

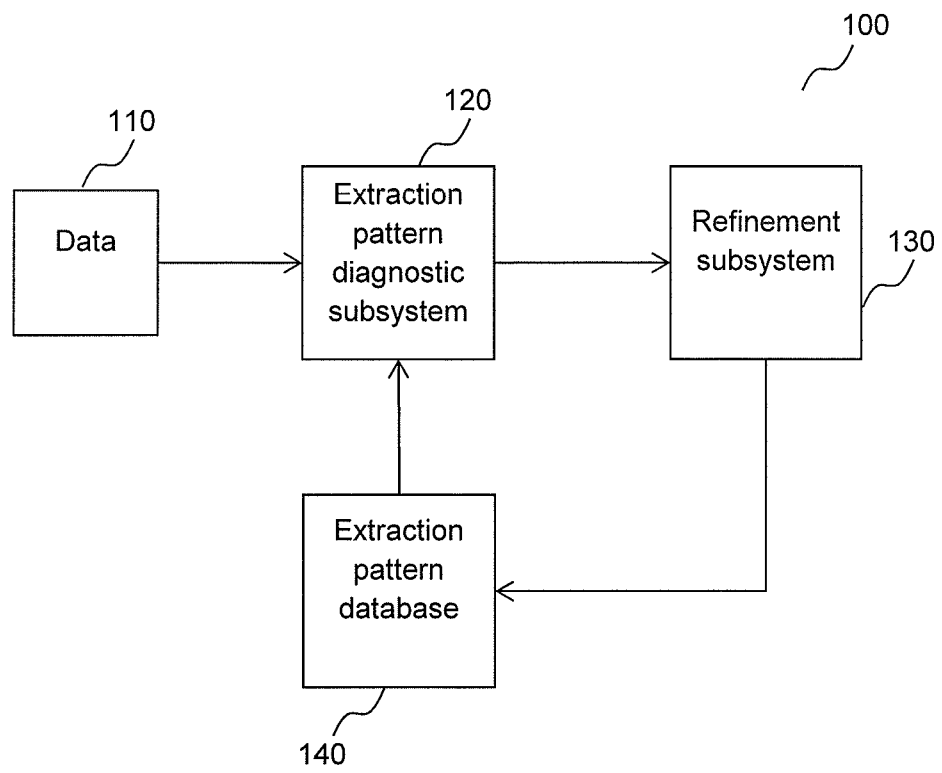


Figure 1

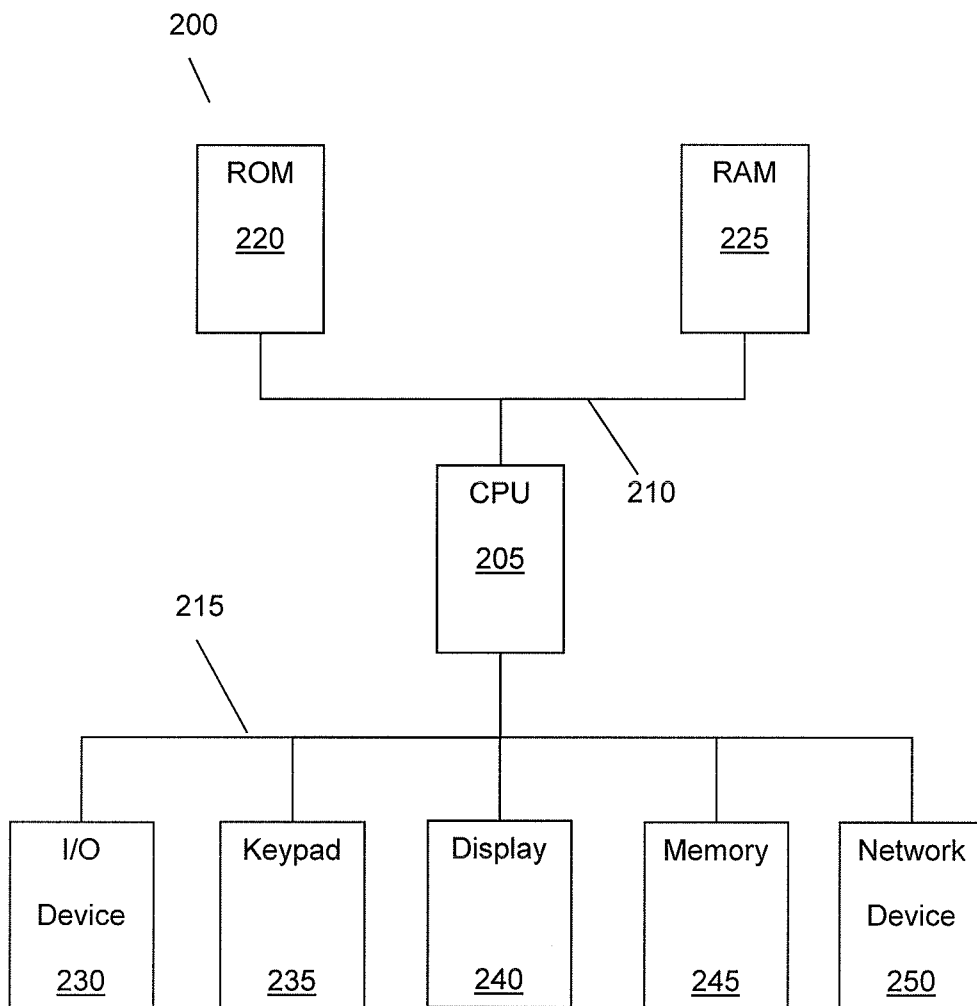


Figure 2

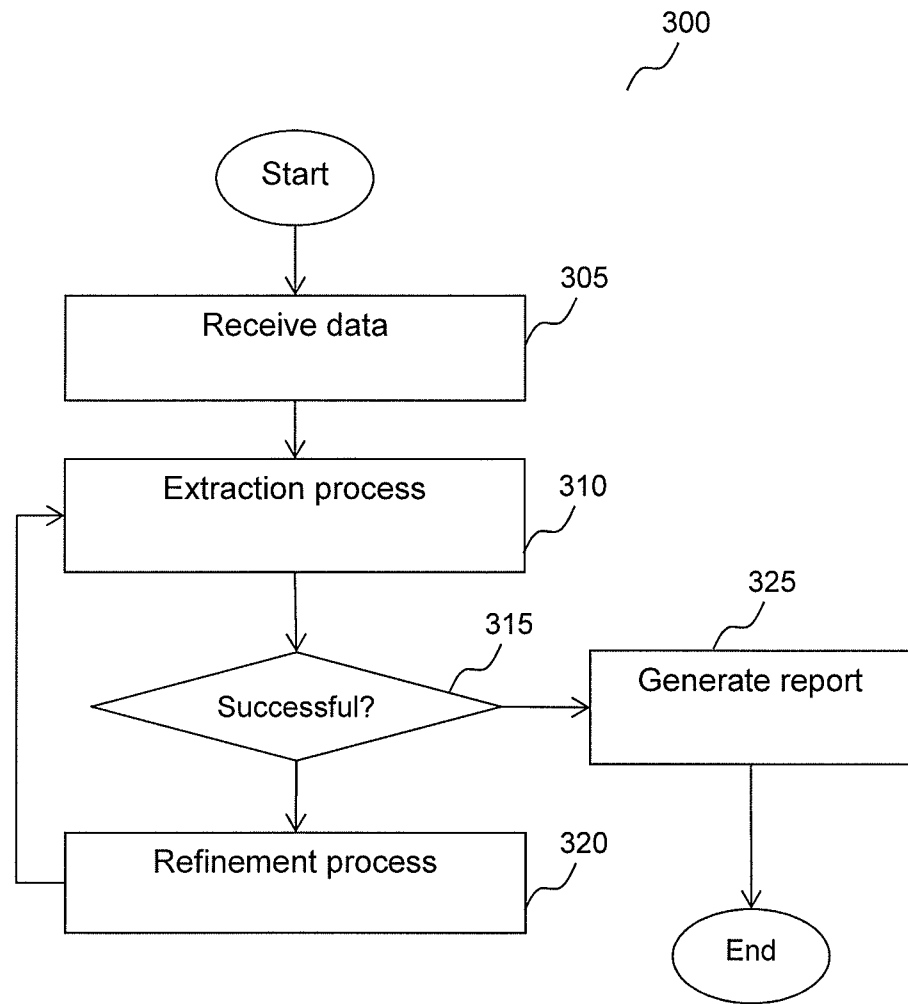


Figure 3

(Source Location) $i ::= 0 \mid 1 \mid \dots$
 (Regular Expression with src loc) $r ::= i:\epsilon \mid i:l \mid i:rr \mid i:r + r \mid i:r^*$

