



A METHOD AND SYSTEM FOR PREDICTING BALLISTIC TIME-OF-FLIGHT AND RANGE OF AN UNGUIDED WEAPON

Field of the Invention

The present invention relates to unguided ballistic weapons, and in particular
5 relates to a method and system for predicting the ballistic time-of-flight and range of an
aerially delivered unguided weapon.

Background of the Invention

When a pilot is preparing to release an unguided weapon, such as a bomb, from
an aircraft, it is imperative that he be provided with accurate and timely data to enable
10 him to assess when to release the bomb so as to maximise the chance of impacting the
selected target. Once the pilot has aligned the aircraft with the target, he requires
information to enable him to determine when he should release the bomb to ensure that
the range of the bomb from the point of release will ensure that the target is hit.

One currently available method for predicting the range and time-of-flight
15 utilises look up tables. Bomb manufacturers routinely provide tables of data providing
the range and time-of-flight for an extensive array of combinations of bomb true air
speed, dive angle and altitude at time of release. When utilising this method, the weapon
delivery and navigation system (WDNS) of the aircraft utilise the look up data tables and
interpolation techniques to predict the range and time-of-flight of the bomb based on the
20 true speed, dive angle and altitude of the aircraft (and hence the unreleased bomb) at any
point in time. Once the range of the bomb equals the range of the selected target, the pilot
is provided with an indication that the bomb is ready for release. Alternatively, the
WDNS may be configured to automatically release the bomb at the appropriate time once
the target and weapon have been selected. This approach in calculating the range and
25 time-of-flight is both rapid and direct, however it requires a tremendous amount of
memory storage to store the ballistic tables for the bomb, rendering the method unsuitable
where minimum memory storage and aircraft weight are desired.

An alternate approach uses numerical integration methods to numerically
integrate a set of equations of motion that describe the kinematic behaviour of the bomb
30 to iteratively plot the trajectory of the bomb, and accordingly determine the range and
time-of-flight. Whilst this method is dynamically attractive, it is potentially slow and
difficult to implement. This method also requires a compromise between accuracy and
speed. An example of such a numerical integration method is disclosed in US Patent No.



3748440. This patent discloses a real-time system for calculating the ballistic time-of-flight and range of a weapon from a known altitude above a target using weighted clock pulses for digital integrators in multi-loop function generators, providing a result in less than about 40 msec of actual time. A delay of 40 msec, however, is an entirely unacceptable delay in calculating the range and time-of-flight data for a high speed aircraft, where the aircraft will have flown several tens of feet during the calculation period, effecting the accuracy of delivery of the bomb.

Object of the Invention

It is an object of the present invention to overcome or substantially ameliorate at least one of the above disadvantages, or at least to provide a useful alternative.

Summary of the Invention

In a first aspect, the present invention provides a method of predicting range and time-of-flight of an aerially delivered unguided weapon comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

In a second aspect, the present invention provides a method of aerially delivering an unguided weapon from an aircraft comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

selecting a target;

aligning said aircraft with said target;

determining the range of said target from said aircraft;

determining altitude, climb angle and true air speed of said aircraft;

calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations; and

releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

In a third aspect, the present invention provides a system for predicting range and time-of-flight of an aerially delivered unguided weapon comprising:

data storage means storing a look-up table of ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

aircraft mounted data acquisition means for determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

a first processor module operating on codified empirical equations approximating the range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and

time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

a second processor module operating on a parameter optimization code to calculate substantially optimal values of said correction constants for said weapon by correlating solutions of said empirical equations with said look-up table ballistic results; and

an aircraft mounted third processor module operating on said empirical equations to calculate predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

In a fourth aspect, the present invention provides a system for aurally delivering an unguided weapon from an aircraft to a selected target comprising:

processor means for calculating predicted range and time-of-flight of said weapon, said processor means including codified empirical equations approximating range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight, said processor means including substantially optimal values of said correction constants for said weapon derived using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

flight control means for aligning said aircraft with said target;

first data acquisition means for determining the range of said target from said aircraft;

second data acquisition means for determining altitude, climb angle and true air speed of said aircraft for use by said processor means in calculating said predicted weapon range and time-of-flight; and

means for releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

Typically, said theoretical time-of-flight equation is:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zint}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{zint} = vertical component of weapon release true air speed

Typically, said theoretical range equation is:

$$r = \frac{mV_{xint}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

where:

r = weapon range,

m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xint} = horizontal component of weapon release true air speed

Preferably, said empirical equations include a modification of the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

Preferably, said empirical equations step include a modification of said drag factor using correction factors based on weapon release true air speed and air density.

