

METHOD FOR IDENTIFYING AND AUTOMATIC MONITORING THE
VIBRATIONAL MODES OF A CIVIL STRUCTURE AND SYSTEM FOR
AUTOMATIC MONITORING THE CIVIL STRUCTURE

5 The present invention relates to a method for identifying
and automatic monitoring the vibrational modes of a civil
structure and system for automatic monitoring the civil
structure, particularly but not exclusively employable
in the field of so-called *Structural Health Monitoring*
10 or SHM, i.e., the monitoring of civil structures or works
such as bridges, viaducts, buildings, and so on.
Nowadays, it is known to monitor the state of a civil
structure or work by arranging a system of sensors
positioned at different points of the structure to be
15 monitored. Such sensors can be vibration sensors,
displacement sensors, and so on.
The detections of the sensors are usually stored in
databases accessible by specialized operators by one or
more processing units, such as for example personal
20 *computers, workstations*, and so on. The detections of
the sensors are generally processed according to known
techniques in order to recognize or identify and classify
the vibrational modes of the civil structure under
investigation. Subsequently, the operators, based on
25 their experience and the observation of the evolution of
the identified vibrational modes, assess the safety
state of the civil structure. The analysis of vibrational
modes or *Operational Modal Analysis* (OMA) has the
advantage of being usable during the dynamic operation
30 of the civil structure without requiring a controlled
test environment with certain loading conditions. This
technique is also referred to as *output-only modal*

analysis as it is processed only based on the response of the civil structure following an uncontrolled and unknown input such as for example the environmental and operating forcing of the structure (wind, traffic, micro-tremors, and so on).

One of the best-known techniques in the field of OMA is the so-called *Frequency Domain Decomposition* or FDD, which is based on a non-parametric algorithm that operates in the frequency domain and starting from a decomposition into singular values (SVD) of the power spectral density (PSD) matrix, is able to determine the modal frequencies and forms of the civil structure under analysis.

In recent years, the need to automate and digitize structural monitoring activities has grown increasingly, not only to accelerate the extrapolation of results but also to make them more easily interpretable by operators and experts in the field. This is where the so-called *Automated Operational Modal Analysis* or AOMA is from: i.e., the technique that allows to perform OMA in a completely automatic manner.

There are various techniques capable of performing automatic identification of the vibrational modes.

In particular, classifying methods based on neural networks are known, capable of automatically classifying vibrational modes.

Such known techniques require a large amount of data and extremely complex and computationally expensive algorithms.

Furthermore, the known techniques still require the intervention of a specialized operator both to control the correct operation of the method for classifying the

vibrational modes of the civil structure and for their monitoring over time. The intervention of the operator implies non-deterministic evaluations and therefore, in principle, non-repeatable and debatable.

- 5 The document Favarelli Elia et al: "Machine Learning for Automatic Processing of Modal Analysis in Damage Detection of Bridges", IEEE Transactions on Instrumentation and Measurement, IEEE, USA, vol. 70, 16 November 2020, pages 1-13, describes an example of a
10 machine learning algorithm for detecting anomalies in a bridge from vibrational data.

The object of the present invention is to overcome the above-mentioned drawbacks and, in particular, to ideate a method for identifying and automatic monitoring the
15 vibrational modes of a civil structure and a system for automatic monitoring such civil structure, capable of providing repeatable and reliable results while at the same time reducing the computational cost compared to the known techniques.

- 20 This and other objects according to the present invention are achieved by making a method for identifying and automatic monitoring the vibrational modes of a civil structure and a system for automatic monitoring such civil structure as set forth in claim 1.

- 25 Further features of the method for identifying and automatic monitoring the vibrational modes of a civil structure and of a system for automatic monitoring such civil structure are the object of the dependent claims. The features and advantages of a method for identifying
30 and automatic monitoring the vibrational modes of a civil structure and of a system for automatic monitoring such civil structure according to the present invention will

be more apparent from the following exemplary and not limiting description, referred to the attached schematic drawings in which:

- Figure 1 is a flowchart of an embodiment of a method
5 for identifying and automatic monitoring the vibrational modes of a civil structure according to the present invention;
- Figure 2A is a flowchart of a possible embodiment of a first numerical processing comprised in the method for
10 identifying and automatic monitoring the vibrational modes of a civil structure according to the present invention;
- Figure 2B is a flowchart of a possible embodiment of a step of performing a *frequency domain decomposition*
15 algorithm comprised in the method for identifying and automatic monitoring the vibrational modes of a civil structure according to the present invention;
- Figure 2C is a flowchart of a possible embodiment of a training process comprised in the method for
20 identifying and automatic monitoring the vibrational modes of a civil structure according to the present invention;
- Figure 2D is a flowchart of a possible embodiment of a monitoring process comprised in the method for
25 identifying and automatic monitoring the vibrational modes of a civil structure according to the present invention;
- Figure 3 is a block diagram of a system for automatic monitoring a civil structure according to the present
30 invention.

