega_-(0)}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}}$ independently of the value of the seismic mass M .

From the figure, we also see, that there are particular accelerations (denoted by Γ_1^{LI} and Γ_2^{LI}) for which the amplitudes of both vibrating frequencies are the same. For accelerations $\Gamma < \Gamma_1^{LI}$, we see that the dominant amplitudes are A_{1-} and A_{2+} ; for $\Gamma_1^{LI} < \Gamma < \Gamma_2^{LI}$ the dominant amplitudes are A_{1+} and A_{2+} ; and finally for $\Gamma_1^{LI} > \Gamma_2^{LI}$ the dominant amplitudes are A_{1+} and A_{2-} . In order avoid the need to refer to the frequency having the dominant amplitude, we will denote this frequency as the “dominant frequency”, it being understood that the dominant frequency may be f^- or f^+ .

Therefore, any method that detects only the dominant frequency will give in the range $\Gamma_1^{LI}, \Gamma_2^{LI}$ a unitary value of frequency equal to f^- or f^+ as a single frequency output for both beams, depending on initial conditions. For the sake of completeness, it should

be noted that positive acceleration denotes acceleration in the direction of increasing x in Fig. 2, while negative acceleration denotes deceleration in the direction of x or acceleration in the opposite direction.

Fig. 4 shows on a common axis the measured output frequencies of the two beams as measured in conventional accelerometers, these being, of course, by definition the dominant frequencies because only these are measured using conventional approaches. It is seen that by using the signal zero-cross detection method, for the range of accelerations $\Gamma_1^{LI}, \Gamma_2^{LI}$ the frequency outputs are the same. It can be seen that the two dominant frequencies f_{1+} and f_{2+} overlap in the lock-in zone. We reiterate that by “dominant frequencies” we mean the frequencies whose amplitudes are dominant. This can be explained by the fact that this method detects only the dominant frequency of each beam. Since we have already established with reference to Fig. 3 that the dominant amplitudes in this range are A_{1+} and A_{2+} the detected frequency will be f_{+} . It may be observed that the measured frequencies in the lock-in zone are f_{-} or f_{+} depending on the initial conditions. For instance, for the case where $a = b$, the lock-in frequency is f_{+} , while for $a = -b$, the lock-in frequency is f_{-} . It can also be seen that boundaries of the lock-in zone are $[-2, 2] g$.

For the sake of abundant clarity, Figs. 5 and 6 show the output frequencies of Beam 1 and Beam 2, respectively. Thus, in Fig. 5 it is seen that for positive acceleration outside of the lock-in zone, the dominant frequency for Beam 1 is f_{1+} and this remains the dominant frequency also inside the lock-in zone. For negative acceleration outside of the lock-in zone, the only measured frequency is f_{1-} . What this means in practice is that if the acceleration to which Beam 1 is subjected falls within a range of -2 to $+5 g$, the only measured frequency will be f_{1+} , while for accelerations less than $-2 g$, the only measured frequency will be f_{1-} . Similarly, Fig. 6 shows that if Beam 2 is subjected to positive accelerations greater than $+2 g$ i.e. outside of the lock-in zone, the only measured frequency will be f_{2-} , while for accelerations less than $+2 g$, the only measured frequency will be f_{2+} . It may be seen that the curves shown in Figs. 5 and 6, respectively, depicting f_{1+} and f_{2+} are congruent in the lock-in zone. It will thus be apparent that when considering the measured frequencies for both beams, the combined frequency-acceleration curves are as shown in Fig. 4, such that the frequency difference $\Delta f = f_{1+} - f_{2+} = 0$ inside the lock-in zone. It is for this reason that conventional methods that measure acceleration as a function of frequency difference based only on the measurement of the dominant frequencies produce zero output in the lock-in zone. This

is clearly seen in Fig. 7, which shows graphically the frequency difference $\Delta f(\Gamma) = f_1(\Gamma) - f_2(\Gamma)$ for the conventional approach where only the dominant frequencies are measured in the lock-in zone. It is seen that in the lock-in zone, defined by the acceleration range $[-2, 2] g$, the frequency difference $\Delta f(\Gamma)$ is zero because the
 5 dominant frequencies for two identical beams are the same. The discontinuous or nonlinear behavior of the frequency difference $\Delta f(\Gamma)$ can also be seen.

Fig. 8 depicts the frequency-acceleration characteristic for Beam 1, where both the dominant and sub-dominant frequencies are shown. The dominant frequencies are the same as in Fig. 5 and are shown in different line types for f_{1+} and f_{1-} , respectively. But
 10 also seen are the respective sub-dominant frequencies, which are not measured in conventional approaches, but critically they are measured in the invention, at least in the lock-in zone. Likewise, Fig. 9 depicts the frequency-acceleration characteristic for Beam 2, where both the dominant and sub-dominant frequencies are shown. The dominant frequencies are the same as in Fig. 6 and are shown in different line types for f_{2+} and
 15 f_{2-} , respectively. But also seen are the respective sub-dominant frequencies, it being reiterated that these frequencies are not measured in conventional approaches, but critically are measured in the invention, at least in the lock-in zone. It is seen that the frequency-acceleration characteristics for both beams are identical, as is to be expected for identical beams. This means, that if the two figures are superimposed as has been
 20 done in Fig. 4, the dominant frequencies for both beams are overlapping in the lock-in zone and so, too, are the sub-dominant frequencies for both beams overlapping in the lock-in zone. Consequently, the frequency difference $\Delta f(\Gamma)$ within the lock-in zone for both dominant and the sub-dominant frequencies is zero; but critically the frequency difference $\Delta f(\Gamma)$ within the lock-in zone between the dominant frequency of either beam
 25 and the sub-dominant frequency for the other beam will not be the same. It is this finding that is used by the invention to obtain a non-zero frequency difference $\Delta f(\Gamma)$ that is indicative of the acceleration within the lock-in zone.