Preferably, said empirical equations include a modification of the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on
 5 weapon release altitude, the vertical component of the weapon release true air speed and air density.

Preferably, said empirical equations includes a modification of the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

10 Preferably, said empirical equations include a modification of the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

Brief Description of the Drawings

A preferred form of the present invention will now be described by way of
 15 example with reference to the accompanying drawings, wherein:

Figure 1 is a schematic side elevation view of an aircraft carrying an unguided bomb and flying above a target.

Figure 2 is a schematic representation of the forces acting on a bomb.

Figure 3 is a schematic representation of the horizontal component of the forces
 20 acting on the bomb of Figure 2.

Figure 4 is a schematic representation of the vertical component of the vertical forces acting on the bomb of Figure 2.

Figure 5 is a flow diagram of a method of predicting time-of-flight and range of a bomb.

25 Detailed Description of the Preferred Embodiments

Figure 1 depicts an aircraft 1 carrying an unguided weapon in the form of a bomb 2 and flying at an altitude z above ground level (AGL). The bomb 2 is carried underneath the aircraft wing or fuselage on a weapon pylon 3. The weapon pylon 3 includes means 4 for releasing the bomb, typically in the well known form of a releasable
 30 detent means and ejectors which force the bomb away from the pylon on release of the detent means in response to a command from the pilot relayed through the WDNS computer 6 of the aircraft. A target 5 is located a range r from the aircraft. The range r can be readily determined by the aircraft's data acquisition systems, such as by a laser

target designator and range finder (LTDR) 7 usually used to designate targets for laser guided weapons.

Once the pilot has selected the target 5, he needs to align the aircraft 1 with the target 5 such that the target 5 is in the same vertical plane as the plane of motion of the aircraft 1. This is achieved in the usual manner with control inputs from the pilot being processed through the flight control computer (FCC) 8 to the various flight control surfaces 9 of the aircraft. To decide when to release the bomb 2, the pilot needs to know how far the bomb 2 will travel (it's range) if it is released at any point in time, and ideally release the bomb 2 when the bomb range is equal to the range of the target 5.

To assess the range of the bomb, the ballistic motion of an aurally delivered unguided weapon can first be greatly simplified by considering the weapon as a fixed weight point-mass moving over a flat non-rotating earth. Such a point-mass is represented in Figure 2. The forces acting on the point-mass are a combination of gravity and aerodynamic drag, and the general system of equations of motion acting on the mass can be represented as follows:

$$m \frac{dV}{dt} = -D - mg \sin \gamma \quad (1)$$

$$mV \frac{d\gamma}{dt} = -mg \cos \gamma \quad (2)$$

$$\frac{dz}{dt} = V \sin \gamma \quad (3)$$

$$\frac{dx}{dt} = V \cos \gamma \quad (4)$$

where:

V = true airspeed,

m = bomb mass,

D = drag,

γ = flight path climb angle,

z = altitude above ground level,

x = horizontal distance,

g = acceleration due to earth's gravity, and

t = time.

The aircraft true airspeed, flight path climb angle and altitude are available from the aircraft's flight data acquisition systems 10, including, for example pitot-static probes, global positioning system (GPS), gyroscopes and processors processing data output from these data sources.

5 When a fairly accurate aerodynamic drag model of the bomb is included in the motion, it renders the above equations extremely nonlinear, and no general solution in closed form has been found that is satisfactory for an airborne flight-control computer.

To simplify these equations, drag (D) is assumed to be a function of true air speed, giving:

$$10 \quad D = bV \quad (5)$$

where:

b = unknown drag factor constant.