With reference to the figures, a method for identifying and automatic monitoring the vibrational modes of a civil

structure, overall indicated by 100, is shown.

Such method 100 for identifying and automatic monitoring is implementable by an electronic calculator or computer provided with a memory for data storage. In particular,
5 the method 100 for identifying and automatic monitoring is actuatable by a program or software for identifying and automatic monitoring loaded in the memory of the electronic calculator or computer.

Such program for identifying and automatic monitoring
10 thus comprises instructions which induce the electronic computer or computer to actuate the method 100 for identifying and automatic monitoring when the electronic computer or computer performs the program.

The program for identifying and automatic monitoring is
15 associated with a system for automatic monitoring a civil structure 200 comprising a plurality of acceleration sensors 210 in a number equal to N positioned in respective points of the civil structure. For example, the civil structure can be a bridge and the acceleration
20 sensors can be accelerometers positioned at points of the span.

The system for automatic monitoring 200 also comprises a central processing unit 220 associated with the acceleration sensors 210 in a wireless and/or wired
25 manner and programmed to actuate the method 100 for identifying and automatic monitoring.

The central processing unit 220, in particular, comprises a database in which the program for identifying and monitoring is stored. The central processing unit
30 220 can be physically accessible to a user or can be reachable by other computers or electronic terminals by any telecommunication network, such as the Internet; in

such case, the method 100 for identifying and automatic monitoring can be actuated remotely in a so-called cloud, i.e., in a remote memory space reachable by the Internet or other networks.

5 The method 100 for identifying and automatic monitoring comprises the initial step of receiving 110 a plurality of detecting data from the plurality of acceleration sensors applied in respective points of the civil structure.

10 Then, the method 100 for identifying and automatic monitoring provides to perform at least once a day the steps that will be described below.

First, a first numerical processing 120 of the detecting data is performed in order to eliminate data related to
15 spurious acceleration peaks and acceleration peaks caused by forcing events, such as for example heavy traffic or the passage of trains in the case of railway bridges.

Therefore, the object of the first numerical processing
20 120 is to bring the detecting data of the civil structure under normal operating conditions as close as possible to those obtainable from the same civil structure in a free oscillation condition, i.e., in ideal non-operating conditions. In order to analyse the vibrational modes of
25 the civil structure, it would be necessary to observe it under ideal conditions of absence of traffic, wind, and other environmental conditions that instead affect the vibrational state of the real civil structure.

Spurious peaks mean the peaks that erroneously appear in
30 the power spectral density of the civil structure due to mathematical estimation errors or excessive noise related to traffic or other factors. Therefore, in

addition to these spurious peaks, the acceleration peaks detected by the acceleration sensors 210 due to the presence of the above-mentioned forcing events are also eliminated.

5 Therefore, the first numerical processing 120 provides filtered detecting data, particularly closer to those obtainable from the observation of the civil structure under ideal non-operating conditions.

This allows the method 100 for identifying and automatic
10 monitoring to be used for a civil structure in operation even during normal operating conditions, i.e., even in traffic conditions under any climatic condition.

After the first numerical processing 110, a *frequency domain decomposition* algorithm 130 is performed on the
15 filtered detecting data, obtaining a curve representing the estimated power spectral density of the civil structure.

Once such curve is obtained, one or more peaks related to respective possible vibrational modes are identified
20 140 on the curve itself, where the peaks are centered around respective modal frequencies. Then, a respective modal form is associated 150 with each modal frequency, where the modal form is an N-dimensional vector having as components the spatial deformation values at the
25 points in which the acceleration sensors 210 are applied. These deformation values are calculated based on the detections of the acceleration sensors 210.

In detail, the modal form can be represented in a three-dimensional Cartesian space as a three-dimensional
30 surface formed by interpolating a plurality of points, where each of the points is the spatial deformation at an acceleration sensor.