It is also seen that outside of the lock-in zone, the dominant and sub-dominant frequencies for each beam form a continuum for both frequencies, i.e. f_+ and f_- . Thus,
 30 referring to Fig. 8 relating to Beam 1, by way of example, the curve representing f_{1+} stops at the negative boundary of the lock-in zone at an acceleration of $-2 g$, and at exactly this point for accelerations less than $-2 g$, the sub-dominant frequency shown in dashed line takes over seamlessly. The same is true in reverse for the curves representing f_{1-} , which are mirror images of the f_{1+} curves about a horizontal axis. The same is true

for Beam 2, except that the respective curves are complementary to those of Beam 1. In other words, f_{2+} is the dominant frequency and extends for negative accelerations, stopping at the positive boundary of the lock-in zone at an acceleration of +2 g. At exactly this point for accelerations greater than +2 g, the sub-dominant frequency takes over
5 seamlessly. Similar but complementary observations can be made concerning the sub-dominant frequencies.

It emerges from Figs. 8 and 9 that if we know the boundaries of the lock-in zone, then we can use these figures to determine the crossover frequencies at the two boundaries of the lock-in zone. For a balanced system whose two beams are identical, the frequency-
10 acceleration curves will be symmetrical about the vertical frequency axis i.e. for zero acceleration. In this case, the dominant crossover frequencies of Beam 1 will be identical and so, too, will the dominant crossover frequencies of Beam 2. It can be seen that for both beams, the crossover frequency is approximately 69,910 Hz. For a non-balanced system, the crossover frequencies at opposing boundaries of the lock-in zone may be
15 different, but the principle is the same. In both Figs. 8 and 9, the lower frequency characteristics shown predominantly in chain-dotted line, because they represent for the most part sub-dominant frequencies, also intersect the boundaries of the lock-in zone at identical frequencies. It can be seen that for both beams, the sub-dominant crossover frequency is approximately 69,810 Hz. If we consider only the upper frequency-acceleration
20 characteristics, we see that within the lock-in zone the dominant frequencies dip, i.e. they are lower than the crossover frequencies. This means that if the measured dominant frequency for Beam 1 is higher than the crossover frequency, it can be inferred that we are not in the lock-in zone. In this case, we do not need to measure the sub-dominant frequencies.

25 With reference to Fig. 9, it can be seen that the same is true for Beam 2. Thus, in either case, if the measured dominant frequency is higher than the crossover frequency, it can be inferred that we are not in the lock-in zone and we can infer that acceleration will be a function of the frequency difference between the two measured dominant frequencies for the respective beams. These frequencies can be measured using a frequency counter
30 in conventional manner, or using any other suitable method. On the other hand, if the measured dominant frequency is less than the crossover frequency, then we can infer that we are in the lock-in zone. In this case, as we have seen, the dominant frequencies of the two beams will be identical and acceleration can only be determined by measuring also the sub-dominant frequencies. Within the lock-in zone, the frequency difference of

interest is then the difference between the dominant frequency for either beam less the sub-dominant frequency for the other beam. It is apparent from Figs. 8 and 9 that for a balanced system having identical beams, the absolute value of the frequency difference for any given acceleration will be the same regardless of for which of the two beams the dominant frequency is used and for which the sub-dominant frequency is used.

Derivation of the Pseudo Lock-In Boundary

From Eq. (10) we see that both beams vibrate at the Eigen-frequencies f_- and f_+ , where $\omega = 2\pi f$. This means that while a single beam attached to a seismic mass vibrates at a single frequency, two beams attached to the same mass will vibrate at a combination of the two Eigen-frequencies f_- and f_+ .

In real world applications, the beams are put in vibration with initial zero velocity i.e. $y_3(0) = y_4(0) = 0$. That is, the beams are taken out of equilibrium by pulling the beam along the transversal axis i.e. $y_1(0) = a$ and $y_2(0) = b$. Equations (10) and (11) may then be simplified as follows:

$$\begin{aligned} y_1(t) &= A_{1+} \cos \omega_+ t + A_{1-} \cos \omega_- t \\ y_2(t) &= A_{2+} \cos \omega_+ t + A_{2-} \cos \omega_- t \end{aligned} \quad (20)$$

We define:

$$\begin{aligned} A_{1+}(\Gamma) &= 2\sqrt{c_1^2} \\ A_{1-}(\Gamma) &= 2\sqrt{c_2^2} \\ A_{2+}(\Gamma) &= 2\sqrt{c_1^2 \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_+^2\right)^2} \\ A_{2-}(\Gamma) &= 2\sqrt{c_2^2 \left(\frac{M}{k^-}\right)^2 \left(\frac{k^+}{M_0} - \omega_-^2\right)^2} \end{aligned} \quad (21)$$

Substituting these definitions into Eq. (11) gives:

$$\begin{aligned}
c_1 &= \frac{a}{2} - \frac{a}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) - \frac{bk^-}{2M(\omega_-^2 - \omega_+^2)} \\
c_2 &= \frac{a}{2} \left(\frac{\frac{k^+}{M_0} - \omega_+^2}{\omega_-^2 - \omega_+^2} \right) + \frac{bk^-}{2M(\omega_-^2 - \omega_+^2)}
\end{aligned} \tag{22}$$