Figure 4

$$\boxed{\bar{i} \vdash r/l \Rightarrow \overline{(r', \bar{i}')}} \quad$$

$$\text{(Eps)} \quad \bar{i} \vdash i:\epsilon/l \Rightarrow \{\} \quad \text{(Lab1)} \quad \bar{i} \vdash i:l/l \Rightarrow \{(i:\epsilon, \bar{i} + i)\} \quad \text{(Lab2)} \quad \bar{i} \vdash i:l/l' \Rightarrow \{\}$$

$$\text{(Seq1)} \quad \frac{\epsilon \in r_1 \quad \bar{i} \vdash r_1/l \Rightarrow \overline{(r'_1, \bar{j})} \quad \bar{i} \vdash r_2/l \Rightarrow \overline{(r'_2, \bar{j}')}}{\bar{i} \vdash i:r_1r_2/l \Rightarrow \{(r'_1r_2, \bar{j}) \mid (r'_1, \bar{j}) \in (r'_1, \bar{j})\} \cup \overline{(r'_2, \bar{j}')}} \quad$$

$$\text{(Seq2)} \quad \frac{\epsilon \notin r_1 \quad \bar{i} \vdash r_1/l \Rightarrow \overline{(r'_1, \bar{j})}}{\bar{i} \vdash i:r_1r_2/l \Rightarrow \overline{(r'_1, \bar{j})}} \quad$$

$$\text{(Choice)} \quad \frac{\bar{i} \vdash r_1/l \Rightarrow \overline{(r'_1, \bar{j})} \quad \bar{i} \vdash r_2/l \Rightarrow \overline{(r'_2, \bar{j}')}}{\bar{i} \vdash i:r_1 + r_2/l \Rightarrow \overline{(r'_1, \bar{j})} \cup \overline{(r'_2, \bar{j}')}} \quad$$

$$\text{(Star)} \quad \frac{\bar{i} \vdash r/l \Rightarrow \overline{(r', \bar{i}')}}{\bar{i} \vdash i:r^*/l \Rightarrow \{(i:r'_i:r^*, \bar{i}') \mid (r', \bar{i}') \in \overline{(r', \bar{i}')} \}} \quad$$

$$\boxed{\bar{i} \vdash r/w \Rightarrow \overline{(r', \bar{i}')}} \quad$$

$$\text{(WordEps)} \quad \bar{i} \vdash r/\epsilon \Rightarrow \{(r, \bar{i})\}$$

$$\text{(WordCon)} \quad \frac{\bar{i} \vdash r/l \Rightarrow \overline{(r', \bar{i}')} \quad \text{for } j \in \{1, \dots, n\} \quad \bar{i}'_j \vdash r'_j/w \Rightarrow \overline{(r''_j, \bar{i}''_j)}}{\bar{i} \vdash r/lw \Rightarrow \bigcup_j \overline{(r''_j, \bar{i}''_j)}} \quad$$

Figure 5

$$\begin{array}{c}
\frac{}{\{3\} \vdash 4.b/b \Rightarrow \{(4.\epsilon, \{3, 4\})\}} \text{(Lab1)} \\
\frac{}{\{3\} \vdash 2.(3.a + 4.b)/b \Rightarrow \{(4.\epsilon, \{3, 4\})\}} \text{(Choice)} \\
\frac{}{\{3\} \vdash 1.2.(3.a + 4.b)^*/b \Rightarrow \{(1.4.\epsilon_1.2.(3.a + 4.b)^*, \{3, 4\})\}} \text{(Star)} \\
\frac{}{\{3\} \vdash 1.3.\epsilon_1.2.(3.a + 4.b)^*/b \Rightarrow \{(1.4.\epsilon_1.2.(3.a + 4.b)^*, \{3, 4\})\}} \text{(Seq1)} \\
\frac{}{\{3\} \vdash 1.3.\epsilon_1.2.(3.a + 4.b)/b \Rightarrow \{(1.4.\epsilon_1.2.(3.a + 4.b)^*, \{3, 4\})\}} \text{(WordCon)} \\
\frac{}{\{3\} \vdash 1.3.\epsilon_1.2.(3.a + 4.b)^*/b \Rightarrow \{(1.4.\epsilon_1.2.(3.a + 4.b)^*, \{3, 4\})\}} \text{(WordCon)} \\
\frac{}{\{3\} \vdash 1.2.(3.a + 4.b)^*/ab \Rightarrow \{(1.4.\epsilon_1.2.(3.a + 4.b)^*, \{3, 4\})\}} \text{(WordCon)}
\end{array}$$

$$\frac{\frac{\frac{\{3\} \vdash 4.b/b \Rightarrow \{(4.\epsilon, \{3, 4\})\}}{\{3\} \vdash 2.(3.a + 4.b)/b \Rightarrow \{(4.\epsilon, \{3, 4\})\}} \quad \text{(Choice)}}{\frac{\{3\} \vdash 1.2.(3.a + 4.b)^*/b \Rightarrow \{(1.4.\epsilon 1.2.(3.a + 4.b)^*, \{3, 4\})\}}{\{3\} \vdash 1.3.\epsilon 1.2.(3.a + 4.b)^*/b \Rightarrow \{(1.4.\epsilon 1.2.(3.a + 4.b)^*, \{3, 4\})\}} \quad \text{(Seq1)}} \quad \text{(WordCon)} \\ \frac{\{3\} \vdash 1.3.\epsilon 1.2.(3.a + 4.b)^*/b \Rightarrow \{(1.4.\epsilon 1.2.(3.a + 4.b)^*, \{3, 4\})\}}{* / ab \Rightarrow \{(1.4.\epsilon 1.2.(3.a + 4.b)^*, \{3, 4\})\}} \quad \text{(WordCon)}$$

Figure 6

$$(\text{Match Error}) \ E ::= E_l^{\bar{i}} \mid E_e^{\bar{i}}$$

$$\boxed{\bar{i} \vdash r/l \Rightarrow (\overline{r'}, \overline{i'}), \overline{E}}$$

$$(\text{Eps}) \quad \bar{i} \vdash i:\epsilon/l \Rightarrow \{\}, \{E_e^{\bar{i}+i}\}$$

$$(\text{Lab1}) \quad \bar{i} \vdash i:l/l \Rightarrow \{(i:\epsilon, \bar{i}+i)\}, \{\} \quad (\text{Lab2}) \quad \bar{i} \vdash i:l/l' \Rightarrow \{\}, \{E_l^{\bar{i}+i}\}$$

$$(\text{Seq1}) \quad \frac{\epsilon \in r_1 \quad \bar{i} \vdash r_1/l \Rightarrow (\overline{r'_1}, \overline{j}), \overline{E_1} \quad \bar{i} \vdash r_2/l \Rightarrow (\overline{r'_2}, \overline{j'}), \overline{E_2}}{\bar{i} \vdash i:r_1 r_2/l \Rightarrow \{(r'_1 r_2, \bar{j}) \mid (r'_1, \bar{j}) \in (r'_1, \bar{j})\} \cup (r'_2, \bar{j'}), \overline{E_1} \cup \overline{E_2}}}$$

$$(\text{Seq2}) \quad \frac{\epsilon \notin r_1 \quad \bar{i} \vdash r_1/l \Rightarrow (\overline{r'_1}, \overline{j}), \overline{E'}}{\bar{i} \vdash i:r_1 r_2/l \Rightarrow (\overline{r'_1}, \overline{j}), \overline{E'}}$$

$$(\text{Choice}) \quad \frac{\bar{i} \vdash r_1/l \Rightarrow (\overline{r'_1}, \overline{j}), \overline{E_1} \quad \bar{i} \vdash r_2/l \Rightarrow (\overline{r'_2}, \overline{j'}), \overline{E_2}}{\bar{i} \vdash i:r_1 + r_2/l \Rightarrow (\overline{r'_1}, \overline{j}) \cup (\overline{r'_2}, \overline{j'}), \overline{E_1} \cup \overline{E_2}}$$

$$(\text{Star}) \quad \frac{\bar{i} \vdash r/l \Rightarrow (\overline{r'}, \overline{i'}), \overline{E'}}{\bar{i} \vdash i:r^*/l \Rightarrow \{(i:r' i:r^*, \bar{i}') \mid (r', \bar{i}') \in (\overline{r'}, \overline{i'})\}, \overline{E'}}$$

$$\boxed{\bar{i}, \overline{E} \vdash r/w \Rightarrow (\overline{r'}, \overline{i'}), \overline{E'}}$$

$$(\text{WordEps}) \quad \bar{i}, \overline{E} \vdash r/\epsilon \Rightarrow \{(r, \bar{i})\}, \overline{E}$$

$$(\text{WordCon}) \quad \frac{\bar{i} \vdash r/l \Rightarrow (\overline{r'}, \overline{i'}), \overline{E'} \quad \text{for } j \in \{1, \dots, n\} \quad \bar{i}'_j, \overline{E}'_j \vdash r'_j/w \Rightarrow (\overline{r''_j}, \overline{i''_j}), \overline{E''_j}}{\bar{i}, \overline{E} \vdash r/lw \Rightarrow \bigcup_j (\overline{r''_j}, \overline{i''_j}), \overline{E'} \cup \bigcup_j \overline{E''_j}}$$