The points of the surface have relative values between -1 and 1.

Therefore, the modal form represents the spatial deformation of the civil structure at a certain modal frequency, or a vibration or resonance frequency of the civil structure.

The Cartesian space can be arbitrarily centered. The modal forms are adapted, scaled, and graphically illustrated based on the selection made by the user.

Therefore, the modal forms are N-dimensional vectors.

Thereby, a plurality or cloud of multi-dimensional points having the modal frequency-modal form pairs as coordinates is obtained. Then, the current value of a counter having an initial value equal to zero is increased by one, obtaining an updated current value; At this point, if the updated current value is below a predefined upper threshold value, the data related to the previously obtained multi-dimensional points are stored in a database.

Otherwise, if the updated current value is equal to a predefined upper threshold value, a training process is performed through which a plurality of labels representing the vibrational modes proper of the structure are identified, and the multi-dimensional points stored in the database are associated with the identified labels.

For example, if the above-described steps are performed once a day, the predefined upper threshold value can be equal to 14. In such case, the training process will be performed based on the data collected in 14 days.

The labels comprise a frequency value and a modal form vector.

The training process 180 performs an unsupervised classification. In other words, the training process 180 finds the features of the labels to be sought, i.e., a frequency value and a modal form vector, which are not initially known; such finding of the features is performed based on similarity, correlation, and distance operations among the points of the set of previously collected and stored data. A label is considered valid only if there is a minimum number of points that are associated with it.

If the updated current value is above the predefined upper threshold value, performing a monitoring process 190 comprising at least the step in which the multi-dimensional points are associated 191 with the labels found in the training process 180, obtaining labelled traces or groups of points, or they are classified 192 as anomalies.

Anomaly means any multi-dimensional point that is not associable with a vibrational mode; therefore, the anomaly can be associated with values altered due to the temperature, or with values altered due to numerical errors, or with values altered due to noise caused, for example, by traffic or measurement systems or structural damage.

Preferably, the first processing step 120 comprises a filtering step 121 in which the accelerations detected by the acceleration sensors 210 are filtered with a band-pass filter, for example, having a filtering band between $0.004 \cdot f_s$ and $0.4 \cdot f_s$ Hz where f_s is the sampling frequency of the sensors, a first step of removing the acceleration peaks 122 due to the presence of events with heavy forcing and a second step of power normalization of the

events 123 in which the acceleration peaks detected by the acceleration sensors 210 due to the presence of events with light forcing are attenuated.

"Heavy" forcing means all those forces that determine stresses on the civil structure not assimilable to white Gaussian noise. They are often determined by loads whose mass is comparable to that of the civil structure (for example, the passage of a TIR on the civil structure).

"Light" forcing means those forces whose effect can be assimilable to oscillations of white Gaussian noise (for example, the passage of cars or the wind hitting the civil structure). A "light" forcing for a civil structure can be "heavy" for another and vice versa, it all depends on the features of the civil structure.

Preferably, the step of performing the *frequency domain decomposition* algorithm 130 comprises the steps of:

- calculating 131 a *Power Spectral Density* matrix PSD of the events recorded where the PSD matrix is square with $N \times N$ size; in particular, the PSD matrix is calculated by the Welch method with Hanning windows of length 211 overlapping between each other by 50%, applied to FFT (*Fast Fourier Transform*) of the signal of length $M = 2^{15}$;
- decomposing 132 the PSD matrix into singular values SVD, obtaining for each frequency a product of three matrixes $U \Lambda U^H$ that are frequency-dependent and the central one Λ of which is diagonal having as components the singular values;
- determining 133 the first eigenvalue Λ_1 (vector $1 \times M/2$) and the first eigenvector U_1 (matrix $N \times M/2$) of the PSD matrix, which are respectively the daily estimation of the power spectral density of the civil structure and its modal forms.