The amplitudes will change with changes in the external acceleration Γ . The lock-in boundary is defined by the following condition:

$$\begin{aligned}
A_{1+}(\Gamma^{LI}) &= A_{1-}(\Gamma^{LI}) \\
A_{2+}(\Gamma^{LI}) &= A_{2-}(\Gamma^{LI})
\end{aligned} \tag{23}$$

We solve conditions (23) for Γ^{LI} to obtain:

$$\begin{aligned}
\Gamma_1^{LI} &= \frac{-bm_0}{\lambda[aM + m_0(a - b)]} \\
\Gamma_2^{LI} &= \frac{am_0}{\lambda[bM + m_0(b - a)]}
\end{aligned} \tag{24}$$

For the special case where $a = b$,

$$\begin{aligned}
\Gamma_1^{LI} &= -\frac{m_0}{\lambda M} = -\frac{\Delta\omega(\Gamma = 0)}{\eta} \\
\Gamma_2^{LI} &= \frac{m_0}{\lambda M} = \frac{\Delta\omega(\Gamma = 0)}{\eta}
\end{aligned} \tag{25}$$

- 5 Equation (25) conforms to the range of the lock-in zone measured by Le Traon, 2006 ([1]).
 Fig. 10, is a graphical representation of the amplitude-acceleration characteristic for a device having beams of different masses as derived using a suitably modified model to the model of Fig. 2, it is seen that if the masses of the two beams are unequal, then the lock-in range will not be centered at zero frequency, but will be centered either in a positive
 10 or negative acceleration position depending on the sizes of the two beams. For small differences between the masses of the beams, Eq. (24) remains a very good approximation.

A Solution to the Lock-In Problem

We have shown in Eq. (12) that, except for an external acceleration equal to zero, both beams vibrate at their two Eigen-frequencies:

$$\begin{aligned}\omega_- &\equiv \sqrt{k_0 \left[\frac{1}{M_0} - \sqrt{\frac{1}{M^2} + \lambda^2 \Gamma^2 \left(\frac{1}{M_0^2} - \frac{1}{M^2} \right)} \right]} \\ \omega_+ &\equiv \sqrt{k_0 \left[\frac{1}{M_0} + \sqrt{\frac{1}{M^2} + \lambda^2 \Gamma^2 \left(\frac{1}{M_0^2} - \frac{1}{M^2} \right)} \right]}\end{aligned}\quad (12)$$

We have also seen that the lock-in effect is a consequence of how we measure the vibration frequencies of the beams. Therefore, in order to solve the problem, we need to determine the boundaries of the lock-in zone, and for accelerations inside the lock-in zone measure “both” vibration frequencies for each beam, and to build an algorithm to determine how to calculate the frequency difference of the frequencies of both beams. Any frequency measurement that provides discrimination between both harmonics, i.e. f_+ and f_- may be employed. For instance, the Fourier Transform may be considered the oldest and well known. The Least Square Method and other estimators such as Kalman and extended Kalman filters are described in the literature. We can also use signal cancellation by determining the dominant frequency using a Phase Lock Loop or a frequency counter, synthesize an equivalent sinusoidal signal of identical frequency and then subtract the synthetic signal from the original to derive the sub-dominant signal. Without loss of generality, any method that can detect the harmonic components of a given signal can be used.

We can write the output signals of the beams in the following general form:

$$\begin{aligned}y_1(t) &= A_{1+} \cos(\omega_+ t + \varphi_+) + A_{1-} \cos(\omega_- t + \varphi_-) \\ y_2(t) &= A_{2+} \cos(\omega_+ t + \varphi_+) + A_{2-} \cos(\omega_- t + \varphi_-)\end{aligned}\quad (26)$$

where A_{1+} and A_{2+} are as defined in Eq. (19) and

$$\begin{aligned}\varphi_+ &= \arctan\left(\frac{e_1}{c_1}\right) \\ \varphi_- &= \arctan\left(\frac{e_2}{c_2}\right)\end{aligned}\quad (27)$$

Fig. 11 summarizes the relationship between the vibration amplitudes of both beams inside and outside the lock-in zone. The line types of the amplitude-acceleration curves for the two beams are the same as Fig. 3. Using these relationships, we can derive the following algorithm to calculate the frequency difference:

$$\begin{aligned}
 & \text{if } \begin{cases} A_{2-} < A_{2+} \\ A_{1-} > A_{1+} \end{cases} \text{ or } \begin{cases} A_{2+} > A_{1+} \\ A_{1-} > A_{2-} \end{cases} \text{ then } \Delta f = f_{1-} - f_{2+} \\
 & \text{if } \begin{cases} A_{2+} < A_{1+} \\ A_{1-} < A_{2-} \end{cases} \text{ or } \begin{cases} A_{2-} > A_{2+} \\ A_{1-} < A_{1+} \end{cases} \text{ then } \Delta f = f_{1+} - f_{2-}
 \end{aligned} \tag{28}$$

As seen in the figure, outside the lock-in zone, both dominant amplitudes have different associated frequencies for each beam. For negative accelerations, the dominant amplitude for beam 1 is A_{1-} with corresponding frequency f_{1-} and for beam 2 is A_{2+} with corresponding frequency f_{2+} . If we subtract the frequency of the second beam from that of the first beam, this leads to $\Delta f = f_{1-} - f_{2+}$. For positive accelerations, outside the lock-in range, the dominant amplitude for beam 1 is A_{1+} with corresponding frequency f_{1+} and for beam 2 is A_{2-} with corresponding frequency f_{2-} . Again, subtracting the frequency of the second beam from that of the first beam leads to $\Delta f = f_{1+} - f_{2-}$. Inside the lock-in zone, we see that the dominant amplitudes of both beams are associated with the same frequencies, whereby the difference will be zero. Therefore, inside the lock-in zone, we use the non-dominant frequencies to compute the frequency difference as follows: we compare the dominant amplitudes of both beams and if $A_{2+} > A_{1+}$ we choose the non-dominant frequency f_{1-} whereby $\Delta f = f_{1-} - f_{2+}$; otherwise we choose the non-dominant frequency f_{2-} whereby $\Delta f = f_{1+} - f_{2-}$.