Figure 7

$$\begin{array}{c}
\frac{}{\{\} \vdash 3.a/a \Rightarrow \{3.\epsilon, \{3\}\}, \{\}} \text{(Lab1)} \\
\frac{}{\{\} \vdash 4.b/a \Rightarrow \{\}, \{E_l^{(4)}\}} \text{(Lab2)} \\
\frac{}{\{\} \vdash 2.(3.a + 4.b)/a \Rightarrow \{3.\epsilon, \{3\}\}, \{E_l^{(4)}\}} \text{(Choice)} \\
\frac{}{\{\} \vdash 1.2.(3.a + 4.b)^*/a \Rightarrow \{1.3.\epsilon 1.2.(3.a + 4.b)^*, \{3\}\}, \{E_l^{(4)}\}} \text{(Star)} \\
\hline
\frac{}{\{3\} \vdash 3.a/c \Rightarrow \{\}, \{E_l^{\{3,3\}}\}} \text{(Lab2)} \\
\frac{}{\{3\} \vdash 4.b/c \Rightarrow \{\}, \{E_l^{\{3,4\}}\}} \text{(Lab2)} \\
\frac{}{\{3\} \vdash 2.(3.a + 4.b)/c \Rightarrow \{\}, \{E_l^{\{3,3\}}, E_l^{\{3,4\}}\}} \text{(Choice)} \\
\frac{}{\{3\} \vdash 1.2.(3.a + 4.b)^*/c \Rightarrow \{\}, \{E_l^{\{3,3\}}, E_l^{\{3,4\}}\}} \text{(Star)} \\
\frac{}{\{3\} \vdash 3.\epsilon/c \Rightarrow \{\}, \{E_\epsilon^{\{3,3\}}\}} \text{(Eps)} \\
\frac{}{\{3\} \vdash 1.3.\epsilon 1.2.(3.a + 4.b)^*/c \Rightarrow \{\}, \{E_l^{\{3,3\}}, E_l^{\{3,4\}}, E_\epsilon^{\{3,3\}}\}} \text{(Seq1)} \\
\frac{}{\{3\}, \{E_\epsilon^{\{4\}}\} \vdash 1.3.\epsilon 1.2.(3.a + 4.b)^*/c \Rightarrow \{\}, \{E_l^{\{3,3\}}, E_l^{\{3,4\}}, E_\epsilon^{\{3,3\}}\}} \text{(WordCon)} \\
\hline
\frac{}{\{\}, \{\} \vdash 1.2.(3.a + 4.b)^*/ac \Rightarrow \{\}, \{E_l^{\{4\}}, E_l^{\{3,3\}}, E_l^{\{3,4\}}, E_\epsilon^{\{3,3\}}\}} \text{(WordCon)}
\end{array}$$