Preferably, the training process 180 comprises the steps of:

- removing 181 the anomalies from the cloud of multi-dimensional points in both the frequency domain and the modal form domain; such removal is actuated by a first clustering algorithm based on density, such as for example DBSCAN; density-based clustering algorithms mean clustering algorithms that identify clusters as dense regions of space, separated by less dense zones, which thus represent noise. This removing operation 181, thus, gives the training process higher robustness against noise.
- Performing N times 182, through a Monte-Carlo simulation, a second hierarchical clustering algorithm adapted to calculate the number and value of centroids (pairs of mean frequency values and modal form vector) optimal for correctly finding the labels; this Monte-Carlo simulation involves selecting as the optimal centroid configuration the one found the most times; performing the second hierarchical clustering algorithm N times makes the training process more robust, i.e., with more repeatable result. Furthermore, such operation of performing 182 the second algorithm eliminates the need for subjective and non-repeatable manual selection of centroids.
- Performing 183 a third hierarchical clustering algorithm to aggregate the multi-dimensional points in cluster and define labels for the classification of vibrational modes; such third algorithm initializes the starting clusters with the optimal centroids calculated in the previous step. Such third hierarchical clustering is based on MAC and frequency distance, where MAC is an

index between 0 and 1 that allows to understand how similar two N-dimensional vectors are. In particular, if two N-dimensional vectors are identical, the MAC value will be equal to 1. The MAC index is used to analyse how similar two modal forms are, as they are nothing more than two N-dimensional vectors, i.e., the first eigenvectors U1.

The third hierarchical clustering algorithm provides to associate an i-th multi-dimensional point to a k-th cluster only if these three conditions are simultaneously satisfied:

- $df_i < dMAX_F_k$ with $df_i = \left| \frac{f_k - f_i}{f_k} \right|$
- $dMAC_i < dMAX_MAC_k$
with $dMAC_i = 1 - MAC_i$ and $MAC_i = \frac{|\phi_i^T \bar{\phi}_k|}{(\phi_i^T \phi_i)(\bar{\phi}_k^T \bar{\phi}_k)}$,
- $d_i < dMAX_k$ with $d_i = df_i + dMAC_i$

where

- f_k and ϕ_k respectively indicate the frequency value and the modal form of the k-th cluster, i.e., the mean frequency and the mean modal form of the k-th vibrational mode;
- f_i and ϕ_i respectively indicate the frequency value and the modal form of the point to be associated;
- df_i indicate the frequency distances between the i-th point to be associated and the frequency value of the k-th cluster;
- $dMAX_F_k$, $dMAX_MAC_k$, $dMAX_k$, are the first acceptance thresholds employed to verify the associability or non-associability of the i-th point to the k-th cluster. Such first acceptance thresholds have initial preset values, for example, $dMAX_F_k=0.008$ $dMAX_MAC_k=0.012$

$dMAX_k=0.02$, which remain fixed until the k -th cluster contains at least one first predefined number of points. The initial values of the first acceptance thresholds are, for example, preset based on scientific literature.

5 After the k -th candidate cluster reaches the first predefined number of points, the first acceptance thresholds become adaptive, i.e., they are iteratively updated based on the distribution of the points to be classified; in particular, updating the first acceptance
10 thresholds is made with a median operation of the distances between the points associated with that given cluster.

Thus, it is possible that such first acceptance thresholds assume completely different values from one
15 civil structure to another or even among different labels of the same civil structure.

In particular, the first acceptance thresholds are updated according to the following expressions:

- $dMAX_{F_k} = \sqrt{\text{med}(|df_k - \text{med}(df_k)|)}$
- $dMAX_{MAC_k} = \sqrt{\text{med}(|dMAC_k - \text{med}(dMAC_k)|)}$
- $dMAX_k = \sqrt{\text{med}(|d_k - \text{med}(d_k)|)}$

where df_k , $dMAC_k$ and dk are the distances between the
25 points belonging to the k -th cluster and the centroid of the k -th cluster, $\text{med}()$ is the function that calculates the median of the points belonging to the k -th cluster. At the end of the third clustering algorithm, only the clusters that have a second predefined number of points
30 are labelled as vibrational modes proper of the structure.

Performing 183 the third algorithm ensures higher

adaptability to different application scenarios since the third algorithm uses clustering thresholds that adapt to the features of the data.

At the end of the training process 180, the selected
5 labels are stored and used in the monitoring process
190.