Applying the assumed drag function of Eqn (5) to Eqn (1) above, and considering motion in the horizontal plane, as depicted in Figure 3, we get:

$$15 \quad m \frac{d^2 x}{dt^2} = -b \frac{dx}{dt} \quad (6)$$

The solution to this Eqn (6), for the forward travel of the bomb at any point in time, can be approximated with the following trial solution:

$$x = a(1 - e^{-kt}) \quad (7)$$

where:

20 a, k = unknown constants.

$$\frac{dx}{dt} = ake^{-kt} = V_x \quad (8)$$

$$\frac{d^2 x}{dt^2} = -ak^2 e^{-kt} \quad (9)$$

Substituting Eqns (9) and (8) into (6), we get:

$$25 \quad \begin{aligned} -mak^2 e^{-kt} &= -bake^{-kt} \\ \Rightarrow k &= \frac{b}{m} \end{aligned} \quad (10)$$

From Eqn (9), at $t = 0$, we get:

$$\begin{aligned}
 V_{xini} &= ak \\
 &= a \frac{b}{m} \\
 \rightarrow a &= \frac{mV_{xini}}{b}
 \end{aligned} \tag{11}$$

Substituting Eqns (10) and (11) into Eqn (7), we get the horizontal distance travelled by the bomb (known as the forward-throw) at any point in time as:

$$x = \frac{mV_{xini}}{b} (1 - e^{-\frac{b}{m}t}) \tag{12}$$

If we now consider Eqn (1) for the component of motion in the vertical plane, as depicted in Figure 4, we get:

$$m \frac{d^2 z}{dt^2} = -b \frac{dz}{dt} - mg \tag{13}$$

The solution to this Eqn (13), for the altitude of the bomb at any point in time, can be approximated with the following trial solution:

$$z = c(1 - e^{-\frac{b}{m}t}) + dt + f \tag{14}$$

where:

c, d, f = unknown constants.

$$\frac{dz}{dt} = \frac{bc}{m} e^{-\frac{b}{m}t} + d = V_z \tag{15}$$

$$\frac{d^2 z}{dt^2} = -\frac{b^2}{m^2} c e^{-\frac{b}{m}t} \tag{16}$$

Substituting Eqns (16) and (15) into Eqn (13), we get:

$$m(-\frac{b^2}{m^2} c e^{-\frac{b}{m}t}) + b(\frac{bc}{m} e^{-\frac{b}{m}t} + d) + mg = 0 \tag{17}$$

From Eqn (17) at $t = 0$, we get:

$$-\frac{b^2}{m} c + \frac{b^2}{m} c + bd + mg = 0 \tag{18}$$

$$\rightarrow d = -\frac{mg}{b} \quad (19)$$

From Eqn (9), at $t = 0$, we get:

$$V_{zini} = \frac{bc}{m} + \left(-\frac{mg}{b}\right) \quad (20)$$

$$\rightarrow c = \frac{m}{b} \left(V_{zini} + \frac{mg}{b}\right) \quad (21)$$

5 Also, at $t = 0$, $z = z_0$, from Eqn (14), we get:

$$f = z_0 \quad (22)$$

Substituting Eqns (19), (21), and (22) into Eqn (14), we get:

$$z = z_0 + \frac{m}{b} \left(V_{zini} + \frac{mg}{b}\right) \left(1 - e^{-\frac{b}{m}t}\right) - \frac{mg}{b}t$$

or

$$10 \quad z = z_0 + \left(\frac{mV_{zini}}{b} + \frac{m^2g}{b^2}\right) \left(1 - e^{-\frac{b}{m}t}\right) - \frac{mg}{b}t \quad (23)$$

Eqn (23) represents the height of the bomb above the ground at t seconds after it is released from the aircraft. Clearly, to obtain an explicit solution for the time-of-flight taken by the bomb before it hits the ground (i.e. $z=0$) using Eqn (23) is nearly impossible.