Figure 8

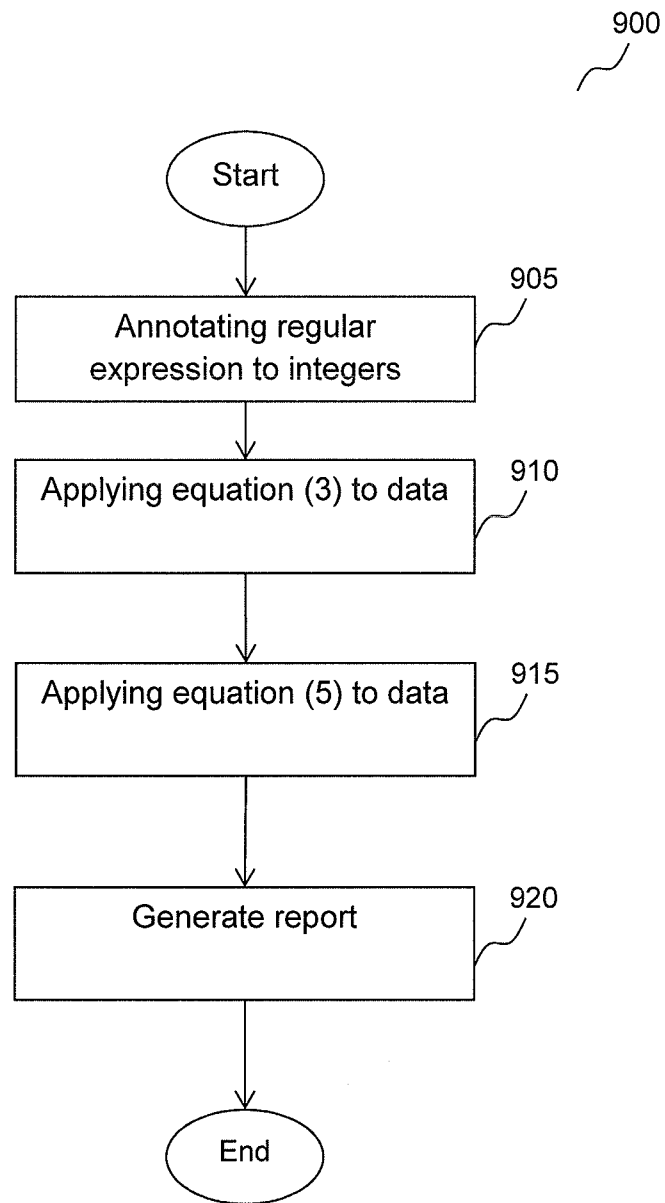


Figure 9

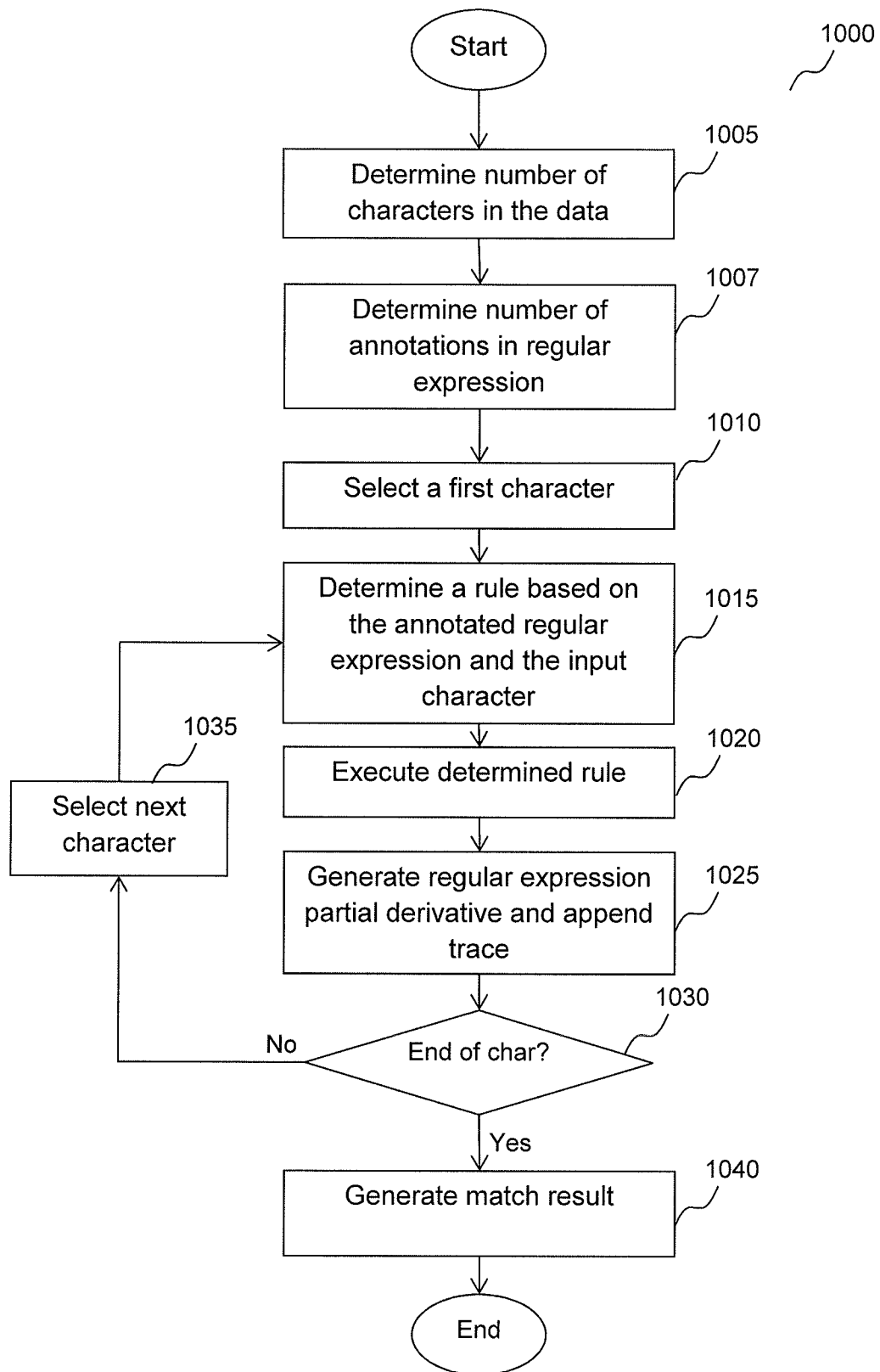


Figure 10

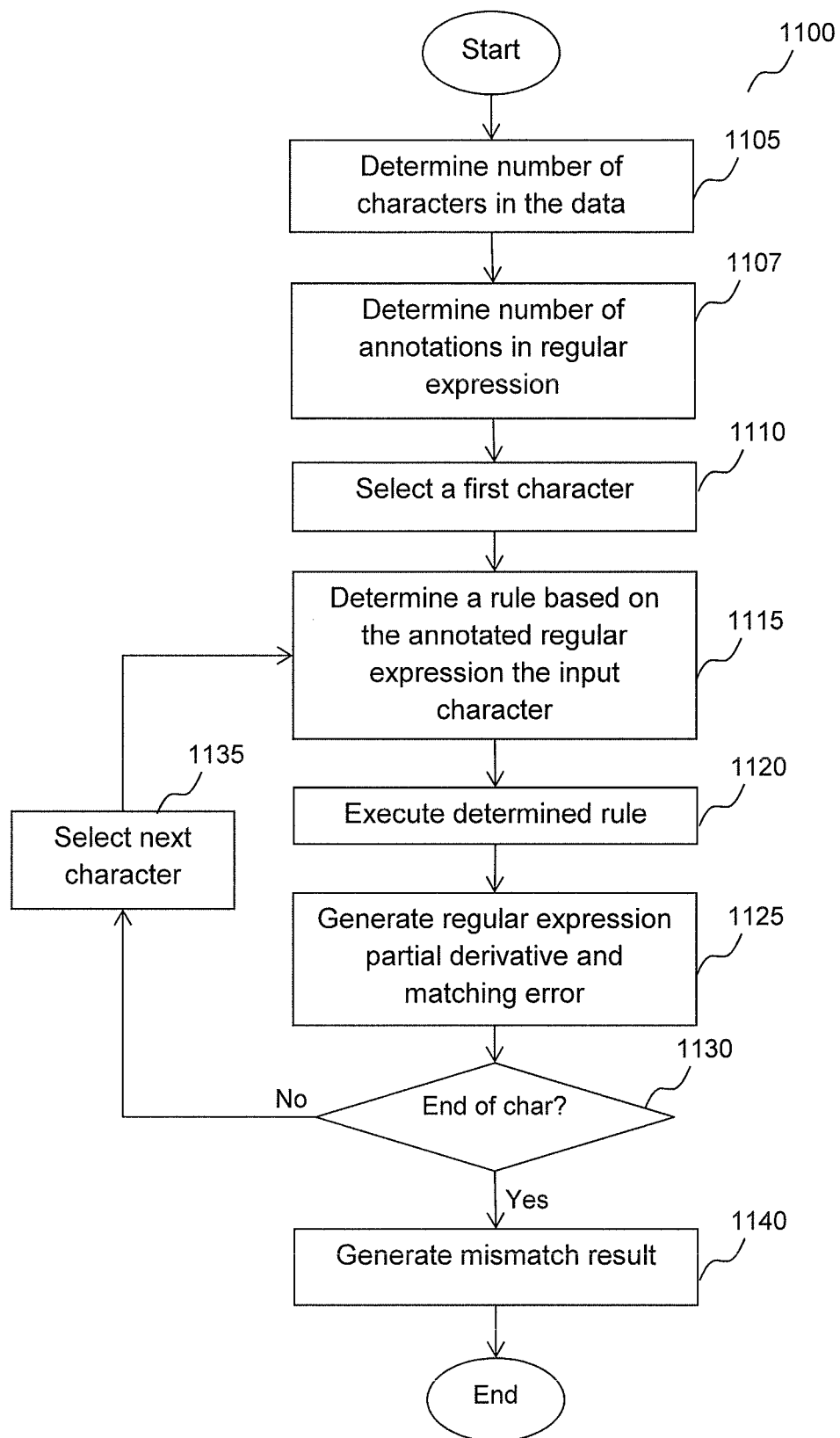


Figure 11

(Refinement Requirement) $\mathcal{R} ::= \{(i, r), \dots, (i, r)\}$

$\mathcal{R}, r \vdash w :: r'$

1210

$$\mathcal{R}, i:\epsilon \vdash w :: i:r \iff \begin{cases} w \in r' \wedge w \in r \wedge \epsilon \in r & \text{if } (i, r') \in \mathcal{R} \\ w \in r \wedge \epsilon \in r & \text{otherwise} \end{cases}$$

1220

$$\mathcal{R}, i:l \vdash w :: i:r \iff \begin{cases} w \in r' \wedge w \in r \wedge l \in r & \text{if } (i, r') \in \mathcal{R} \\ w \in r \wedge l \in r & \text{otherwise} \end{cases}$$

1230

$$\mathcal{R}, i:r_1 r_2 \vdash w :: i:r'_1 r'_2 \iff \begin{cases} \begin{matrix} w \in r' \wedge w = w_1 w_2 \\ \mathcal{R}_{\overline{r_1}}, r_1 \vdash w_1 :: r'_1 \wedge \mathcal{R}_{\overline{r_2}}, r_2 \vdash w_2 :: r'_2 \end{matrix} & \text{if } (i, r') \in \mathcal{R} \\ \begin{matrix} w = w_1 w_2 \wedge \mathcal{R}_{\overline{r_1}}, r_1 \vdash w_1 :: r'_1 \\ \mathcal{R}_{\overline{r_2}}, r_2 \vdash w_2 :: r'_2 \end{matrix} & \text{otherwise} \end{cases}$$

1240

$$\mathcal{R}, i:r_1 + r_2 \vdash w :: i:r'_1 + r_2 \iff \begin{cases} w \in r' \wedge \mathcal{R}_{\overline{r_1}}, r_1 \vdash w_1 :: r'_1 & \text{if } (i, r') \in \mathcal{R} \\ \mathcal{R}_{\overline{r_1}}, r_1 \vdash w_1 :: r'_1 & \text{otherwise} \end{cases}$$

1250

$$\mathcal{R}, i:r_1 + r_2 \vdash w :: i:r_1 + r'_2 \iff \begin{cases} w \in r' \wedge \mathcal{R}_{\overline{r_2}}, r_2 \vdash w_2 :: r'_2 & \text{if } (i, r') \in \mathcal{R} \\ \mathcal{R}_{\overline{r_2}}, r_2 \vdash w_2 :: r'_2 & \text{otherwise} \end{cases}$$