The monitoring process 190 comprises the steps of:

- for each of the multi-dimensional points obtained at the end of the step of performing the FDD algorithm 130,
- 10 associating said multi-dimensional point to the k-th label if the following conditions are simultaneously satisfied:

- $df_i < dTRACCE_F_k$ where $df_i = |\overline{f_{traces,k}} - f_i|$

- $dMAC_i < dTRACCE_MAC_k$

15 with $dMAC_i = 1 - MAC_i$, $MAC_i = \frac{|\phi_i^T \cdot \overline{\phi_{traces-k}}|^2}{(\phi_i^T \cdot \phi_i) \cdot (\overline{\phi_{traces-k}^T} \cdot \overline{\phi_{traces-k}})}$

- $d_i < dTRACE_k$ where $d_i = df_i + dMAC_i$

where

- $\overline{f_{traces-k}}$ e $\overline{\phi_{traces-k}}$ are respectively the frequency and the mean modal form of the k-th label;
- 20 - ϕ_i^T is the modal form associated with the i-th frequency f_i , T indicates the transposition mathematical operation.

- $dTRACE_F_k$, $dTRACE_MAC_k$ and $dTRACE_k$ are the second acceptance thresholds used to associate the i-th multi-
25 dimensional point with the k-th label. These second acceptance thresholds are adaptive and are calculated by the following expressions:

- $dTRACE_F_k = \sqrt{std(df_k)} \left(1 + \frac{1}{\log h}\right)$
- 30 • $dTRACE_MAC_k = \sqrt{std(dMAC_k)} \left(1 + \frac{1}{\log h}\right)$

- $dTRACE_k = dTRACE_{F_k} + dTRACE_{MAC_k}$

where

- h is a correction coefficient that considers how many points have already been associated with the k-th label;

5 - std is a function that calculates the standard deviation, i.e., the square root of the variance.

Preferably, the monitoring process 190 comprises a purifying step in which the civil structure temperature effects are removed 191 from each labelled trace.

10 The temperature of the civil structure can be measured by first temperature probes associated with the civil structure or can be approximated by the ambient temperature measured by second temperature probes configured for detecting of air temperatures surrounding
15 the civil structure.

Each labelled trace comprises a plurality of multi-dimensional points each having its own modal frequency. Such multi-dimensional points of a single labelled trace are related to successive detection time instants. The
20 temperature, by acting on the materials of the structural elements of the civil structure, affects the temporal trend of the multi-dimensional points of each labelled trace. This affection must be eliminated in order to properly finding potential damage or degradation to the
25 civil structure.

The purification is carried out by performing a machine learning algorithm capable of determining the expected value of the modal frequencies starting from the known value of the temperature obtained as reported above.

30 The expected value of the modal frequencies determined by the machine learning algorithm is then subtracted from the value of the respective modal frequencies

determined in the step of identifying 140 the peaks following the step of performing the *frequency domain decomposition* algorithm 130. Such subtraction makes the removal of the temperature effect from the labelled
 5 traces.

Such *machine learning* algorithm is selected as the optimal statistical model at the end of a training step between the statistical models belonging to a group comprising at least the linear regression, the
 10 autoregressive model ARX, the *Principal Component Analysis* (PCA), and the *Support Vector Machine* (SVM) regressor.

In particular, the training of the machine learning algorithm is carried out using the multi-dimensional
 15 points of each labelled trace obtained from detecting of the acceleration sensors 210 carried out in a predetermined period of time, for example, one year; these latter multi-dimensional points are referred to as the training set.

20 The multi-dimensional points belonging to the training set are employed as input for all the statistical models belonging to the group mentioned above.

Selecting the optimal statistical model is made based on a score assigned to the regression carried out with each
 25 model of the group. This score, referred to as R^2 or determination coefficient, is defined as

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

where y_i represents the i -th frequency value, $\hat{y}_i$ the model prediction of the i -th frequency value, and $\bar{y}$ is the

mean value of the trace. Note that the determination coefficient is calculated on the test set to avoid fitting the model to its own statistical fluctuations (the phenomenon of overfitting).

- 5 The statistical model that is marked as optimal is the one with the highest determination coefficient among those of the models belonging to the group.

Once the optimal trained model is selected, such model is used to perform the purifying step.

- 10 Preferably, the monitoring process comprises, after the purifying step, the step in which an anomaly finding process 194 (or *anomaly detection*) is actuated.

Therefore, the object of this process is to report if the purified traces undergo significant variations.

- 15 These variations could represent a signal of degradation of one or more structural elements and therefore should be reported to the civil work maintainer.

- It should be noted that the anomalies that can be found in the anomaly finding process 194, i.e., after the
20 purifying step 193, represent damage to the civil structure.

