

"A digital communication system and a receiver for use in such a system"

REPLACED SHEETS		SG
☒	Description : paged ...1... to paged ...9...	
☒	Claim(s) : paged ...10... to paged ...12...	
☒	Drawing(s) : paged ...13... to paged ...23...	
☒	Abstract : paged ...24... to paged ...25...	
DATE OF RECEIPT		

This Sheet Has Been Replaced



The present invention relates to a digital communication system comprising a transmitter for transmitting digital messages representing symbols modulated onto at least one carrier, which symbols are representable as symbol vectors in a signal constellation which is rotationally invariant over a predetermined rotation angle, the system further comprising a transmission channel and a receiver for receiving the symbols transmitted through the channel, which receiver comprises a demodulator for demodulating the received symbols into received messages, the demodulator using an estimate for a channel state of the channel. Such a system can be a Digital Television Broadcasting System in which a digital signal is broadcasted to digital TV receivers, or any other digital communication system.

The present invention further relates to a receiver for use in such a system.

A system of this kind is known from the International Patent Application WO 92/05646. In this system OFDM (Orthogonal Frequency Division Multiplexed) signals are transmitted by a transmitter to a receiver through a transmission channel. On each carrier of this multi-carrier system, symbols to be transmitted are QPSK (Quadrature Phase Shift Keying) modulated. An OFDM signal is obtained by supplying a QPSK symbol stream to an Inverse Discrete Fourier Transformer. The QPSK signals can be considered as two bit binary messages representing symbols on a carrier. The symbols can be represented as symbol vectors in a signal constellation which is rotationally invariant over a predetermined angle. In the example given the signal configuration consists of four constellation points, representing two bit binary messages "00", "10", "11", and "10". In case of differential encoding, starting from an initial symbol state, the messages are encoded as a phase difference between two successive symbols. Then, in the receiver, it is not necessary to know the exact phase, but, starting from a first detected symbol, the succeeding symbols are detected from relative phases of symbols. The transmitted symbols are subject to noise and to channel distortion. Therefore, the received symbols are amplitude and phase shift modulated QPSK signals. In a



complex plane, the received symbols are scattered around four points of a four point signal constellation, or, stated otherwise, when many symbols are received the received symbols can be plotted as four clouds of end points of received symbol vectors in the complex plane. Before proper demodulation, a position of two notional perpendicular axes, which are symmetrical to centres of the signal clouds, must be found. For this reason, in the known receiver, a separate phase recovery method is disclosed for each subcarrier of the received OFDM signal. In this method, with demodulation, a channel state is estimated, which varies with time. More particularly a channel phase shift is estimated recursively, i.e. with each of the received symbols, the position of the notional axes with respect to the symbol clouds is updated. Starting with a phase reference signal which is set equal to the first received signal, a reference phase which is set equal to an argument of the phase reference signal, and notional axes at 45° and 135° to the signal vector, a next received symbol is rotated with a multiple of 90° such that it falls within 45° sectors at both sides of the reference phase. I.e., a minimum squared Euclidean distance is determined, when detecting the symbol. From the number of 90° rotations, the transmitted binary message can be detected. The phase reference signal is updated with the received rotated vector, and, consequently, the reference phase. This method is shown to be Maximum Likelihood in most practical cases. In digital communication systems, also symbols representable by other signal constellations are transmitted. E.g. for Digital Video Broadcasting a so-called 2R(2,2)-QAM constellation can be used. Such a constellation is a 2-resolution configuration, i.e. a constellation representing symbols of which a first group of 2 bits for encoding the symbols represents constellation points in respective quadrants of the signal constellation, and of which a second group of 2 bits for encoding the symbols represents constellation points within the quadrants. The first group of bits are so-called high priority bits, and the second group of bits are so-called low priority bits. When symbols representing video signals are transmitted using such a 2R(2,2)-QAM signal constellation, at receiver side, depending on the signal-to-noise ratio, the video signals can be detected with low resolution by only detecting the high priority bits and the video signals can be detected with high resolution by also detecting the low priority bits. The 2R(2,2)-QAM constellation is rotationally invariant as to rotation of clusters of constellation points in the respective quadrants over a multiple of 90° . In this sense, also other signal constellations, which can be multiresolution or not, are rotationally invariant, such as a 64-QAM constellation. Such signal constellations is described in the handbook "Digital Communication", E.A. Lee et al., Kluwer Academic Publishers, 1990, Chapter 6.5. "Signal Set Design", pages 198-206. In the same handbook transmitters and receivers are described

for transmitting and receiving of symbols from such constellations.