It may be inferred from Fig. 11, that other algorithms can be defined by using the inequalities shown in the figure. Specifically, after determining the respective crossover points for each beam where the respective amplitudes (A^- , A^+) at both the first and second frequencies (f^- , f^+) are the same, the frequency-acceleration characteristic for the accelerometer can be determined by subjecting the accelerometer to a range of accelerations and measuring a corresponding differential frequency Δf equal to the difference in frequencies for each beam as follows:

- In a region bound by the crossover points, the differential frequency is set to a difference in the respective frequency for one beam where the amplitude is higher and the respective frequency for the other beam where the amplitude is lower.
- For all points outside the region bound by the crossover points the differential frequency is set to a difference in the respective frequency for each beam for which the amplitude is higher for both beams or lower for both beams.

Figs. 12 and 13 depict the frequency difference Δf as a function of the external acceleration Γ when Δf is computed according to the above procedure. The two curves are drawn to different scales, Fig. 13 in effect showing a magnified portion of Fig. 12 corresponding to a narrow range of accelerations between $-5 g$ and $+5 g$. It is seen that there is zero acceleration for which the measured frequency difference is zero. When the acceleration is zero, the frequency difference is discontinuous and non-linear but this, of course, does not adversely affect the accuracy of the accelerometer. It is also seen that a major part of the frequency-acceleration characteristic is approximately linear. Specifically, when the absolute value of differential frequency exceeds 200 Hz, corresponding to an acceleration of approximately $\pm 5 g$, acceleration is substantially a linear function of the differential frequency. This avoids the need to store the complete frequency-acceleration characteristic and facilitates fast computation of acceleration for much of the range once the differential frequency has been measured. Furthermore, if the measured amplitude corresponds to an acceleration outside of the lock-in zone, then the differential frequency can easily be calculated as the difference between the dominant frequencies of the two beams. In this case, determination of differential frequency and hence acceleration are fast and require very little processing.

A further enhancement of the invention relates to linearizing the frequency-acceleration characteristic shown in Fig. 12 so as to avoid the discontinuity at zero acceleration. Fig. 12 depicts the frequency difference $\Delta f(t)$ as a function of the external acceleration Γ as obtained using the algorithm described above. We can therefore re-write Eq. (13) while replacing ω by $2\pi f$ as follows:

$$\Delta f(\Gamma) = \begin{cases} -\sqrt{\frac{k_0}{2\pi^2 m_0}} \sqrt{1 + \frac{m_0}{M} - \sqrt{(1 - \lambda^2 \Gamma^2) \left(1 + 2\frac{m_0}{M}\right)}} & \text{for } \Gamma < 0 \\ 0 & \text{for } \Gamma = 0 \\ \sqrt{\frac{k_0}{2\pi^2 m_0}} \sqrt{1 + \frac{m_0}{M} - \sqrt{(1 - \lambda^2 \Gamma^2) \left(1 + 2\frac{m_0}{M}\right)}} & \text{for } \Gamma > 0 \end{cases} \quad (29)$$

As can be seen from Eq. (29) and Fig. 13, there is a discontinuity at zero acceleration, where the frequency difference is indeed zero. It is seen that while the lock-in problem has been minimized or even solved, there is a discontinuity and a high non-linearity of the frequency-acceleration relationship around zero. We now discuss a second embodiment that overcomes the non-linearity and discontinuity.

Preventing Discontinuity and Non-Linearity in the Lock-in Zone

The following analysis serves to obtain a continuous and a much more linear behavior inside the lock-in zone and also to improve the linearity outside the lock-in zone. We start by replacing ω by $2\pi f$ in Eq. (16) to obtain:

$$\Delta f(\Gamma) \approx \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}} \sqrt{\lambda^2 \Gamma^2 + \frac{m_0^2}{M^2}} \quad (30)$$

Eq. (30) can be written as:

$$\Delta^2 f(\Gamma) \approx \frac{1}{4\pi^2} \frac{k_0}{m_0} \left(\lambda^2 \Gamma^2 + \frac{m_0^2}{M^2} \right) \approx \frac{1}{4\pi^2} \frac{k_0}{m_0} \lambda^2 \Gamma^2 + \frac{1}{4\pi^2} \frac{k_0}{m_0} \frac{m_0^2}{M^2} \quad (31)$$