1260

$$\mathcal{R}, i:r^* \vdash w :: i:r'^* \iff \begin{cases} \begin{matrix} w \in r'' \wedge w \neq \epsilon \wedge w = w_1 \dots w_n \wedge \\ \text{for } i \in \{1, \dots, n\} \mathcal{R}_{\overline{r}}, r \vdash w_i :: r' \end{matrix} & \text{if } (i, r'') \in \mathcal{R} \\ \begin{matrix} w \neq \epsilon \wedge w = w_1 \dots w_n \wedge \\ \text{for } i \in \{1, \dots, n\} \mathcal{R}_{\overline{r}}, r \vdash w_i :: r' \end{matrix} & \text{otherwise} \end{cases}$$

1270

$$\mathcal{R}, i:r^* \vdash \epsilon :: i:r'^* \iff \begin{cases} \epsilon \in r'' & \text{if } (i, r'') \in \mathcal{R} \\ True & \text{otherwise} \end{cases}$$

Figure 12

(Sub-matching Environment) $\Delta ::= \{(i, w), \dots (i, w)\}$

$$\boxed{w \in r \Rightarrow \Delta}$$

(Eps) $\epsilon \in {}_i:\epsilon \Rightarrow \{(i, \epsilon)\}$ (Lit) $l \in {}_i:l \Rightarrow \{(i, l)\}$

(Seq) $\frac{w = w_1 w_2 \quad w_1 \in r_1 \Rightarrow \Delta_1 \quad w_2 \in r_2 \Rightarrow \Delta_2 \quad w_1, w_2 \text{ is the left maximal word match}}{w \in {}_i:r_1 r_2 \Rightarrow \Delta_1 \uplus \Delta_2 \uplus \{(i, w)\}}$

(ChoiceL) $\frac{w \in r_1 \Rightarrow \Delta_1}{w \in {}_i:r_1 + r_2 \Rightarrow \Delta_1 \uplus \{(i, w)\}}$ (ChoiceR) $\frac{w \notin r_1 \quad w \in r_2 \Rightarrow \Delta_2}{w \in {}_i:r_1 + r_2 \Rightarrow \Delta_2 \uplus \{(i, w)\}}$

(Star) $\frac{w = w_1 \dots w_n \quad w_i \in r \Rightarrow \Delta_i \quad w_1, \dots, w_n \text{ is the left maximal word match}}{w \in {}_i:r^* \Rightarrow \biguplus_i \Delta_i \uplus \{(i, w)\}}$

Figure 13

(Label) $l ::= 'a' \mid 'b' \mid \dots$
 (Level) $L ::= W \mid S$
 (Refinement Environment) $\Theta ::= \{(i, op), \dots, (i, op)\}$
 (Refinement Operation) $op ::= i:l+\overset{\circ}{L} \mid i:l+\vec{L} \mid +^\epsilon \mid id_L$
 (Source Location Aliases) $\mathcal{A} ::= \{i \mapsto j, \dots, i \mapsto j\}$

$$\boxed{\mathcal{R} \vdash_{\mathcal{A}}^L r/l \Rightarrow (\mathcal{R}, r, \Theta, \mathcal{A})}$$

$$(EpsS) \frac{(i, r) \in \mathcal{R} \quad j \text{ is a fresh src loc}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:\epsilon/l \Rightarrow \{(\mathcal{R} \oplus \{(i, r')\}, j:\epsilon, \{(i, j:l+\overset{\circ}{S})\}, \mathcal{A}) \mid r' \in r/pl\}}$$

$$(EpsW) \frac{i \notin \text{dom}(\mathcal{R}) \quad j \text{ is a fresh src loc} \quad L' = \max(L, W)}{\mathcal{R} \vdash_{\mathcal{A}}^L i:\epsilon/l \Rightarrow \{(\mathcal{R}, j:\epsilon, \{(i, j:l+\overset{\circ}{L'})\}, \mathcal{A})\}}$$

$$(LitS1) \frac{(i, r) \in \mathcal{R}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:l/l \Rightarrow \{(\mathcal{R} \oplus \{(i, r')\}, i:\epsilon, \{(i, id_S)\}, \mathcal{A}) \mid r' \in r/pl\}}$$

$$(LitS2) \frac{(i, r) \in \mathcal{R} \quad l \neq l' \quad j \text{ is a fresh src loc}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:l/l' \Rightarrow \{(\mathcal{R} \oplus \{(i, r')\}, i:\epsilon, \{(i, j:l'+\vec{S})\}, \mathcal{A}) \mid r' \in r/pl\}}$$

$$(LitW1) \frac{i \notin \text{dom}(\mathcal{R}) \quad L' = \max(L, W)}{\mathcal{R} \vdash_{\mathcal{A}}^L i:l/l \Rightarrow \{(\mathcal{R}, i:\epsilon, \{(i, id_{L'})\}, \mathcal{A})\}}$$

$$(LitW2) \frac{i \notin \text{dom}(\mathcal{R}) \quad l \neq l' \quad L' = \max(L, W) \quad j \text{ is a fresh src loc}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:l/l' \Rightarrow \{(\mathcal{R}, i:\epsilon, \{(i, j:l'+\overset{\rightarrow}{L'})\}, \mathcal{A})\}}$$

$$(SeqS1) \frac{\begin{array}{l} (i, r) \in \mathcal{R} \quad \epsilon \in r_1 \quad \mathcal{R}_{\overline{r_1}} \vdash_{\mathcal{A}}^S r_1/l \Rightarrow V_1 \quad \mathcal{R}_{\overline{r_2}} \vdash_{\mathcal{A}}^S r_2/l \Rightarrow V_2 \\ V'_1 = \{(\mathcal{R}'_1 \cup \mathcal{R}_{\overline{r_2}} \cup \{(i, r')\}, i:(r'_1 r_2), \Theta, \mathcal{A}) \mid (\mathcal{R}'_1, r'_1, \Theta, \mathcal{A}) \in V_1, r' \in r/pl\} \\ V'_2 = \{(\mathcal{R}'_2 \cup \{(i, r')\}, r'_2, \Theta, \mathcal{A}) \mid (\mathcal{R}'_2, r'_2, \Theta, \mathcal{A}) \in V_2, r' \in r/pl\} \end{array}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:(r_1 r_2)/l \Rightarrow V'_1 \cup V'_2}$$

$$(SeqS2) \frac{\begin{array}{l} (i, r) \in \mathcal{R} \quad \epsilon \notin r_1 \quad \mathcal{R}_{\overline{r_1}} \vdash_{\mathcal{A}}^S r_1/l \Rightarrow V_1 \quad \mathcal{R}_{\overline{r_2}} \vdash_{\mathcal{A}}^S r_2/l \Rightarrow V_2 \\ V'_1 = \{(\mathcal{R}'_1 \cup \mathcal{R}_{\overline{r_2}} \cup \{(i, r')\}, i:(r'_1 r_2), \Theta, \mathcal{A}) \mid (\mathcal{R}'_1, r'_1, \Theta, \mathcal{A}) \in V_1, r' \in r/pl\} \\ V'_2 = \{(\mathcal{R}'_2 \cup \{(i, r')\}, r'_2, \Theta \cup \Theta', \mathcal{A}) \mid (\mathcal{R}'_2, r'_2, \Theta, \mathcal{A}) \in V_2, \Theta' \in mkfin(r_1)\} \end{array}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:(r_1 r_2)/l \Rightarrow V'_1 \cup V'_2}$$