15 Accordingly, it is necessary to derive an approximate solution for the ballistic time-of-flight (t_f).

Firstly, let:

$$l = \frac{mV_{zini}b + m^2g}{b^2} \quad (24)$$

From Eqn (23), we get:

$$z = z_0 + l - le^{-\frac{b}{m}t} - \frac{mg}{b}t \quad (25)$$

20 At $t = t_f$, $z = 0$. Eqn (25) gives:

$$0 = z_0 + l - le^{-\frac{b}{m}t_f} - \frac{mg}{b}t_f \quad (26)$$

or

$$\frac{mg}{b}t_f + le^{-\frac{b}{m}t_f} - z_0 - l = 0 \quad (27)$$

Eqn (27), can be simplified by making the following approximation:

$$e^{-\frac{b}{m}t_f} = 1 - \eta\sqrt{t_f} \quad (28)$$

where:

5 $\eta = \text{unknown constant.}$

Substituting Eqn (28) into Eqn (22), we obtain:

$$\frac{mg}{b}t_f - l\eta\sqrt{t_f} - z_0 = 0 \quad (29)$$

Eqn (29) represents a readily solvable quadratic equation in $\sqrt{t_f}$. Solving this equation, we get:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2 \quad (30)$$

Eqn (30) thus provides the predicted ballistic time-of-flight (t_f).

Accordingly, Eqns (12) and (30) are theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of the bomb when it is considered to be a point-mass moving over a flat non-rotating earth.

15 As these equations are theoretical, and do not account for a complex aerodynamic drag model, the results utilising these approximate theoretical solutions do not provide sufficient accuracy to be relied upon in practice. Accordingly, Eqns (12) and (30) are modified utilising various correction factors in an attempt to provide empirical solutions which more closely match known actual results.

20 At any point in time prior to release of the bomb 2 from the aircraft 1, the aircraft true air speed (V_T), altitude ($z=h$) and climb angle (γ) are known from the flight control computer (FCC) 8, determined from the various flight data acquisition systems 10 of the aircraft. With the bomb being fixed to the aircraft prior to release, the true air speed, altitude and climb angle of the bomb will clearly equal that of the aircraft.

25 The true air speed (V_T) can be resolved into vertical and horizontal components as follows:

$$V_{xmut} = V_T \cos \gamma \quad - \text{horizontal component} \quad (31)$$

$$V_{zmut} = V_T \sin \gamma \quad - \text{vertical component} \quad (32)$$

The air density (ρ) at a given altitude (in metres) can be approximated by the following:

$$\rho = \rho_0 e^{-\frac{h}{10000}} \quad (33)$$

where:

ρ_0 = air density at sea level.

In modifying the approximate theoretical results of Eqns (12) and (30), correction factors are applied based on parameters which are expected to effect the theoretical result. These parameters are expected to include altitude, climb angle (or dive angle, being equal to $-\text{climb angle}$), true air speed, air density and time-of-flight (for the range calculation).

A first group of correction factors are applied to the vertical velocity component (V_{zmut}) as follows:

$$V_z = V_{zmut} \left(1 + \beta_1 \cos \gamma + \frac{\beta_2}{\sqrt{\rho}} + \beta_3 \frac{V_T}{250} + \beta_4 \frac{h}{1500} \right) \quad (34)$$

Where $\beta_1, \beta_2, \beta_3$ and β_4 are the first through fourth correction constants.

The velocity correction factor has a division of the true air speed (V_T) by 250 m/s in an attempt to reduce the order of magnitude of the velocity expression to single digits. Similarly, the altitude correction constant has a division by 1500 m.

A second group of correction factors are applied to calculate the drag factor (b) as follows:

$$b = 50 + \beta_5 \rho + \beta_6 \rho \frac{V_T}{250} + \beta_7 \left(\frac{V_T}{250} \right)^{\beta_8} \quad (35)$$

Where $\beta_5, \beta_6, \beta_7$ and β_8 are the fifth through eighth correction constants.

From Eqn (28) we have an unknown constant η , which we relabel as the ninth correction constant (β_9).

To simplify expression of Eqn (30), we then let:

$$p_1 = \beta_9 l \quad (36)$$

$$p_2 = \sqrt{p_1^2 + \beta_{10} \frac{4mgh}{b}} \quad (37)$$

Where β_{10} is the tenth correction constant.

Eqn (30) can thus be rewritten as:

$$t_f = \left(\frac{p_1 + p_2}{2mg/b} \right)^2 \quad (38)$$

5 Further correction factors are then added to provide an empirical equation approximating the time-of-flight of the bomb as follows:

$$t_f = \left(\frac{p_1 + p_2}{2mg/b} \right)^2 + \beta_{11} \frac{h}{1500} + \beta_{12} \left(\frac{h}{1500} \right)^{\beta_{13}} + \beta_{14} \rho + \beta_{15} V_z \sqrt{p} + \beta_{16} V_z \quad (39)$$

Where β_{11} , β_{12} , β_{13} , β_{14} , β_{15} and β_{16} are the eleventh through sixteenth correction constants.