Fig. 14 shows graphically dominant and sub-dominant frequency versus acceleration for a single beam showing that the curves veer toward each other at zero acceleration and then separate. The function $\Delta^2 f(\Gamma)$ has a minimum when the acceleration, $\Gamma = 0$. We shall refer to this specific acceleration as the “Veering acceleration”, because it is at this acceleration that the curves veer toward each other, and denote it by Γ_v . More generally, we define the Veering acceleration Γ_v as the input acceleration for which $\Delta f(\Gamma)$ is minimal, as shown for example in Figs. 8 and 9. For the case of equal beams and vibration amplitudes, the Veering acceleration Γ_v is located mid-point between the respective accelerations Γ_1^{LI} and Γ_2^{LI} at opposite boundaries of the lock-in zone, one of which is negative while the other is positive, i.e. $\Gamma_v = \frac{1}{2}(\Gamma_1^{LI} + \Gamma_2^{LI})$. It will be recalled from Fig. 3 that Γ_1^{LI} and Γ_2^{LI} are the accelerations for which the respective amplitudes of both dominant and sub-dominant frequencies for each beam are equal. Therefore, Γ_v can be determined if the opposite boundaries of the lock-in zone are known. We call the minimum of $|\Delta f(\Gamma)|$ the “Veering frequency difference width” and we denote it by Δf_v , and therefore Eq. (30) leads to $\Delta f_v \approx \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}} \frac{m_0}{M}$ and Eq. (31) can now be rewritten as:

$$\Delta^2 f(\Gamma) \approx \frac{1}{4\pi^2} \frac{k_0}{m_0} \lambda^2 \Gamma^2 + \Delta^2 f_v \quad (32)$$

Reordering terms in Eq. (32) leads to:

$$\pm \sqrt{\Delta^2 f(\Gamma) - \Delta^2 f_v} \approx \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}} \lambda \Gamma \quad (33)$$

Finally we denote by $\hat{\Delta}f(\Gamma)$ the new linearized frequency-acceleration relationship, given by:

$$\hat{\Delta}f(\Gamma) = \pm \sqrt{\Delta^2 f(\Gamma) - \Delta^2 f_v} \approx \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}} \lambda \Gamma \quad (34)$$

where $\pm$ refers to positive/negative accelerations. In order to determine the sign of the external input acceleration we use the frequency difference i.e. $\Delta f(\Gamma)$. For example, when $\Delta f > 0$ the positive sign is chosen. Strikingly, Eq. (34) has a linear behavior with respect to Γ . We see from Fig. 11 that the sub-dominant amplitude for each beam is zero at the veering acceleration, such that both beams vibrate at a single frequency. This frequency will always be the dominant frequency, which may be f^+ or f^- depending on whether the two beams vibrate in phase or in anti-phase and since both beams vibrate at the dominant frequency, $\Delta f(\Gamma) = 0$.

Motivated by Eq. (34) we construct a linearization formula as follows:

$$\hat{\Delta}f(\Gamma) = \begin{cases} -\sqrt{(\Delta^2 f(\Gamma) - \Delta^2 f_v)} & \text{for } \Delta f < 0 \\ 0 & \text{for } \Delta f = 0 \\ \sqrt{(\Delta^2 f(\Gamma) - \Delta^2 f_v)} & \text{for } \Delta f > 0 \end{cases} \quad (35)$$

We have noted above that the Veering frequency difference width Δf_v is given by:

$$\Delta f_v \approx \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0} \frac{m_0}{M}}$$

Using the values specified previously, namely:

$$M = 2 \times 10^{-5} \text{ kg}, m_0 = 2 \times 10^{-8} \text{ kg}, k_0 = 3850 \text{ kg s}^{-2} \text{ gives:}$$

$$\Delta f_v \approx \frac{5}{\pi} \sqrt{1925} = 69.829 \text{ Hz}$$

As a first approximation, we can substitute this value in Eq. (35). However, it assumes that the values of M , m_0 and k_0 as provided in the manufacturer's specification are accurate. In practice, there are likely to be slight discrepancies owing to manufacturing tolerances, and it is therefore recommended to estimate the Veering frequency difference

width Δf_v for each accelerometer prior to use. This can be done by the manufacturer or the end-user, and we suggest two different approaches as follows:

i) From Figs. 12 and 13 it can be seen that $\Delta f(\Gamma)$ changes substantially linearly with acceleration outside the lock-in range but levels off to a non-zero value as the acceleration approaches zero. This is particularly noticeable in Fig. 12 which shows a magnified detail of the characteristic shown in Fig. 13. Measuring $\Delta f(\Gamma)$ close to the Veering acceleration Γ_v from either the left (negative accelerations) or from the right (positive accelerations), gives a good estimation of Δf_v . Thus, with reference to Fig. 12 it is seen that the curves intersect the frequency axis at $\Delta f(\Gamma) = 70$ Hz, which actually conforms very closely to the analytical value based on the nominal manufacturer's spec. In practice, once the differential frequency-acceleration characteristic is obtained, the Veering frequency difference can be measured on either side of the frequency axis i.e. for either negative or positive acceleration where it intersects the frequency axis. Alternatively, it can be measured on both sides of the frequency axis, and an average taken of the absolute values of the two measurements.

ii) A second method exploits the fact that by operating the accelerometer without correcting for lock-in with the beams vibrating first in-phase and secondly in anti-phase, it is possible to estimate Δf_v . Then, we first operate the accelerometer with the beams vibrating in-phase and measure the dominant frequency when the accelerometer is exposed to the Veering acceleration. The procedure is repeated with the beams vibrating in anti-phase. Since we are inside the lock-in zone, the vibration frequency of both beams will be the same in both cases. Finally Δf_v is determined as the difference between the measured in-phase and anti-phase frequencies. Figs. 15 and 16 show graphically dominant frequency versus acceleration for both beams when vibrated in-phase and anti-phase, respectively. It can be seen from Fig. 15 that the curves intersect the frequency axis at approximately 69,900 Hz for in-phase vibrations, while for anti-phase vibrations as shown in Fig. 16, they intersect the frequency axis at 69,830 Hz. In this particular example, the difference is again 70 Hz.