This Sheet Has Been Replaced



It is an object of the present invention to provide a digital communication system in which received symbols representable by rotationally invariant signal constellations can be easily detected.

To this end the digital telecommunication system according to the present invention is characterized in that the signal constellation is partitioned into clusters of constellation points which coincide when at least once rotated over the predetermined rotation angle, the clusters representing a first group of bits in the messages and constellation points within the clusters representing at least a second group of bits in the messages, and in that the first group of bits is demodulated by rotating a cluster containing a received symbol over an integer number of times of the predetermined rotation angle such that the received symbol is located in a partition of the signal constellation containing the estimate for the channel state and by determining the first group of bits from the rotation, and the second group of bits is demodulated by translating the rotated cluster to an origin of the signal constellation and by determining the second group of bits from the translated cluster, the estimate being updated with each received symbol. By rotating the cluster containing the received symbol to the partition containing the channel state estimate and by updating the channel state estimate with each received symbol, information about the first group of bits is achieved. The channel state estimate can be determined as according to the method disclosed in WO 92/05646, then, or by subjecting the received signal to a M^{th} -order power loop, known per se, for a $360^\circ/M$ rotationally invariant constellation, M being a positive integer greater than 2. By translating the rotated cluster to the origin of the signal configuration, information about the second group of bits is obtained. After translation to the origin, the method disclosed in WO 92/05646 can be used for obtaining the information about the second group of bits. The estimate for the channel state can be fine adjusted by using amplitude and phase information about the thus recovered symbol. Accordingly, a very simple detection mechanism is obtained.

The present invention is based upon the insight that with detection of the second group of bits, the phase information in the estimate for the channel state is preserved when translating a cluster to the origin of the signal constellation.

In an embodiment of a digital communication system according to the present invention the constellation points within the clusters represent a third group of bits

within the second group of bits, the second group of bits being representative for a rotationally invariant further cluster within the cluster, and wherein the third group of bits is demodulated as according to the second group of bits. By translating the third group of bits to the origin of the signal constellation in two steps, first the rotationally invariant cluster, and then the further cluster within the cluster, the same simple symbol recovery can be applied as for the first group of bits, and the second group of bits. The clusters within clusters are rotationally invariant after translation to the origin. For still more complicated signal configurations the same detection mechanism can be applied. The constellations can then be made to be beyond 2-resolution. Depending on the signal-to-noise ratio, the receiver can decide up to which resolution the symbols are detected. E.g., when too high a BER (Bit Error Rate) is observed in the receiver for a particular resolution, the receiver can only detect the symbols with a lower resolution.

In an embodiment of a digital communication system according to the present invention a phase reference signal is recursively determined from the rotated received symbols, and a centre of the rotated cluster is estimated by dividing the phase reference signal through the number of received symbols, the rotated cluster being translated through vectorial subtraction of a currently rotated received symbol and the cluster constellation points with the estimated centre. It is thus achieved that the translation vector is updated with each received vector, resulting in an accurate translation with each received symbol.

In a further embodiment of a digital communication system according to the present invention the second group of bits is determined from the rotated and translated cluster by determining a minimum squared Euclidean distance between the rotated and translated received symbol and the constellation points of the rotated and translated cluster. Thus, a simple decision criterion is obtained for detecting the second group of bits. For a (2,2)-QAM system, i.e. a signal constellation with 16 constellation points, a minimum Euclidean distance is achieved by determining whether a rotated and translated symbol is located in a sector of two notional axes which form 45° angles with the determined reference phase.

In a further embodiment of a digital communication system according to the present invention the phase reference signal is updated for each received symbol by subtracting therefrom a vector corresponding to the determined second group of bits. Herewith the estimate for the channel state is both updated with phase and amplitude information of the received symbols. With this fine phase and amplitude recovery an accurate reference phase is obtained.

This Sheet Has Been Replaced





Further embodiments are claimed in the dependent claims. When the first group of bits is non-differentially encoded, from the received a priori known reference symbol an initial decoding value is derived. When differential encoding for the first group of bits is applied, the first received symbol, not necessarily known a priori, serves as a
5 reference symbol for detecting successive symbols.