10 A further group of correction factors is applied to the horizontal velocity component (V_{xint}):

$$V_x = V_{xint} \left(1 + \beta_{17} \sin \gamma + \frac{\beta_{18}}{\sqrt{\rho}} + \beta_{19} \frac{V_T}{250} + \beta_{20} \frac{h}{1500} \right) \quad (40)$$

Where β_{17} , β_{18} , β_{19} and β_{20} are the seventh through twentieth correction constants.

15 To simplify Eqn (12), and to add a further correction constant, we let:

$$p_3 = 1 - e^{-\frac{b}{m} t_f} + \beta_{21} \quad (41)$$

Where β_{21} is a twenty-first correction factor.

Substituting this into Eqn (12), and adding further correction factors, we arrive at an empirical equation approximately the range or forward-throw of the bomb as follows:

$$r = \left\{ \frac{mV_x}{b} p_3 + \beta_{22} V_x t_f + \beta_{23} t_f^2 + \beta_{24} V_x \sqrt{\rho} + \beta_{25} \frac{V_T h}{1500} + \beta_{26} \frac{h}{1500} + \right. \\ \left. \beta_{27} \left(\frac{h}{1500} \right)^2 + \beta_{28} \sqrt{\rho} + \beta_{29} (V_x \rho)^2 + \beta_{30} V_x t_f^2 + \beta_{31} V_T (\cos \gamma)^{\beta_{32}} + \beta_{33} V_x \right\} \quad (42)$$

Where $\beta_{22}, \beta_{23}, \beta_{24}, \beta_{25}, \beta_{26}, \beta_{27}, \beta_{28}, \beta_{29}, \beta_{30}, \beta_{31}, \beta_{32}, \beta_{33}$ are the twenty-second through thirty-third correction constants.

All of the above equations are expressed in SI units, however it will be usual to convert the range result of Eqn 42 into British units of feet for use by the pilot.

5 The thirty-three correction constants β_1 to β_{33} now need to be determined to provide the best fit of results utilising the above Eqns to fit the tabulating results provided by the manufacturer for a given weapon. These correction constants can be selected by solving the following parameter optimization problem utilising any standard optimization technique:

$$10 \quad J = w_1 \sum_1^n \left(\frac{t_{\text{empirical}} - t_{\text{table}}}{20} \right)^2 + w_2 \sum_1^n \left(\frac{r_{\text{empirical}} - r_{\text{table}}}{1500} \right)^2 \quad (43)$$

where:

J = performance index to be minimised with the optimization process,

w_1, w_2 = weightage factors to be selected dependent upon the relative importance of range accuracy versus time-of-flight accuracy,

15 $t_{\text{empirical}} - t_{\text{table}}$ = time-of-flight error between empirical fit values utilising the above Eqns and the tabulated values,

$r_{\text{empirical}} - r_{\text{table}}$ = range error between empirical fit values utilising the above Eqns and tabulated values,

n = number of data points compared.

20 The result from solving this optimization problem, which has the time-of-flight and range errors normalised by 20 seconds and 1500 metres respectively, is the set of optimal correction constants β_1 to β_{33} . These optimal correction constants are bomb specific.

25 As accuracy of the range calculation is much more critical than the accuracy of the time-of-flight calculation, the second weightage factor (w_2) should typically be selected to be much greater than the first weightage factor (w_1). These weightage factors can be changed through trial and error if desired in a further attempt to minimise the range and time-of-flight errors in calculating the optimal correction constants.

30 With the empirical equations (39) and (42) approximating the time-of-flight and range of an unguided weapon 2 having been derived manually, these empirical equations are next implemented to predict time-of-flight and range by first codifying the empirical equations into a first process module 11 as depicted in the flow diagram of Figure 5. The person skilled in the art will be well versed in standard methods to code such an equation

into the first processor module 11. A look-up table of ballistic results of range and time-of-flight data for the weapon of interest is stored in a data storage means 12. Again, the person skilled in the art will be well versed in standard data storage devices. A second processor module 13 operates on a standard parameter optimization code utilizing standard optimization methods such as a least square regression method, Newton's method or a Taylor series reduction method. Again the person skilled in the art will be well versed in such numerical optimization methods. The parameter optimization code is used to calculate substantially optimal values of the correction constants β_1 to β_{33} using the parameter optimization Eqn (43).