Figure 14

$$\begin{array}{c}
\text{(SeqW1)} \quad \frac{i \notin \text{dom}(\mathcal{R}) \quad \epsilon \in r_1 \quad \mathcal{R}_{\overline{r_1}} \vdash_{\mathcal{A}}^L r_1/l \Rightarrow V_1 \quad \mathcal{R}_{\overline{r_2}} \vdash_{\mathcal{A}}^L r_2/l \Rightarrow V_2 \quad \mathcal{R}'_2 = \mathcal{R}_{\overline{r_2}}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:(r_1 r_2)/l \Rightarrow \{(\mathcal{R}'_1 \cup \mathcal{R}'_2, i:(r'_1 r_2), \Theta) \mid (\mathcal{R}'_1, r'_1, \Theta, \mathcal{A}) \in V_1\} \cup V_2} \\
\\
\text{(SeqW2)} \quad \frac{i \notin \text{dom}(\mathcal{R}) \quad \epsilon \notin r_1 \quad \mathcal{R}_{\overline{r_1}} \vdash_{\mathcal{A}}^L r_1/l \Rightarrow V_1 \quad \mathcal{R}'_2 = \mathcal{R}_{\overline{r_2}} \cup \{(i, r')\} \quad V'_1 = \{(\mathcal{R}'_1 \cup \mathcal{R}'_2, i:(r'_1 r_2), \Theta, \mathcal{A}) \mid (\mathcal{R}'_1, r'_1, \Theta, \mathcal{A}) \in V_1\} \quad V'_2 = \{(\mathcal{R}'_2 \cup \{(i, r')\}, r'_2, \Theta \cup \Theta', \mathcal{A}) \mid (\mathcal{R}'_2, r'_2, \Theta, \mathcal{A}) \in V_2, \Theta' \in \text{mkfin}(r_1)\}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:(r_1 r_2)/l \Rightarrow V'_1 \cup V'_2} \\
\\
\text{(ChoiceS)} \quad \frac{(i, r) \in \mathcal{R} \quad \mathcal{R} \vdash_{\mathcal{A}}^S r_1/l \Rightarrow V_1 \quad \mathcal{R} \vdash_{\mathcal{A}}^S r_2/l \Rightarrow V_2 \quad V'_1 = \{(\mathcal{R}'_1 \cup \{(i, r')\}, r'_1, \Theta, \mathcal{A}) \mid (\mathcal{R}'_1, r'_1, \Theta, \mathcal{A}) \in V_1, r' \in r/p_l\} \quad V'_2 = \{(\mathcal{R}'_2 \cup \{(i, r')\}, r'_2, \Theta, \mathcal{A}) \mid (\mathcal{R}'_2, r'_2, \Theta, \mathcal{A}) \in V_2, r' \in r/p_l\}}{\mathcal{R} \vdash_{\mathcal{A}}^L i:(r_1 + r_2)/l \Rightarrow V'_1 \cup V'_2} \\
\\
\text{(ChoiceW)} \quad \frac{i \notin \text{dom}(\mathcal{R}) \quad \mathcal{R} \vdash_{\mathcal{A}}^L r_1/l \Rightarrow V_1 \quad \mathcal{R} \vdash_{\mathcal{A}}^L r_2/l \Rightarrow V_2}{\mathcal{R} \vdash_{\mathcal{A}}^L i:(r_1 + r_2)/l \Rightarrow V_1 \cup V_2} \\
\\
\text{(StarS)} \quad \frac{(i, r_0) \in \mathcal{R} \quad \text{relabel}(r) = (r', \mathcal{A}') \quad \mathcal{A}'(\mathcal{R}) \vdash_{\mathcal{A} \cup \mathcal{A}'}^S r'/l \Rightarrow V}{\mathcal{R} \vdash_{\mathcal{A}}^L i:r^*/l \Rightarrow \{(\mathcal{R}' \oplus (i, r'_0), i:(r'' i:r^*), \Theta, \mathcal{A}'') \mid (\mathcal{R}', r'', \Theta, \mathcal{A}'') \in V, r'_0 \in r_0/p_l\}} \\
\\
\text{(StarW)} \quad \frac{i \notin \text{dom}(\mathcal{R}) \quad \text{relabel}(r) = (r', \mathcal{A}') \quad \mathcal{A}'(\mathcal{R}) \vdash_{\mathcal{A} \cup \mathcal{A}'}^L r'/l \Rightarrow V}{\mathcal{R} \vdash_{\mathcal{A}}^L i:r^*/l \Rightarrow \{(\mathcal{R}', i:(r''(i:r^*)), \Theta, \mathcal{A}'') \mid (\mathcal{R}', r'', \Theta, \mathcal{A}'') \in V\}} \\
\\
\boxed{\mathcal{R} \vdash_{\mathcal{A}}^L r/w \Rightarrow \{\overline{R, r, \Theta, \mathcal{A}}\}} \\
\\
\text{(WordCon)} \quad \frac{\mathcal{R} \vdash_{\mathcal{A}}^L r/l \Rightarrow \{\overline{R, r, \Theta, \mathcal{A}}\} \quad \text{for } i \in \{1, \dots, n\} \quad \mathcal{R}_i \vdash_{\mathcal{A} \cup \mathcal{A}_i}^L r_i/w \Rightarrow V_i \quad V'_i = \{(\mathcal{R}_j, r_j, \Theta_i \cup \Theta_j, \mathcal{A}_j) \mid (\mathcal{R}_j, r_j, \Theta_j, \mathcal{A}_j) \in V_i\}}{\mathcal{R} \vdash_{\mathcal{A}}^L r/lw \Rightarrow V'_1 \cup \dots \cup V'_n} \\
\\
\text{(WordEps)} \quad \mathcal{R} \vdash_{\mathcal{A}}^L r/\epsilon \Rightarrow \{(\mathcal{R}, r, \{\}, \mathcal{A})\}
\end{array}$$

Figure 15

$$\begin{aligned}
& \Theta(\mathcal{A})(i:\epsilon) &= \text{let } \{j_1, \dots, j_n\} = \mathcal{A}^+(i) \\
& \quad \quad \quad ops = \Theta(i) \cup \Theta(j_1) \dots \cup \Theta(j_n) \\
& \quad \quad \quad ts = \{j:l | j:l + \vec{L} \in ops\} \\
& \quad \quad \quad ss = \{\Theta(\mathcal{A})(j:l) | j:l + \vec{L} \in ops\} \\
& \quad \quad \quad \{k_1, \dots, k_n\} = makeFreshSrcLocs(n) \\
& \quad \quad \quad r = k_2:(mkChoice(ts) + k_1:(i:\epsilon) + mkApp(ss)) \\
& \quad \quad \quad \text{in } r \\
& \Theta(\mathcal{A})(i:l) &= \text{let } \{j_1, \dots, j_n\} = \mathcal{A}^+(i) \\
& \quad \quad \quad ops = \Theta(i) \cup \Theta(j_1) \dots \cup \Theta(j_n) \\
& \quad \quad \quad es = \{+^\epsilon | +^\epsilon \in ops\} \\
& \quad \quad \quad ts = \{j:l | j:l + \vec{L} \in ops\} \\
& \quad \quad \quad ss = \{\Theta(\mathcal{A})(j:l) | j:l + \vec{L} \in ops\} \\
& \quad \quad \quad \{k_1, \dots, k_n\} = makeFreshSrcLocs(n) \\
& \quad \quad \quad r = k_4:(mkChoice(ts) + k_3:(i:l)(k_2:(((k_1:\epsilon) + mkApp(ss)) \\
& \quad \quad \quad \text{in if } |es| > 0 \text{ then } k_6:(k_5:\epsilon + r) \text{ else } r \\
& \Theta(\mathcal{A})(i:rr') &= i:(\Theta(\mathcal{A})(r)(\Theta(\mathcal{A})(r')))) \\
& \Theta(\mathcal{A})(i:r + r') &= i:(\Theta(\mathcal{A})(r) + (\Theta(\mathcal{A})(r')))) \\
& \Theta(\mathcal{A})(i:r^*) &= i:(\Theta(\mathcal{A})(r)^*) \\
mkChoice(\{r_1, \dots, r_m\}) &= \text{let } \{i_1, \dots, i_{m-1}\} = makeFreshSrcLocs(m-1) \\
& \quad \text{in } i_1:(r_1 + \dots (i_{m-1}:(r_{m-1} + r_m))) \\
mkApp(\{r_1, \dots, r_m\}) &= \text{let } \{i_1, \dots, i_{m-1}\} = makeFreshSrcLocs(m-1) \\
& \quad \text{in } i_1:(r_1 \dots (i_{m-1}:(r_{m-1}r_m)))
\end{aligned}$$

Figure 16

$$\begin{aligned}
& \text{refineAlgo}(\mathcal{R})(r)(w) = \\
& \quad \text{let } \mathcal{R} \vdash_{\{\}}^W r/w \Rightarrow V \\
& \quad \quad \bar{\Theta} = \{\Theta(\mathcal{A}) | (\mathcal{R}, r, \Theta, \mathcal{A}) \in V, \epsilon \in \Theta(\mathcal{A})(r)\} \\
& \quad \quad \bar{\Theta}' = \{mkfin(r)(\mathcal{A}) \cdot \Theta(\mathcal{A}) | (\mathcal{R}, r, \Theta, \mathcal{A}) \in V, \epsilon \notin \Theta(\mathcal{A})(r), \text{let } i = src(r)\} \\
& \quad \quad \Theta = min_{\prec} \bar{\Theta} \cup \bar{\Theta}' \\
& \quad \text{in } \Theta(r) \\
& \quad \text{where } src(i:_) = i \\
mkfin(i:\epsilon) &= \{ \} \\
mkfin(i:l) &= \{ (i, +^\epsilon) \} \\
mkfin(i:r_1 r_2) &= mkfin(r_1) \cup mkfin(r_2) \\
mkfin(i:r_1 + r_2) &= mkfin(r_1) \\
mkfin(i:r^*) &= \{ \}
\end{aligned}$$