The present invention will now be described, by way of example, with reference to the accompanying drawings, wherein

10 Fig. 1, shows a digital communication system according to the present invention,

Fig. 2 shows a 2R(2,2)-QAM signal constellation,

Fig. 3 shows detection of a received symbol in a receiver in a system according to the present invention, and

15 Fig. 4 shows an embodiment of a demodulator according to the present invention.

Throughout the figures, the same reference numerals are used for the same features.

20 Fig. 1 shows a digital communication system 1 according to the present invention comprising a transmitter 2 for transmitting digital messages representing complex symbols s_i to a receiver 3 via a transmission channel 4. The digital messages can be binary messages but also another logic is possible such as ternary and quaternary logic, for instance.
25 The symbols s_i are modulated onto a carrier. The modulated symbols are quadrature modulated 2R(2,2)-QAM signals, i.e. the symbols from a complex-valued alphabet are representable as symbol vectors in a signal constellation. For allowing two-resolution detection, in the signal constellation, constellation points are non-uniformly spaced. In each of the four quadrants of the signal constellation, four constellation points are spaced
30 relatively close together, whereas the so formed clusters of constellation points are spaced further apart from each other. For further details with respect to signal constellations referred is to said Chapter 6.5 of said handbook of Lee et al. Modulated is a source bit stream B_k , which is first coded by means of a coder 5, in an embodiment, a first group of two bits, or the so-called HP (high priority) bits, being differentially encoded, and a second group of



bits, or the so-called LP (low priority) bits, being non-differentially encoded. As can be seen in Fig. 2, the HP-bits in the respective quadrants are "00", "01", "11", and "10", the bits "00" being in the first quadrant, and the LP-bits "00", "01", "11", and "10" being rotationally invariant as to the four quadrants. In the signal constellation, a cluster distance d_b , and a distance d_g between constellation points within a cluster CL are indicated. The coder 5 supplies two symbol sequences $\text{Re}\{s_i\}$ and $\text{Im}\{s_i\}$ to respective transmit filters 6 and 7, of which outputs are fed to multipliers 8 and 9 which multiply the respective output signals of the filters 8 and 9 with quadrature carriers $\cos(\omega_c t)$ and $\sin(\omega_c t)$, ω_c being the carrier frequency and t being time. Such a transmitter 2 is well-known in the art. Referred is for example to said handbook of Lee, pages 167-172. The symbols can also be modulated onto multiple carriers as so-called OFDM signals (Orthogonal Frequency Division Multiplexed) signals, such as described in said Patent Application WO 92/05646. Then, the output of the coder 5 is subjected to an N-points Inverse Discrete Fourier Transform, before modulation onto a carrier, N being an integer. At receiver side, the present invention is then applied to each subcarrier of the OFDM signal. The carrier modulated in phase and quadrature signals are combined in an adder 10, and then transmitted to the receiver 3 via the transmission channel 4. The receiver 3 receives a signal which is distorted by the channel 4, i.e. the received signal is an phase and amplitude modulated version of the transmitted signal, further disturbed with noise. It is assumed that the noise is AWGN (Additive White Gaussian Noise). In the receiver 3, the received signal is first subjected to carrier demodulation with respective multipliers 11 and 12, followed by respective receive filters 13 and 14. At outputs of the receive filters 13 and 14 a complex carrier demodulated signal r_i is available in the form of a real signal part $\text{Re}\{r_i\}$ and imaginary signal part $\text{Im}\{r_i\}$. The signal r_i can thus be represented as a signal vector in a signal constellation. Such a receiver front end is well-known and for further details as to this kind of receivers, referred is to said handbook of Lee et al., pages 184-186. The signal parts $\text{Re}\{r_i\}$ and $\text{Im}\{r_i\}$ are sampled by means of respective analog-to-digital converters 15 and 16, digital output signals of which are supplied to a demodulator 17 which comprises a signal processor 18. The actual demodulation is carried out by the signal processor 18, which is programmed to implement the present invention, and which contains non-volatile memory for storing the programs for implementing the present invention, and volatile memory for storing program variables. At an output of the digital signal processor 18, a bitstream comprising the HP-bits and the LP-bits is available.