Example II:

We will apply the linear enhancement to the accelerometer of Example I having a seismic mass $M = 2 \times 10^{-5} \text{ kg}$, beams of mass $m_0 = 2 \times 10^{-8} \text{ kg}$, spring constant

$k_0 = 3850 \text{ kg s}^{-2}$, and $\lambda = \frac{0.0005}{g}$. The initial conditions are $y_1(0) = y_2(0) = 1\mu\text{m}$ and $y_3(0) = y_4(0) = 0$. We use Eq. (10) to calculate the output signals $y_1(t)$ and $y_2(t)$.

We first estimate the Veering frequency difference width by measuring the dominant frequencies when the accelerometer is operated in-phase and anti-phase as described above with reference to Figs. 15 and 16.

For this example $\Gamma_v = 0$ and from the data of Figs. 15 and 16 it can be estimated Δf_v as follows:

$$\Delta f_v = f_{in-phase}(\Gamma_v) - f_{anti-phase}(\Gamma_v) \quad (36)$$

This gives:

$$\Delta f_v = 69898.742 \text{ Hz} - 69828.948 \text{ Hz} = 69.794 \text{ Hz} \quad (37)$$

Using the data of Fig. 12 we calculate Eq. (35), then

$$\hat{\Delta f}(\Gamma) = \begin{cases} -\sqrt{(\Delta^2 f(\Gamma) - (69.794)^2)} & \text{for } \Delta f < 0 \\ 0 & \text{for } \Delta f = 0 \\ \sqrt{(\Delta^2 f(\Gamma) - (69.794)^2)} & \text{for } \Delta f > 0 \end{cases} \quad (38)$$

Fig 17 shows frequency-difference versus acceleration using the frequency-difference, $\Delta f(\Gamma)$, of Fig. 12 as compared with the enhanced linear relationship according to a second embodiment using the corrected frequency-difference, $\hat{\Delta f}(\Gamma)$, obtained from Eq. (38). For the avoidance of doubt, it is emphasized that in both embodiments, the frequencies depend on whether we are outside or inside the lock-in range. Outside the lock-in range, $\Delta f(\Gamma)$ is the difference between either the dominant or the sub-dominant frequencies for both beams, while inside the lock-in zone, it is the difference between the dominant frequency of one beam and the sub-dominant frequency for the other beam. In order to quantify the enhancement we perform a linear fitting of both curves of Fig. 17. The linear functions obtained from the fits are:

$$\begin{aligned} p_1(\Gamma) &= 35.422\Gamma + 4.585 \times 10^{-11} \\ p_2(\Gamma) &= 34.932\Gamma + 4.988 \times 10^{-11} \end{aligned} \quad (39)$$

where $p_1(\Gamma)$ and $p_2(\Gamma)$ correspond to the first embodiment and the enhanced linearized version, respectively. It is seen from Eq. (39) that the biases for both cases (4.585×10^{-11} and 4.988×10^{-11}) are almost zero in accordance with the obtained null

theoretical value. The respective scale factor for both cases is summarized in Table 2 below:

	Analytical estimation by Eq. (18)	First embodiment	Second embodiment
Scale Factor [Hz/g]	34.914	35.422	34.932

**Table 2: Comparison of scale factors with analytical value
for both embodiments**

5 The above table shows that the linearized differential frequency-acceleration curve according to the enhanced second embodiment has a scale factor value that is closer to the analytical value than is the scale factor of the first embodiment.

Fig. 19 shows graphically the deviation from linearity for both $\hat{\Delta}f(\Gamma)$ and $\Delta f(\Gamma)$ based on the data of Fig. 17, it being clearly seen that there is negligible deviation for $\hat{\Delta}f(\Gamma)$.

Table 3 shows the standard deviation for the data of Fig. 17. It is seen that the deviation for the linearized version is orders of magnitude smaller than the deviation obtained for the first embodiment.

	First embodiment	Second embodiment
Residual (Standard Deviation) [Hz/g]	16.74	0.0014

Practical Implementations

15 Our discussion so far has been largely analytical. We have shown that for a two-beam accelerometer or indeed any accelerometer that can be modelled mathematically in the form of a system having two beams operating in a differential push-pull mode, each beam will vibrate with two frequencies, one of which is of higher amplitude and to which we therefore refer as the dominant frequency; and the other of which is of lower amplitude and to which we refer as the non-dominant frequency. We have further established that there is a respective point for both beams during both negative and positive acceleration, where the amplitudes of both frequencies for each beam are identical. We call these the crossover points and they define the extent or boundary of the lock-in range. To avoid doubt, the amplitudes of the respective frequencies at the crossover points may be different for the two beams if the beams are not identical. But regardless of the physical

properties of the beams such as mass and stiffness, which may give rise to differences in the amplitudes of the two frequencies for each beam at their respective crossover points, it is always the case that the respective crossover points for both beams define opposite extremes of the lock-in range.

5 The above paragraph is merely a brief repetition of what has been described in detail with particular reference to Figs. 3 and 11 of the drawings. Specifically, determination of the lock-in range (or what might perhaps better, if not somewhat pedantically, may be referred to as the pseudo lock-in range) is one aspect of the present invention and in some applications is a prerequisite to correcting for the effect of frequency lock-in. In other
10 applications, the actual correction, which requires measurement of both dominant and non-dominant frequencies in the lock-in range may be independent of prior determination of the lock-in range. Furthermore, correction of the differential frequency within the lock-in range does not actually require prior knowledge of the boundary of the lock-in range. Specifically, all that is required is first to measure the dominant frequencies of both beams and compute
15 the difference Δf . If Δf is zero, this means that either the acceleration is zero or, more commonly, we are in the lock-in zone. In this case, we need to determine also the non-dominant frequency for one of the beams and compute Δf based on the difference between the respective dominant and non-dominant frequencies for the two beams, as explained previously.