Figure 17

$$\begin{array}{c}
\frac{|\theta_1| < |\theta_2|}{\theta_1 \prec \theta_2} \\
\\
\begin{array}{ccc}
\frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id} > |\theta_2|_{id}} & \frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id_S} = |\theta_2|_{id_S} \quad |\theta_1|_{id_W} = |\theta_2|_{id_W}} & \frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id_S} = |\theta_2|_{id_S} \quad |\theta_1|_{id_W} = |\theta_2|_{id_W}} \\
\theta_1 \prec \theta_2 & \frac{|\theta_1|_{id_W} = |\theta_2|_{id_W}}{|\theta_1|_{-+\vec{s}} > |\theta_2|_{-+\vec{s}}} & \frac{|\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}}}{|\theta_1|_{-+\vec{w}} > |\theta_2|_{-+\vec{w}}} \\
\\
\frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id_-} = |\theta_2|_{id_-} \quad |\theta_1|_{id_S} > |\theta_2|_{id_S}} & \frac{|\theta_1|_{-+\vec{s}} > |\theta_2|_{-+\vec{s}}}{\theta_1 \prec \theta_2} & \frac{|\theta_1|_{-+\vec{w}} > |\theta_2|_{-+\vec{w}}}{\theta_1 \prec \theta_2} \\
\theta_1 \prec \theta_2 & &
\end{array} \\
\\
\begin{array}{c}
|\theta_1| = |\theta_2| \\
|\theta_1|_{id_S} = |\theta_2|_{id_S} \quad |\theta_1|_{id_W} = |\theta_2|_{id_W} \\
|\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} = |\theta_2|_{-+\vec{w}} \\
\frac{|\theta_1|_{-+\vec{s}} > |\theta_2|_{-+\vec{s}}}{\theta_1 \prec \theta_2}
\end{array} \\
\\
\begin{array}{ccc}
\frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id_S} = |\theta_2|_{id_S} \quad |\theta_1|_{id_W} = |\theta_2|_{id_W}} & \frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id_S} = |\theta_2|_{id_S} \quad |\theta_1|_{id_W} = |\theta_2|_{id_W}} & \frac{|\theta_1| = |\theta_2|}{|\theta_1|_{id_S} = |\theta_2|_{id_S} \quad |\theta_1|_{id_W} = |\theta_2|_{id_W}} \\
|\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} = |\theta_2|_{-+\vec{w}} & |\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} = |\theta_2|_{-+\vec{w}} & |\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} = |\theta_2|_{-+\vec{w}} \\
|\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} > |\theta_2|_{-+\vec{w}} & |\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} = |\theta_2|_{-+\vec{w}} & |\theta_1|_{-+\vec{s}} = |\theta_2|_{-+\vec{s}} \quad |\theta_1|_{-+\vec{w}} = |\theta_2|_{-+\vec{w}} \\
\theta_1 \prec \theta_2 & \theta_1 \prec \theta_2 & \theta_1 \prec \theta_2
\end{array}
\end{array}$$

Figure 18

$$\begin{array}{c}
\frac{\mathcal{R} \vdash_{\{\}}^W 2: < / < \Rightarrow (\mathcal{R}, 2:\epsilon, \{(2, id_W)\}, \{\})}{\mathcal{R} \vdash_{\{\}}^W 1:2: < 3:4:(5:p+6:s)7: > / < \Rightarrow \{(\mathcal{R}, 1:2:\epsilon 3:4:(5:p+6:s)7: >, \{(2, id_W)\}, \{\})\}} \quad \text{(LitW1)} \\
\frac{\quad}{\mathcal{R} \vdash_{\{\}}^W 1:2: < 3:4:(5:p+6:s)7: > / < \Rightarrow \{(\mathcal{R}, 1:2:\epsilon 3:4:(5:p+6:s)7: >, \{(2, id_W)\}, \{\})\}} \quad \text{(SeqW2)} \\
\frac{\quad}{\mathcal{R} \vdash_{\{\}}^W 1:2: < 3:4:(5:p+6:s)7: > / < i > \Rightarrow \left\{ \begin{array}{l} (\mathcal{R}, 2:3:4:(5:p+6:s)7: >, \{(2, id_W), (2, 8:i+\overset{\circ}{W}), (2, 9: > +\overset{\circ}{W}), \{\}) \\ (\mathcal{R}, 3:5:\epsilon 7: >, \{(2, id_W), (5, 8:i+\vec{S}), (5, 10: > +\overset{\circ}{W}), \{\}) \\ (\mathcal{R}, 7:\epsilon, \{(2, id_W), (5, 8:i+\vec{S}), (7, id_W)\}, \{\}) \\ (\mathcal{R}, 3:6:\epsilon 7: >, \{(2, id_W), (6, 8:i+\vec{S}), (6, 10: > +\overset{\circ}{W}), \{\}) \\ (\mathcal{R}, 7:\epsilon, \{(2, id_W), (6, 8:i+\vec{S}), (7, id_W)\}, \{\}) \end{array} \right\}} \quad \text{Sub derivation tree I} \\
\frac{\quad}{\quad} \quad \text{(WordCon)}
\end{array}$$

Figure 19

$$\begin{array}{c}
\text{Sub derivation tree V} \\
\hline
\mathcal{R} \vdash_{\{\}}^W 2:\epsilon/i \Rightarrow \{(\mathcal{R}, 2:\epsilon, \{(2, s, i + \overset{\circ}{w}\}), \{\})\} \quad (\text{EpsW}) \\
\hline
\mathcal{R} \vdash_{\{\}}^W 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: > /i \Rightarrow \left\{ \begin{array}{l} (\mathcal{R}, 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: >, \{(6, s, i + \overset{\circ}{w}\}), \{\}), \\ (\mathcal{R}, 3:5:\epsilon \tau_i: >, \{(5, s, i + \overset{\circ}{s}\}), \{\}), \\ (\mathcal{R}, 3:6:\epsilon \tau_i: >, \{(6, s, i + \overset{\circ}{s}\}), \{\}) \end{array} \right\} \quad (\text{SeqW2}) \\
\hline
\mathcal{R} \vdash_{\{\}}^W 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: > /i \Rightarrow \left\{ \begin{array}{l} (\mathcal{R}, 2:3:4:(5:p + 6:s)\tau_i: >, \{(2, s, i + \overset{\circ}{w}\}), (2, 9: > + \overset{\circ}{w}\}), \{\}), \\ (\mathcal{R}, 3:5:\epsilon \tau_i: >, \{(5, s, i + \overset{\circ}{s}\}), (5, 10: > + \overset{\circ}{w}\}), \{\}), \\ (\mathcal{R}, 7:\epsilon, \{(5, s, i + \overset{\circ}{s}\}), (7, id_W)\}, \{\}), \\ (\mathcal{R}, 3:6:\epsilon \tau_i: >, \{(6, s, i + \overset{\circ}{s}\}), (6, 10: > + \overset{\circ}{w}\}), \{\}), \\ (\mathcal{R}, 7:\epsilon, \{(6, s, i + \overset{\circ}{s}\}), (7, id_W)\}, \{\}) \end{array} \right\} \quad (\text{WordCon})
\end{array}$$

Figure 20

Sub derivation tree I

$$\begin{array}{c}
\text{Sub derivation tree I} \\
\hline
\mathcal{R} \vdash_{\{\}}^W 2:\epsilon/ > \Rightarrow \{(\mathcal{R}, 2:\epsilon, \{(6, 9: > + \overset{\circ}{w}\}), \{\})\} \quad (\text{EpsW}) \\
\hline
\mathcal{R} \vdash_{\{\}}^W 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: > / > \Rightarrow \{(\mathcal{R}, 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: >, \{(2, 9: > + \overset{\circ}{w}\}), \{\})\} \quad (\text{SeqW1}) \\
\hline
\mathcal{R} \vdash_{\{\}}^W 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: > / > \Rightarrow \{(\mathcal{R}, 1:2:\epsilon 3:4:(5:p + 6:s)\tau_i: >, \{(2, 9: > + \overset{\circ}{w}\}), \{\})\} \quad (\text{WordCon})
\end{array}$$