Fig. 3 shows detection of the received symbol r_i in the receiver 3 in the



system 1 according to the present invention by means of the digital signal processor 18, the symbol r_i originating from the transmitted symbol s_i representable in a 2R(2,2)-QAM signal constellation according to Fig. 2. For detection of the HP-bits, the channel state can be estimated using the method as disclosed in WO 92/05646, or alternatively with a M^{th} order power loop, known per se, for a $360^\circ/M$ rotationally invariant constellation, M being a positive integer value greater than 2. When e.g. applying a 4^{th} order power loop, a phase ambiguity being a multiple of $\pi/2$ is eliminated by first multiplying the argument of the received symbol by four and then dividing the result by four, after first having eliminated a modulo 2π phase from the multiplied argument. With HP-bit detection, the clusters of signal constellation points are considered as single constellation points. The 2R(2,2)-QAM constellation is then considered as a QPSK constellation, corresponding to 4 point signal constellation. At a time instant i , the complex 4 bit encoded symbol r_i , 2 HP-bits and 2 LP-bits, is received, the symbol r_i corresponding to the i th transmitted symbol s_i . After demodulation of the symbol r_i , an updated reference phase ϕ_{i+1} is determined, being an argument of a recurrently determined phase reference signal f_i . Starting with the first received symbol r_1 , which is an a priori known reference symbol in case the HP-bits are non-differentially encoded, initial values $f_1 = r_1$, and $\phi_2 = \arg r_1$, $\arg$ being the argument. Then, with the second received symbol r_2 , and accordingly with further received symbols, the cluster containing the received symbol r_i is rotated over $k_i \cdot \pi/2$, k_i containing information about the HP-bits; such that the rotated vector b_i is within sectors of 45° with respect to ϕ_i . In the given 2R(2,2)-QAM configuration, k_i is an element of the set $\{0, 1, 2, 3\}$. In case of differential encoding of the HP-bits, the HP-bits are encoded from a phase difference $\Delta\varphi$ of the received symbols r_i and r_{i-1} , in the given example a phase difference 0 corresponding to the bits "00", a phase difference $\pi/2$ corresponding to the bits "01", a phase difference π corresponding to the bits "11", and a phase difference $3\pi/2$ corresponding to the bits "10", the phase difference $\Delta\varphi$ being $\{(2\pi - k_i \cdot \pi/2) - (2\pi - k_{i-1} \cdot \pi/2) \text{ modulo } 2\pi\}$. In case of non-differential encoding, a priori known reference symbols, e.g. r_1 , determine the absolute decoding reference in the signal constellation, e.g. a reference symbol "00" corresponds to $k_1 = 0$, herewith determining decoding as to the other three constellation points. The thus rotated cluster and rotated received symbol b_i , in the example given by $\pi/2$ corresponding to $k_i = 1$, contains information about the two LP-bits. According to the present invention, based upon the reference phase estimate ϕ_i , a four point cluster or constellation CL_x is constructed with a centre point x_c , for which the following relations hold:

$$x_{00} = \{(d_b + 2d_g)/\sqrt{2}\} \angle \phi, \angle \text{ being the angle in the complex plane,}$$

$$\begin{aligned}
 x_{01} &= \{((d_b + (1+j)d_g)/\sqrt{2}) \angle \phi, j=\sqrt{-1}, \\
 x_{10} &= \{((d_b + (1-j)d_g)/\sqrt{2}) \angle \phi, \\
 x_{11} &= \{d_b/\sqrt{2} \angle \phi, \text{ and} \\
 x_c &= \{d_b/\sqrt{2} \angle \phi.
 \end{aligned}$$

This Sheet Has Been Replaced



- 5 According to the present invention, the cluster CL_x and the rotated received signal b_i are translated to the origin O of the complex plane, indicated with TR. The translation TR is implemented by means of vectorial subtraction, i.e. by determining $b_i' = b_i - x_c$. Then, for demodulating the LP-bits, the method disclosed in WO 92/05646 for detecting the HP-bits is applied to the rotated and translated cluster containing the LP-bits, i.e. in $b_i'' = b_i'$
- 10 $\angle k_i' \cdot \pi/2$, k_i' is determined such that the most likely transmitted symbol x_i has minimum squared Euclidean distance E_d with b_i . For $k_i' = 0$, LP-bits "00" are detected, for $k_i' = 1$, LP-bits "10" are detected, for $k_i' = 2$, LP-bits "11" are detected, and for $k_i' = 3$, LP-bits "01" are detected. Using x_i , an estimated centre a_i of the cluster b_i belongs to, is determined as $a_i = b_i + (x_c - x_i)$. Then, the phase reference signal is updated in the recurrent relation f_i
- 15 $= f_{i-1} + a_i$. The reference phase ϕ_i is updated as $\phi_{i+1} = \arg f_i$. Using a_i , in addition to b_i as used with demodulation of an QPSK signal only or an assumed QPSK signal, gives a more accurate estimate of ϕ_{i+1} .