The process for finding anomalies comprises the steps of:

- performing a plurality of anomaly point finding
25 techniques;
- if the anomaly points are present as a result of at least two of the finding techniques, an alarm is sent to the user.

- In particular, the process for finding anomalies can
30 involve the use of two different techniques whose results are compared and mutually validated.

The first technique is the control chart. It is a tool

composed of a set of points (one for each day) and one or more upper limit thresholds that find at least one control interval.

Any point that falls outside the control interval could
 5 potentially indicate an abnormal behaviour of the civil structure.

Each point on the control chart represents a statistical test called Shewhart's T^2 . It is defined as

$$T^2 = \frac{n}{n+1} (\vec{x} - \vec{\bar{x}})^T S^{-1} (\vec{x} - \vec{\bar{x}})$$

10 where n is the number of points in the training set, $\vec{x}$ is the vector containing all the traces for a given day, $\vec{\bar{x}}$ is the vector of mean traces calculated over the training period, $(\cdot)^T$ is the matrix transposition operator, and S^{-1} is the inverse of the covariance matrix
 15 of the traces.

The control level or upper limit threshold for a certain confidence value α (conventionally fixed at 95%) is defined as

$$UCL = p \frac{n-1}{n-p} F_{p,n-p}(\alpha)$$

20 where F is the Fisher-Snedecor distribution with p and $n-p$ degrees of freedom, and p is the number of traces. Once the control chart is constructed, it can be observed if there are time periods in which the T^2 variable repeatedly exceeds the control limit threshold. These
 25 are the periods in which structural anomalies may have potentially occurred.

The second anomaly finding technique is *changepoint detection*. It consists of analysing the purified

- labelled traces and finding which and where the points in which a significant variation occurs in the time series of multi-dimensional points compared to its past are. These points, referred to as *changepoints*,
5 represent the time instants in which the time series of multi-dimensional points deviate from their usual behaviour. This analysis is performed only after properly optimizing and calibrating the parameters that control the finding of the *changepoints*.
- 10 Once the results are obtained, it is analysed if the periods and anomaly points are present in both the used techniques.
- If the anomaly points are present as a result of both techniques, an alarm is sent to the user.
- 15 At this point, a task of the operator will be to investigate and evaluate the report to understand if the anomaly is due to non-linear effects of environmental factors or if the structure is undergoing degradation attributable to damage.
- 20 From the description carried out, the features of the method for identifying and automatic monitoring the vibrational modes of a civil structure object of the present invention are clear, as well as the related advantages.
- 25 Indeed, such method provides a settable duration training process that identifies the vibrational modes of the civil structure, making the subsequent monitoring step quick, accurate, and with reduced computational cost.
- 30 Furthermore, the monitoring process is completely automatic and does not require the intervention of operators to classify the detected vibrational modes.

The monitoring process provided by the method according to the invention is further capable of detecting the anomalies in the detected vibrational modes and consequently alerting the user; this is very important
5 in order to find any potential damage and/or degradation of the civil structure as early as possible and promptly intervene with targeted maintenance operations.

Finally, it is clear that the method for identifying and automatic monitoring the vibrational modes of a civil
10 structure thus conceived is susceptible of a number of modifications and variants, all falling within the invention; furthermore, all details are replaceable with technically equivalent elements. In practice, the used materials, as well as the dimensions, can be any
15 depending on the technical needs.

CLAIMS

- 1) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure comprising the steps of:
- 5 - receiving (110) a plurality of detecting data from a plurality of acceleration sensors (210) in a number equal to N applied in respective points of said civil structure;
- performing at least once a day the steps of:
- 10 - performing a first numerical processing (120) of said detecting data in order to eliminate data related to spurious acceleration peaks and acceleration peaks caused by heavy forcing events, said first numerical processing providing filtered
- 15 detecting data;
- performing a *frequency domain decomposition* algorithm (130) on said filtered detecting data obtaining a representative curve of the estimated power spectral density of the civil structure;
- 20 - identifying (140) on said curve one or more peaks related to respective possible vibrational modes, said peaks being centered around respective modal frequencies;
- associating (150) to each modal frequency a
- 25 respective modal form, where said modal form is an N-dimensional vector having as components the spatial deformation values at the points in which the acceleration sensors are applied, obtaining a plurality or cloud of multi-dimensional points
- 30 having the modal frequency-modal form pairs as coordinates;
- increasing (160) by one the current value of a

counter having initial value equal to zero
obtaining an updated current value;
- if the updated current value is below a predefined
upper threshold value, storing (170) in a database
5 the data related to the previously obtained multi-
dimensional points;
- if the updated current value is equal to a
predefined upper threshold value, performing a
training process (180) in which a plurality of
10 labels representing the vibrational modes proper of
the civil structure are identified and the multi-
dimensional points stored in said database are
associated with the identified labels;
- if the updated current value is above a predefined
15 upper threshold value, performing a monitoring
process (190) comprising at least the step in which
the multi-dimensional points are associated (191)
with the labels found in the training process (180)
or classified (192) as anomalies.