20 In an alternative implementation, we can determine the crossover frequency as explained previously, i.e. the upper frequency where the dominant frequency intersects the boundary of the lock-in zone. Once we know this, we can assess based on whether at any given acceleration to be measured, the dominant frequency is larger or smaller than the crossover frequency within any specified tolerance threshold. The threshold establishes a
25 predetermined margin of error wherein the sub-dominant frequency is measured even if nominally the measured dominant frequency is larger than the crossover frequency. If it is larger, then we are outside of the lock-in range and acceleration will be a function of the difference between the respective dominant frequencies of the two beams. Only if the measured frequency is smaller than the crossover frequency, do we need to measure also
30 the sub-dominant frequency. In such an implementation, we can avoid measuring the sub-dominant frequency altogether if the measured dominant frequency is larger than the crossover frequency.

Such an implementation saves resources resulting in faster computation, but at the price that pre-calibration is required to determine the boundary of the lock-in range and crossover frequencies. This can be done by a manufacturer, for example, or by the end-user prior to actual use by subjecting the accelerometer to a range of accelerations and measuring
5 corresponding amplitudes such that for each beam there are corresponding first and second frequencies (f^- , f^+) whose amplitudes vary with acceleration at which the respective amplitude is dominant. The respective crossover points for each beam are determined where the respective amplitudes (A^- , A^+) at both the first and second frequencies (f^- , f^+) are the same. It will thus be apparent that pre-calibration to determine the boundary of the lock-in
10 zone serves to facilitate subsequent fast measurement of acceleration through the major part of the useful range of the accelerometer. Specifically, for all accelerations for which the corresponding dominant frequency of the two beams exceeds the crossover frequency, we do not need to measure the sub-dominant frequency for a calibrated accelerometer. Calibration thus serves a valuable, industrially applicable use that is independent of actual
15 use of the accelerometer.

Figs. 20 and 21 are flow charts showing alternative methods for determining the boundary of the lock-in zone during calibration of the accelerometer. In both cases, to simplify the flow charts only the positive boundary of the lock-in zone is calibrated and an identical procedure is performed to determine the negative boundary. Thus, in Fig. 20
20 the device is subjected to varying accelerations typically increasing positively from zero within a given range. The boundary is established by measuring the acceleration at which the amplitudes of the dominant and sub-dominant frequencies are equal.

In Fig. 21, the device is subjected to varying accelerations extending beyond the expected acceleration at the crossover boundary and decreasing in absolute value until
25 the respective dominant frequencies of both beams are first equal. At this point, we have hit the boundary of the lock-in zone. If we were to decrease positive acceleration or increase negative acceleration (i.e. make it less negative), we would remain inside the lock-in zone and hit the opposite boundary when the measured dominant frequencies of the two beams are different. We mention this as yet a further alternative approach that
30 could be adopted to determine the boundaries of the lock-in zone. In both cases, the crossover points correspond to the positive and negative accelerations having highest

maximum absolute value for which the dominant or measured vibration frequencies of the two beams are equal.

Finally, for the sake of completeness, although we have not discussed damping, it can be shown that the same principles apply also to damped oscillation of the accelerometer. In a practical system, damping will occur but the AC voltage 14 applied to the
5 respective pairs of electrodes 15 and 15' across the two beams may be adjusted using a feedback control to counter the effects of damping.

CLAIMS:

1. A method for determining a “lock-in zone” for a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that the two beams vibrate at frequencies that are indicative of acceleration, said method comprising:
- 5
- (a) obtaining for each of the beams a respective pair of amplitude-acceleration characteristics by subjecting the accelerometer to a range of accelerations and measuring corresponding amplitudes such that for each beam there are corresponding first and second frequencies (f^- , f^+) at which the respective amplitude is dominant at different accelerations throughout said range; and
 - 10 (b) determining respective crossover points for each beam where the respective amplitudes (A^- , A^+) at both the first and second frequencies (f^- , f^+) for the respective beam are the same and which define opposing boundaries of the lock-in zone within which the respective dominant amplitudes of the first and second frequencies cross over such that the frequency for which amplitude is higher outside the crossover region has a smaller amplitude within the crossover region and *vice versa*.
 - 15
2. A method for calibrating a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that the two beams vibrate at frequencies that are indicative of acceleration, said method comprising:
- 20
- (a) obtaining for each of the beams a respective pair of amplitude-acceleration characteristics by subjecting the accelerometer to a range of known accelerations and measuring corresponding signal amplitudes at each acceleration such that for each beam there are corresponding first and second frequencies (f^- , f^+) at which the respective amplitude is dominant at different accelerations throughout said range; and
 - 25 (b) determining respective crossover points for each beam where at each crossover point the respective amplitudes (A^- , A^+) at both the first and second frequencies (f^- , f^+) are the same.
 - 30