- In case of QAM signalling, the average amplitude of the received signals r_i is also estimated. Then, the centre x_c of the cluster CL_x is estimated from the phase reference signal f_i as f_i/i .
- 20 The determination of f_i and the estimation of x_c can be made adaptive to tackle small time variances in ϕ , by alternatively determining $f_i = \alpha \cdot f_{i-1} + (1-\alpha) \cdot a_i$ for $i > 1$ with $f_1 = (1-\alpha) \cdot r_1$, in which the centre x_c of the cluster CL_x is estimated as $f_i/(1-\alpha^i)$, α being in the interval (0,1). For $\alpha = 0$, non-coherent reception is provided, whereas for α approaching 1 coherent reception is provided. Such an adaptation can be necessary if the phase recovery is
- 25 disturbed by channel properties such as multipath fading and frequency instabilities or incorrect frequency adjustment at in the receiver 3. Then, ϕ is a time varying function, it being assumed that ϕ is constant during transmission of the symbol s_i . The parameter α determines the adaptation speed of ϕ . With decreasing α , the adaptation speed increases, but then, the BER (Bit Error Rate) also increases. Adaptation also has the advantage that
- 30 overflow in f_i registers is avoided during calculation. The phase reference signal f_i not necessarily has to be determined recursively, but can also be determined from a number n of previously determined values of b_i or a_i , according to the formula $f_i = \sum b_j$, $\sum$ being a summing sign, the summing being taken from $j = i-n+1$ to $j = i$, j and i being integer values. Then, again, $\phi_i = \arg f_i$. E.g., $n = 10$. The previous values are stored in a shift

register (not shown), the oldest value being shifted out, with a new value each time. The present invention can be recursively applied for clusters within clusters, for signal constellations in which clusters itself form rotationally invariant signal constellations.

Although the present invention is preferably applied to multi-resolution signal constellations for which there are clear signal-to-noise differences as to respective resolutions, the present invention can also be used for single resolution systems. 2-Resolution can be applied, for instance, for Digital Video Broadcasting. For portable receivers only the HP-bits can be detected, and for fixed receivers, the LP-bits can also be detected, i.e. a normal video mode, and an enhanced video mode is provided, then.

Fig. 4 shows an embodiment of the demodulator 17 according to the present invention for a 2R(2,2)-QAM signal constellation, implemented with the signal processor 18. In a computation block 40, the signal b_i determined which is within 45° sectors with respect to ϕ_i , and from k_i thus determined, the HP-bits are determined. For determining k_i , rotation blocks 41, 42, and 43 are provided for respective rotation of the received signal r_i with $\pi/2$, π , and $3\pi/2$. In a computation block 44 the LP-bits are determined, together with the most likely transmitted symbol x_i . Inputs to the computation block 44 for the LP-bits and x_i are respective computation blocks 45, 46, 47, and 48 for x_{00} , x_{01} , x_{10} , and x_{11} , according to the formulas given. In a summing block 50, to which x_i , b_i , and further an output of a computation block 49 for computing x_c are supplied, the variable a_i is determined. For adaptive calculation of f_i and thus of ϕ_{i+1} , multiplying blocks 51 and 52, a summing block 53, and a block 54 for determining the argument of f_i are provided. In case of estimation of x_c a division block is provided for calculating f_i/i , or, in case of adaptivity, $f_i/(1-\alpha^i)$, instead of the calculation block 49.