The first and second processor modules 11, 13 can be a single processor module, with these operations to be being carried out in a processor, such as in a PC, on the ground, providing substantially optimal values of the correction constants for loading into the aircraft. When the aircraft 1 is loaded with a given unguided weapon 2, the optimal correction constants for that weapon will be loaded into an aircraft mounted third processor module, which would typically consist of the WDNS computer 6. Alternatively, the optimal correction constants for various unguided weapons to be used with a given aircraft type might already be preloaded into the WDNS computer 6. Loading of a particular weapon will then activate retrieval of the preloaded correction constants for that weapon. The WDNS computer 6 also operates on the empirical equations to calculate predicted range and time-of-flight of the weapon by applying the aircraft climb angle, true air speed and altitude, available from the aircraft flight data acquisition systems 10, and the optimal correction constant values for the weapon to the empirical equations.

With the correction constants having already calculated, the calculation of range and time-of-flight is almost instantaneously calculated directly from the empirical equations, enabling instant feedback to the pilot.

Once the pilot has selected a target 5, aligned the aircraft 1 with the target 5 and armed the unguided weapon 2, the WDNS computer 6 will continually update the predicted range and time-of-flight of the weapon and compare the same with the continually updated range of the target provided by the laser target designator and range finder (LTDR). When the predicted range of the weapon equates with that of the selected target, a signal will be provided to the pilot to indicate that the weapon 2 should be released so as to maximize the chance of hitting the target. The WDNS computer 6 could be configured such that the predicted time prior to release is continually calculated based on the difference between the target range and the predicted weapon range, and the

horizontal component of the aircraft true air speed (V_x) so as that the pilot effectively gets a count down to when the weapon should be released. This will minimise reaction delay times of the pilot. In another alternative configuration, the WDNS computer 6 will be configured to automatically release the weapon 2 at the appropriate time when the
 5 predicted weapon range and target range coincide once the weapon 2 has been armed by the pilot.

Whilst the calculation of the optimal values of the correction constants will typically be carried out in separate processors on the ground, it is envisaged that the optimal correction constants calculations might be carried out within the WDNS
 10 computer, prior to the commencement of any relevant bombing missions.

Example

The empirical equations (39) and (42) derived above were applied to a MK 82 bomb having a weight of 500 pounds (mass of 227.27 kg). Table 1 provides a look-up-table data of known ballistic range and time-of-flight for an array of combinations of
 15 altitude, dive angle and true air speed (all expressed in British units) as supplied by the original equipment manufacturer. Solution of the optimization problem of Eqn (43), utilising weightage factors of $w_1 = 8000$ and $w_2 = 213000$, provided the following table of correction constants (in row then column order):

20	$\beta_{1-33} =$	100*[0.00291082347602	0.00066850452991	-0.00138340193913
		-0.00638158812863	1.67519490029754	0.14592396993844
		-1.97903705273431	-0.00090418306503	0.00256277380455
		0.00436642397023	-0.03447759688978	0.14735863202910
		0.00719707351935	0.00231809175132	-0.00303710633834
25		0.00349536751854	0.00135259750495	0.00062295741717
		-0.00202951535514	0.00107244877289	0.00028695252947
		0.00202603830698	0.00144499816415	-0.02368766941289
		0.00208232078074	-1.08872607704417	-2.41869503298574
		0.27630999987104	0.00002857107493	-0.00007086918037
30		-0.00521386015853	0.03964192332155	0.02113397094226]

Utilising these correction factors provided an average error for the bomb range of 390.48 mm (1.2811 ft), a root mean square (RMS) error of 531.82 mm (1.7448 ft) and a maximum error of 1893.5 mm (6.2124 ft). The average error for the time-of-flight was 0.0362 secs, with an RMS error of 0.0465 secs and a maximum error of 0.1571 secs.