3. A method for calibrating a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that the two beams vibrate at frequencies that are indicative of acceleration, said method comprising:
- 5 (a) subjecting the accelerometer to a range of known positive and negative accelerations such that for each beam there are corresponding first and second frequencies (f^- , f^+) such that for any given acceleration one is dominant and one is sub-dominant;
- (b) for each known acceleration measuring the respective dominant frequency of
10 each beam; and
- (c) determining the respective crossover points at which the dominant vibration frequencies of the two beams are equal and the absolute values of positive and negative accelerations for both beams are maximum.
4. The method according to claim 2 or 3, further including:
- 15 (d) obtaining a frequency-acceleration characteristic for the accelerometer by subjecting the accelerometer to a range of accelerations and measuring a corresponding differential frequency Δf equal to the difference in frequencies for each beam such that in a region bound by said crossover points the differential frequency is set to a difference in the respective frequency for one
20 beam where the amplitude is higher and the respective frequency for the other beam where the amplitude is lower and for all points outside the region bound by the crossover points the differential frequency is set to a difference in the respective frequency for each beam for which the amplitude is higher for both beams or lower for both beams.
- 25 5. The method according to claim 4, further including approximating at least part of the pre-calibrated differential frequency-acceleration characteristic to a linear function within a specified range of differential frequencies.
6. A method for determining acceleration using a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential
30 push-pull mode, such that each of the two beams vibrates at corresponding first and second frequencies (f^- , f^+) depending on whether the respective beam is in compression or

tension, and wherein the vibrating beam accelerometer is pre-calibrated using the method of claim 2 or 3, said method comprising:

- (a) subjecting the accelerometer to an acceleration and measuring the frequency of vibration for each beam;
- 5 (b) if the frequency exceeds a crossover frequency corresponding to the dominant frequency of the two beams at the crossover points plus a predetermined margin of error, measuring a corresponding differential frequency Δf equal to a difference in the respective frequency for each beam for which the amplitude is higher for both beams or lower for both beams;
- 10 (c) if the frequency is smaller than the crossover frequency, then measuring a corresponding differential frequency Δf equal to a difference in the respective frequency for one beam where the amplitude is higher and the respective frequency for the other beam where the amplitude is lower; and
- (d) determining acceleration from the measured differential frequency Δf and a
15 pre-calibrated frequency-acceleration characteristic.

7. A method for determining acceleration using a vibrating beam accelerometer that can be modelled mathematically as a system having two beams operating in a differential push-pull mode, such that each of the two beams vibrates at corresponding first and second frequencies (f^- , f^+) depending on whether the respective beam is in compression or
20 tension, said method comprising:

- (a) subjecting the accelerometer to an acceleration and measuring the respective dominant frequency of vibration for each beam;
- (b) if the measured dominant frequencies of the two beams are equal within a predetermined margin of error:
 - 25 i) determining a corresponding sub-dominant frequency of vibration for each beam;
 - ii) measuring a differential frequency Δf equal to a difference in the respective frequency for one beam where the frequency is dominant and the respective frequency for the other beam where the frequency is sub-
30 dominant; and
- (c) if the measured dominant frequencies of the two beams are unequal within a predetermined margin of error:

- 5 i) measuring differential frequency Δf equal to a difference in the
 respective frequency for each beam where the frequency is dominant
 for both beams or sub-dominant for both beams; and
- (d) determining acceleration from the measured differential frequency Δf and a
 pre-calibrated frequency-acceleration characteristic.

8. The method for determining acceleration according to claim 6 or 7, wherein deter-
 mining acceleration includes:

- 10 i) approximating at least part of the pre-calibrated differential frequency-
 acceleration characteristic to a linear function within a specified range
 of differential frequencies; and
- ii) if the differential frequency falls within the specified range, computing
 the acceleration using the linear function.

9. The method according to any one of claims 5 to 8, wherein the linear function is:

$$\Delta f \approx \lambda \cdot \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}} \Gamma$$

- 15 where: Δf = measured frequency difference,
 Γ = acceleration,
 m_0 = mass of each beam,
 k_0 = stiffness of each beam at zero acceleration,
 λ = a known scale factor.

20 10. The method according to any one of claims 6 to 9, wherein the differential-
 frequency-acceleration characteristic is a function $\hat{\Delta f}(\Gamma)$ of differential frequency Δf
 given by:

$$\hat{\Delta f}(\Gamma) = \begin{cases} -\sqrt{(\Delta^2 f(\Gamma) - \Delta^2 f_v)} & \text{for } \Delta f < 0 \\ 0 & \text{for } \Delta f = 0 \\ \sqrt{(\Delta^2 f(\Gamma) - \Delta^2 f_v)} & \text{for } \Delta f > 0 \end{cases}$$

25 where: Δf_v is the minimal absolute difference between the dominant and sub-dominant
 frequency of each of the two beams.

11. The method according to claim 10, where Δf_v is estimated by determining from differential-frequency (Δf) versus acceleration (Γ) when a measured rate of change of differential-frequency (Δf) with acceleration (Γ) is minimal.

12. The method according to claim 11, wherein respective measurements are made
5 for both negative and positive accelerations and Δf_v is set to an average of the absolute values of the two measurements.

13. The method according to claim 10, where Δf_v is estimated by measuring the dominant frequencies of both beams when the accelerometer is operated both in-phase and anti-phase, and computing Δf_v as:

$$\Delta f_v = f_{in-phase}(\Gamma_v) - f_{anti-phase}(\Gamma_v)$$

10 14. The method according to claim 10, where Δf_v is computed analytically as:

$$\Delta f_v = \frac{1}{2\pi} \sqrt{\frac{k_0}{m_0}} \cdot \frac{m_0}{M}$$

where: where: Δf = measured frequency difference,

m_0 = mass of each beam,

k_0 = stiffness of each beam at zero acceleration,

15 λ = a known scale factor, and

M is the mass of the seismic mass.