This Sheet Has Been Replaced



CLAIMS:

This Sheet Has Been Replaced



1. A digital communication system (1) comprising a transmitter (2) for transmitting digital messages representing symbols (s_i) modulated onto at least one carrier, which symbols (s_i) are representable as symbol vectors in a signal constellation which is rotationally invariant over a predetermined rotation angle, the system further comprising a transmission channel (4) and a receiver (3) for receiving the symbols transmitted through the channel, which receiver (3) comprises a demodulator (17) for demodulating the received symbols (r_i) into received messages (HP-bits, LP-bits), the demodulator (17) using an estimate for a channel state of the channel (4), characterized in that the signal constellation is partitioned into clusters (CL) of constellation points which coincide when at least once rotated over the predetermined rotation angle, the clusters representing a first group of bits (HP-bits) in the messages and constellation points within the clusters representing at least a second group of bits (LP-bits) in the messages, and in that the first group of bits (HP-bits) is demodulated by rotating a cluster containing a received symbol (r_i) over an integer number of times of the predetermined rotation angle such that the received symbol (r_i) is located in a partition of the signal constellation containing the estimate (ϕ_i) for the channel state and by determining the first group of bits (HP-bits) from the rotation, and the second group of bits (LP-bits) is demodulated by translating (TR) the rotated cluster (CL_x) to an origin (O) of the signal constellation and by determining the second group of bits (LP-bits) from the translated cluster, the estimate being updated with each received symbol (r_i).
2. A digital communication system (1) according to claim 1, wherein the constellation points within the clusters represent a third group of bits within the second group of bits, the second group of bits being representative for a rotationally invariant further cluster within the cluster, and wherein the third group of bits is demodulated as according to the second group of bits.
3. A digital communication system (1) according to claims 1 or 2, wherein a phase reference signal (f_i) is recursively determined from the rotated received symbols (b_i), and a centre (x_c) of the rotated cluster is estimated by dividing the phase reference signal (f_i) through the number of received symbols (i), the rotated cluster being translated (TR) through vectorial subtraction of a currently rotated received symbol (b_i) and the cluster constellation





points with the estimated centre (x_c).

4. A digital communication system (1) according to claim 3, wherein the second group of bits (LP-bits) is determined from the rotated and translated cluster by determining a minimum squared Euclidean distance (E_d) between the rotated and translated received symbol and the constellation points of the rotated and translated cluster.

5. A digital communication system (1) according to claim 4, wherein the phase reference signal (f_i) is updated for each received symbol (r_i) by subtracting therefrom a vector (a_i) corresponding to the determined second group of bits.

6. A digital communication system (1) according to claims 3, 4, or 5, wherein after determining the second group of bits, a reference phase (ϕ_{i+1}) for the signal constellation is determined from an argument of the phase reference signal (f_i).

7. A digital communication system (1) according to any one of claims 1 to 6, wherein the signal constellation is a multi-resolution constellation (2R(2,2)-QAM).

8. A digital communication system (1) according to any one of the claims 1 to 7, wherein the first group of bits (HP-bits) are differentially encoded.

9. A digital communication system (1) according to any one of the claims 1 to 7, wherein the first group of bits (HP-bits) are non-differentially encoded, and the transmitted symbols (s_i) comprise at least one reference symbol, which is a priori known in the receiver.

10. A receiver (3) for receiving digital messages representing symbols (s_i) modulated onto at least one carrier, which symbols (s_i) are representable as symbol vectors in a signal constellation which is rotationally invariant over a predetermined rotation angle, which receiver (3) comprises a demodulator (17) for demodulating the received symbols (r_i) into received messages (HP-bits, LP-bits), the demodulator (17) using an estimate for a channel state of a channel through which the symbols are transmitted, characterized in that the signal constellation is partitioned into clusters of constellation points which coincide when at least once rotated over an integer number of times of the predetermined rotation angle, the clusters representing a first group of bits (HP-bits) in the messages and constellation points within the clusters representing at least a second group of bits (LP-bits) in the messages, and in that the first group of bits (HP-bits) is demodulated by rotating a cluster containing a received symbol (r_i) over the predetermined rotation angle such that the received symbol (r_i) is located in a partition of the signal constellation containing the estimate (ϕ_i) for the channel state and by determining the first group of bits (HP-bits) from the rotation, and the second group of bits (LP-bits) is demodulated by translating (TR) the rotated cluster to an origin (O)



of the signal constellation and by determining the second group of bits (LP-bits) from the translated cluster, the estimate (ϕ_{i+1}) being updated with each received symbol (r_i).

This Sheet Has Been Replaced