20 2) Method (100) for identifying and automatic monitoring
the vibrational modes of a civil structure according to
claim 1 wherein the first processing step (120)
comprises:
- a filtering step (121) in which the accelerations
25 detected by the acceleration sensors (210) are filtered
with a band-pass filter;
- a first step of removing the acceleration peaks (122)
in which the peaks detected by the acceleration sensors
(210) due to the presence of events with heavy forcing
30 are removed;
- a second step of power normalization of the events
(123) in which the acceleration peaks detected by the

acceleration sensors (210) due to the presence of events with light forcing are attenuated.

3) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to
5 claim 1 or 2 wherein the step of performing the *frequency domain decomposition* algorithm or FDD (130) comprises the steps of:

- calculating (131) a Power Spectral Density PSD matrix of the events recorded where the PSD matrix is square
10 with $N \times N$ size;
- decomposing (132) the singular value SVD PSD matrix obtaining for each frequency a product of the three matrixes $U \Lambda U^H$;
- determining (133) the first eigenvalue λ_1 and the first
15 eigenvector U_1 of the PSD matrix which are respectively the daily estimation of the power spectral density of the civil structure and its modal forms.

4) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to
20 one or more of the preceding claims wherein the training process (180) comprises the steps of:

- removing (181) the anomalies from the cloud of multi-dimensional points in both the frequency domain and the modal form domain by a first clustering algorithm;
- 25 - performing (182) for N times a second hierarchical clustering algorithm adapted to calculate the optimal number and value of centroids for correctly finding the labels;
- performing (183) a third hierarchical clustering
30 algorithm to aggregate the multi-dimensional points in cluster and define the labels for the classification of the vibrational modes.

5) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to claim 4 wherein the third hierarchical clustering algorithm provides to associate an i-th multi-dimensional point to a k-th cluster only if these three conditions are simultaneously satisfied:

- $df_i < dMAX_F_k$ with $df_i = \left| \frac{f_k - f_i}{f_k} \right|$
- $dMAC_i < dMAX_MAC_k$
with $dMAC_i = 1 - MAC_i$ and $MAC_i = \frac{|\phi_i^T \bar{\phi}_k|}{(\phi_i^T \phi_i)(\bar{\phi}_k \phi_k^T)}$,
- 10 • $d_i < dMAX_k$ with $d_i = df_i + dMAC_i$

where

- f_k and ϕ_k respectively indicate the frequency value and the modal form of the k-th cluster;
- f_i and ϕ_i respectively indicate the frequency value and the modal form of the i-th multi-dimensional point to be associated;
- 15 - df_i indicate the frequency distances between the i-th multi-dimensional point to be associated and the frequency value of the k-th cluster;
- 20 - $dMAX_F_k$, $dMAX_MAC_k$, $dMAX_k$, are first acceptance thresholds.

6) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to claim 5 wherein the first acceptance thresholds have the initial preset values which remain fixed until the k-th cluster contains at least one first predefined number of points.

7) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to claim 6 wherein after the k-th candidate cluster reaches

said predefined number of points, the first acceptance thresholds become adaptive, i.e., they are updated according to the following expressions:

- $dMAX_{F_k} = \sqrt{\text{med}(|df_k - \text{med}(df_k)|)}$
- $dMAX_{MAC_k} = \sqrt{\text{med}(|dMAC_k - \text{med}(dMAC_k)|)}$
- $dMAX_k = \sqrt{\text{med}(|d_k - \text{med}(d_k)|)}$

where df_k , $dMAC_k$ and d_k are the distances between the points belonging to the k-th cluster and the centroid of the k-th cluster, med is the function that calculates the median of the points belonging to the k-th cluster.

8) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to one or more of the preceding claims wherein the monitoring process (190) comprises the steps of:

- for each of the multi-dimensional points obtained at the end of the step of performing the FDD algorithm (130), associating said multi-dimensional point with the k-th label if the following conditions are simultaneously satisfied:

- $df_i < dTRACCE_{F_k}$ where $df_i = |\overline{f_{traces,k}} - f_i|$
- $dMAC_i < dTRACCE_{MAC_k}$

with $dMAC_i = 1 - MAC_i$, $MAC_i = \frac{|\phi_i^T \cdot \overline{\phi_{traces-k}}|^2}{(\phi_i^T \cdot \phi_i) \cdot (\overline{\phi_{traces-k}^T} \cdot \overline{\phi_{traces-k}})}$

- $d_i < dTRACE_k$ where $d_i = df_i + dMAC_i$

where

- $\overline{f_{traces-k}}$ and $\overline{\phi_{traces-k}}$ are respectively the frequency and the mean modal form of the k-th labelled;
- ϕ_i^T is the modal form associated with the i-th frequency f_i , T indicates the transposition mathematical operation
- $dTRACE_{F_k}$, $dTRACE_{MAC_k}$ and $dTRACE_k$ are the second

acceptance thresholds used to associate the i-th multi-dimensional point with the k-th label. These second acceptance thresholds are adaptive and are calculated by the following expressions:

5

$$\begin{aligned} \bullet \quad dTRACE_{F_k} &= \sqrt{std(df_k)} \left(1 + \frac{1}{\log h}\right) \\ \bullet \quad dTRACE_{MAC_k} &= \sqrt{std(dMAC_k)} \left(1 + \frac{1}{\log h}\right) \\ \bullet \quad dTRACE_k &= dTRACE_{F_k} + dTRACE_{MAC_k} \end{aligned}$$

where

- 10 - h is a correction coefficient that considers how many points have already been associated with the k-th label;
 - std is a function that calculates the standard deviation, i.e., the square root of the variance.

9) Method (100) for identifying and automatic monitoring
 15 the vibrational modes of a civil structure according to one or more of the preceding claims wherein the monitoring process (190) further comprises the steps of:
 - removing (193) the civil structure temperature effects from each labelled trace obtaining respective purified
 20 labelled traces;

- actuating an anomaly finding process (194) on each purified labelled trace.

10) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure
 25 according to claim 9 wherein the step (193) of removing the temperature effects comprises the steps of:

- performing a machine learning algorithm capable of determining the expected value of the modal frequencies starting from the known value of the temperature;
 30 - subtracting the previously determined expected value of the modal frequencies from the value of the respective modal frequencies determined in the step of identifying

the peaks (140).

11) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to claim 10 wherein the *machine learning*
5 algorithm is selected as the optimal statistical model at the end of a training step between the statistical models belonging to a group comprising at least the linear regression, the autoregressive model ARX, the *Principal Component Analysis* (PCA) and the *Support*
10 *Vector Machine* (SVM) repressor.

12) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to claim 11 wherein selecting the optimal machine learning algorithm comprises the steps of:
15 - performing all the statistical models of said group considering as input the multi-dimensional points of each labelled traces obtained from detecting of the acceleration sensors (210) carried out in a predetermined period of time;
20 - calculating a determination coefficient R^2 as

$$R^2 = 1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}$$

where y_i represents the i-th frequency value, $\hat{y}_i$ the model prediction of the i-th frequency value, and $\bar{y}$ is the mean value of the traces;

25 - marking as optimal the statistical model having the highest coefficient of determination.

13) Method (100) for identifying and automatic monitoring the vibrational modes of a civil structure according to one or more of preceding claims 9 to 12

wherein the process for finding the anomalies comprises the steps of:

- performing a plurality of anomaly point finding techniques;

- 5 - if the anomaly points are present as a result of at least two of the finding techniques, an alarm is sent to the user.

14) Computer program loadable in the memory of a computer and comprising instructions which induce the computer to
10 actuate the method (100) for identifying and automatic monitoring according to one or more of the preceding claims, when the computer performs the program.

15) System for automatic monitoring a civil structure (200) comprising:

- 15 - a plurality of acceleration sensors (210) in a number equal to N positionable in respective points of the civil structure;
- a central processing unit (220) associated with the acceleration sensors (210) in a wireless and/or wired
20 manner and programmed to actuate the method (100) for identifying and automatic monitoring one or more of claims 1 to 13.