Figure 21

Sub derivation tree II

$$\begin{array}{c}
\frac{\mathcal{R} \vdash_{\{\}}^W 5:\epsilon / > \Rightarrow \{(\mathcal{R}, 5:\epsilon, \{(5, 10: > +\overset{\circ}{w}\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:5:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:5:\epsilon\tau_7: >, \{(5, 10: > +\overset{\circ}{w}\}), \{\})\}} \text{(EpsW)} \\
\frac{\mathcal{R} \vdash_{\{\}}^W 3:5:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:5:\epsilon\tau_7: >, \{(5, 10: > +\overset{\circ}{w}\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W \tau_i: > / > \Rightarrow \{(\mathcal{R}, \tau_i:\epsilon, \{(7, idw)\}), \{\})\}} \text{(LitW1)} \\
\frac{\mathcal{R} \vdash_{\{\}}^W \tau_i: > / > \Rightarrow \{(\mathcal{R}, \tau_i:\epsilon, \{(7, idw)\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:5:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:5:\epsilon\tau_7: >, \{(5, 10: > +\overset{\circ}{w}\}), \{\})\}} \text{(SeqW1)} \\
\frac{\mathcal{R} \vdash_{\{\}}^W 3:5:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:5:\epsilon\tau_7: >, \{(5, 10: > +\overset{\circ}{w}\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:5:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:5:\epsilon\tau_7: >, \{(5, 10: > +\overset{\circ}{w}\}), \{\})\}} \text{(WordCon)}
\end{array}$$

Figure 22

Sub derivation tree III

$$\begin{array}{c}
\frac{\mathcal{R} \vdash_{\{\}}^W 6:\epsilon / > \Rightarrow \{(\mathcal{R}, 6:\epsilon, \{(6, 10: > +\overset{\circ}{w}\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:6:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:6:\epsilon\tau_7: >, \{(6, 10: > +\overset{\circ}{w}\}), \{\})\}} \text{(EpsW)} \\
\frac{\mathcal{R} \vdash_{\{\}}^W 3:6:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:6:\epsilon\tau_7: >, \{(6, 10: > +\overset{\circ}{w}\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W \tau_i: > / > \Rightarrow \{(\mathcal{R}, \tau_i:\epsilon, \{(7, idw)\}), \{\})\}} \text{(LitW1)} \\
\frac{\mathcal{R} \vdash_{\{\}}^W \tau_i: > / > \Rightarrow \{(\mathcal{R}, \tau_i:\epsilon, \{(7, idw)\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:6:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:6:\epsilon\tau_7: >, \{(6, 10: > +\overset{\circ}{w}\}), \{\})\}} \text{(SeqW1)} \\
\frac{\mathcal{R} \vdash_{\{\}}^W 3:6:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:6:\epsilon\tau_7: >, \{(6, 10: > +\overset{\circ}{w}\}), \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:6:\epsilon\tau_7: > / > \Rightarrow \{(\mathcal{R}, 3:6:\epsilon\tau_7: >, \{(6, 10: > +\overset{\circ}{w}\}), \{\})\}} \text{(WordCon)}
\end{array}$$

Figure 23

Sub derivation tree IV

$$\begin{array}{c}
\frac{\mathcal{R} \vdash_{\{\}}^S 5:p/i \Rightarrow \{(\mathcal{R}, 5:\epsilon, \{(5, s:i+\vec{s})\}, \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 4:(5:p+6:s)/i \Rightarrow \{(\mathcal{R}, 5:\epsilon, \{(5, s:i+\vec{s})\}, \{\}), (\mathcal{R}, 6:\epsilon, \{(6, s:i+\vec{s})\}, \{\})\}} \quad \frac{\mathcal{R} \vdash_{\{\}}^S 6:s/i \Rightarrow \{(\mathcal{R}, 6:\epsilon, \{(6, s:i+\vec{s})\}, \{\})\}}{\mathcal{R} \vdash_{\{\}}^W 3:4:(5:p+6:s)7_i > /i \Rightarrow \{(\mathcal{R}, 3:5:\epsilon7_i >, \{(5, s:i+\vec{s})\}, \{\}), (\mathcal{R}, 3:6:\epsilon7_i >, \{(6, s:i+\vec{s})\}, \{\})\}} \\
\text{(LitW2)} \quad \text{(LitW2)} \quad \text{(ChoiceS)} \quad \text{(SeqW2)}
\end{array}$$

Sub derivation tree V

Figure 24
20

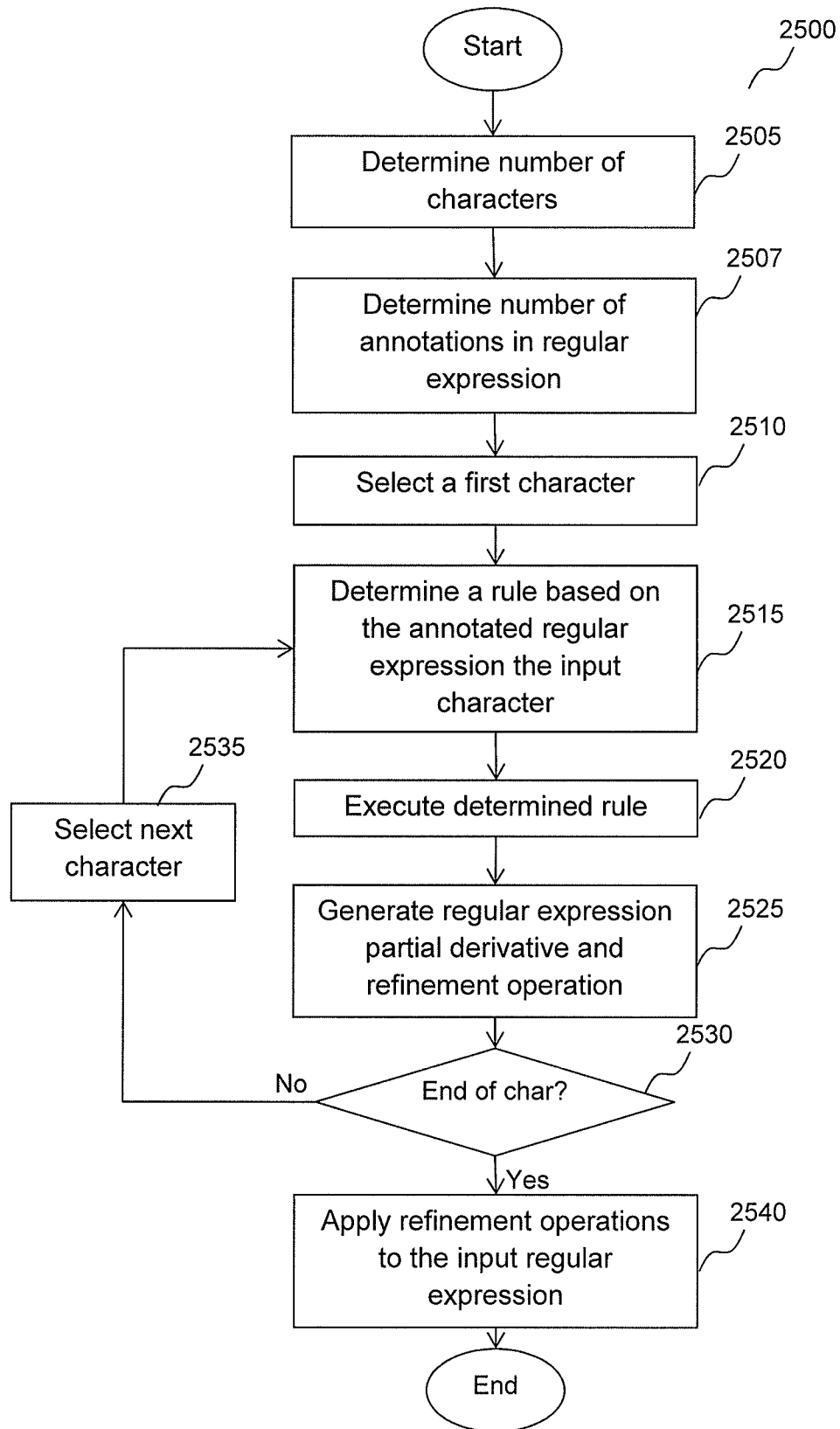


Figure 25