It can thus be seen that the empirical calculations provide an extremely accurate prediction of range and time-of-flight over the full spectrum of typical combinations of altitude, dive angle and true air speed as set out in Table 1.

The methods described have also been applied to the tabulated ballistic data of
 5 four other bombs, with results as follows:

	Time-of-flight Error(s)			Range Error		
	Avg	RMS	Max	Avg	RMS	Max
mk82hd1	0.0341	0.0480	0.1983	500.79mm (1.6430ft)	673.27mm (2.2089ft)	2967.1mm (9.7347ft)
mk83	0.0654	0.0840	0.4017	703.81mm (2.3091ft)	3950.37mm (3.1180ft)	4419.6mm (14.5001ft)
mk831	0.0559	0.0744	0.4375	563.64mm (1.8492ft)	730.06mm (2.3952ft)	2916.2mm (9.5676ft)
mk84	0.0974	0.1233	0.4124	1006.1mm (3.3009ft)	1016.5mm (4.3349ft)	4402.5mm (14.4440ft)

CLAIMS:

1. A method of predicting range and time-of-flight of an aerially delivered unguided weapon comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

2. The method of claim 1 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{xmi}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{xmi} = vertical component of weapon release true air speed

- 5 3. The method of either of claims 1 and 2 wherein said theoretical range equation is:

$$r = \frac{mV_{xmi}}{b} (1 - e^{-\frac{b}{m}t_f})$$

where:

r = weapon range,

10 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xmi} = horizontal component of weapon release true air speed

4. The method of any one of claims 1 to 3 wherein said modifying step
15 includes modifying the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

5. The method of either of claims 2 and 3 wherein said modifying step includes modifying said drag factor using correction factors based on weapon release true
20 air speed and air density.

6. The method of any one of claims 1 to 5 wherein said modifying step includes modifying the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

- 25 7. The method of any one of claims 1 to 6 wherein said modifying step includes modifying the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

8. The method of any one of claims 1 to 7 wherein said modifying step
30 includes modifying the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

9. A method of aurally delivering an unguided weapon from an aircraft comprising the steps of:

providing theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass;

5 modifying said theoretical equations utilising a plurality of correction factors to provide empirical equations approximating the range and time-of-flight of said weapon, each said correction factor comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

10 calculating substantially optimal values of said correction constants for said weapon using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

15 selecting a target;

aligning said aircraft with said target;

determining the range of said target from said aircraft;

determining altitude, climb angle and true air speed of said aircraft;

20 calculating predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations; and

releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

25 10. The method of claim 9 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

30 m = weapon mass,

z_0 = weapon release altitude,

b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zmit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{zmit} = vertical component of weapon release true air speed

- 5 11. The method of either of claims 9 and 10 wherein said theoretical range equation is:

$$r = \frac{mV_{xmit}}{b} (1 - e^{-\frac{b}{m}t_f})$$

where:

r = weapon range,

10 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xmit} = horizontal component of weapon release true air speed

- 15 12. The method of any one of claims 9 to 11 wherein said modifying step includes modifying the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

- 20 13. The method of either of claims 10 and 11 wherein said modifying step includes modifying said drag factor using correction factors based on weapon release true air speed and air density.

14. The method of any one of claims 9 to 13 wherein said modifying step includes modifying the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

- 25 15. The method of any one of claims 9 to 14 wherein said modifying step includes modifying the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

- 30 16. The method of any one of claims 9 to 15 wherein said modifying step includes modifying the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

17. A system for predicting range and time-of-flight of an aerially delivered unguided weapon comprising:

data storage means storing a look-up table of ballistic results of range and time-of-flight for said weapon over an array of combinations of weapon release altitude, climb angle and true air speed;

aircraft mounted data acquisition means for determining altitude, climb angle and true air speed of said aircraft prior to release of said weapon; and

a first processor module operating on codified empirical equations approximating the range and time-of-flight of said weapon, said empirical equations being derived by modifying theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight;

a second processor module operating on a parameter optimization code to calculate substantially optimal values of said correction constants for said weapon by correlating solutions of said empirical equations with said look-up table ballistic results; and

an aircraft mounted third processor module operating on said empirical equations to calculate predicted range and time-of-flight of said weapon by applying said aircraft climb angle, true air speed and altitude and said optimal correction constant values to said empirical equations.

18. The system of claim 17 wherein said first and third processor modules comprise a single processor module.

19. The system of claim 17 wherein said first and second processor modules comprise a single processor module.

20. The method of any one of claims 17 to 19 wherein said theoretical time-of-flight equation is:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

5 b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{xmit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{xmit} = vertical component of weapon release true air speed

10 21. The system of any one of claims 17 to 20 wherein said theoretical range equation is:

$$r = \frac{mV_{xmit}}{b} (1 - e^{-\frac{b}{m}t_f})$$

where:

r = weapon range,

15 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xmit} = horizontal component of weapon release true air speed

20 22. The system of any one of claims 17 to 21 wherein said empirical equations include a modification of the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

23. The system of either of claims 20 and 21 wherein said empirical equations include a modification of said drag factor using correction factors based on
25 weapon release true air speed and air density.

24. The system of any one of claims 17 to 23 wherein said empirical equations include a modification of the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

30 25. The system of any one of claims 17 to 24 wherein said empirical equations include a modification of the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

25. The system of any one of claims 17 to 24 wherein said empirical equations include a modification of the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.

5 26. A system for aerially delivering an unguided weapon from an aircraft to a selected target comprising:

processor means for calculating predicted range and time-of-flight of said weapon, said processor means including codified empirical equations approximating range and time-of-flight of said weapon, said empirical equations being derived by
 10 modifying theoretical equations approximating the theoretical range and time-of-flight of ballistic trajectory motion in a vertical plane of a point-mass moving over a flat non-rotating earth as a function of weapon release altitude, climb angle and true air speed and weapon mass, said empirical equations being derived from said theoretical equations utilising a plurality of correction factors each comprising a function of a correction
 15 constant and a parameter selected from the group of weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight, said processor means including substantially optimal values of said correction constants for said weapon derived using a computational parameter optimization process correlating solutions of said empirical equations with known ballistic results of range and time-of-flight for said weapon over an
 20 array of combinations of weapon release altitude, climb angle and true air speed;

flight control means for aligning said aircraft with said target;

first data acquisition means for determining the range of said target from said aircraft;

25 second data acquisition means for determining altitude, climb angle and true air speed of said aircraft for use by said processor means in calculating said predicted weapon range and time-of-flight; and

means for releasing said weapon when said predicted range of said weapon is substantially equal to said range of said target.

27 The system of claim 26 wherein said theoretical time-of-flight equation
 30 is:

$$t_f = \left[\frac{l\eta + \sqrt{l^2\eta^2 + \frac{4mg}{b}z_0}}{\frac{2mg}{b}} \right]^2$$

where:

t_f = weapon time-of-flight,

m = weapon mass,

z_0 = weapon release altitude,

5 b = unknown drag factor constant,

η = unknown constant,

$$l = \frac{mV_{zmit}b + m^2g}{b^2}$$

g = acceleration due to earth's gravity, and

V_{zmit} = vertical component of weapon release true air speed

10 28. The system of either of claims 26 and 27 wherein said theoretical range equation is:

$$r = \frac{mV_{xmit}}{b} \left(1 - e^{-\frac{b}{m}t_f} \right)$$

where:

r = weapon range,

15 m = weapon mass,

b = unknown drag factor constant,

t_f = weapon time-of-flight,

V_{xmit} = horizontal component of weapon release true air speed

20 29. The system of any one of claims 26 to 28 wherein said empirical equations include a modification of the vertical component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

30. The system of either of claims 27 and 28 wherein said empirical equations step include a modification of said drag factor using correction factors based on
25 weapon release true air speed and air density.

31. The system of any one of claims 26 to 31 wherein said empirical equations include a modification of the time-of-flight calculated with said theoretical time-of-flight equation using correction factors based on weapon release altitude, the vertical component of the weapon release true air speed and air density.

30 32. The system of any one of claims 26 to 32 wherein said empirical equations includes a modification of the horizontal component of weapon release true air speed using correction factors based on weapon release altitude, climb angle and true air speed and air density.

34. The system of any one of claims 26 to 33 wherein said empirical equations include a modification of the weapon range using correction factors based on weapon release altitude, climb angle and true air speed, air density and weapon time-of-flight.