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COMPLETE SPECIFICATION

Method and Apparatus for Linear Processing of Seismic Data

We, TEXAS INSTRUMENTS INCORPORATED, of 13500 North Central Expressway, Dallas, Texas, United States of America, a corporation organized and existing under the laws of the State of Delaware, United States of America, do hereby declare the invention for which we pray that a patent may be granted to us, and the method by which it is to be performed, to be particularly described in and by the following statement:—

This invention relates generally to seismology and seismic prospecting and more particularly to linear processing of seismic data.

In geophysical prospecting, it is common practice to use an array of seismometers to detect the seismic disturbance from an explosion detonated at or below the surface of the earth. Sometimes the seismometer outputs are summed to produce a composite trace for enhancing the seismic signal containing subsurface structure information, the purpose being to give prominence to the important features of the received seismic signal and reduce or remove those undesired features which obscure the important parts of the signal. In the past, both direct and weighted summations of seismometer array outputs have been used along with frequency filtering applied to the sum output according to single channel Wiener least mean square error theory. Also, special array geometries have been used in an attempt to enhance the seismic data. Additionally, narrow band pass frequency filters have been used prior to summation.

These prior techniques, however, do not possess the capability of operating in an optimum manner over the entire frequency range of the desired seismic signal. That is, these techniques perform well over a limited frequency range but are only partially successful since the desired seismic signal includes a wide range of frequencies.

According to this invention, a method of processing seismological and seismic prospecting data for separating desired and undesired signals having different apparent velocities and overlapping frequency and wave number spectra, includes the steps of:

- (a) generating a plurality of different seismic signal traces,
- (b) modifying the said plurality of seismic traces each in a manner different from any other trace by relatively weighting and time shifting the traces as a function of desired signal frequency, the time shifts and weights for one of the said traces being dependent upon the undesired signal in at least one other of the said traces to which weights and time shifts are applied,
- (c) and thereafter summing all of the said modified traces to pass the said desired signal and suppress the said undesired signal.

For example, the method may include the steps of:

- (a) generating a plurality N of different seismic signal traces,

$$[S_1(t) + N_1(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

- (b) modifying the said N traces by relatively weighting and time delaying them

as a function of frequency, the modifying operator $Y_i(f)$ for the i th trace

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being given by the matrix equation:

$$\{Y_i(f)\} = [S_{ii}(f) + N_{ii}(f)]^{-1} \{S_{ki}(f)\}$$

where $S_{ij}(f)$ and $N_{ij}(f)$ are the cross-power spectra for the desired and undesired signals, respectively, between the i th and j th of the seismometer channels supplying the data, and $S_{ki}(f)$ is the cross-power spectrum of the desired signal between the k th and j th of the seismometer channels ($i=1$ to N , $j=1$ to N) and k =the specified seismometer

channel from which the desired signal is to be obtained (where $N>1$).

- (c) summing the said modified traces to produce a trace which is the best estimate of the said desired signal in the least mean square error sense.

This invention also provides a method of processing seismological and seismic prospecting data from seismometer channels which includes the steps of:

- (a) generating a plurality of different seismic signal traces

$$[S_i(t) + N_i(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

- (b) modifying each of the said seismic traces by applying relative weight and time shifts to the said traces, the modifying operator $Y_i(f)$ for the i th trace being given by the matrix equation:

$$\{Y_i(f)\} = [S_{ii}(f) + N_{ii}(f)]^{-1} \{S_{ki}(f)\}$$

where $S_{ij}(f)$ and $N_{ij}(f)$ are the cross-power spectra for the desired and undesired signals between the i th and j th channels, $S_{ki}(f)$ is the cross-power spectra of the desired signal between the k th and j th channels, $i=1$ to N , $j=1$ to N , and k =the specified channel from which the desired signal is to be obtained;

- (c) dividing the seismic trace in at least one of the said channels into a plurality of time separated seismic traces each having the same frequency range, and applying a plurality of weights to the said time separated seismic traces; and

- (d) summing the said modified traces to pass the said desired signal and to reject the said undesired signal.

A processor according to the invention for seismological and seismic prospecting data for separating desired and undesired signals, includes a plurality of means operable to receive different seismic signal traces

$$[S_i(t) + N_i(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

filters having complex responses $Y_i(f)$, $Y_n(f)$, $\dots$, $Y_N(f)$ connected in respective transmission channels (1, $\dots$, n , $\dots$, N) operable to feed the respective seismic traces to signal summing means, the said filters being operable to apply relative weights and time shifts to the seismic traces in the said transmission channels and having complex frequency responses given by the matrix equation:

$$\{Y_i(f)\} = [S_{ii}(f) + N_{ii}(f)]^{-1} \{S_{ki}(f)\}$$

where $S_{ij}(f)$ and $N_{ij}(f)$ are the cross-power spectra for the desired and undesired signals between the i th and j th channels, $S_{ki}(f)$ is the cross-power spectrum of the desired signal between the k th and j th channels, $i=1$ to N , $j=1$ to N , and k =the specified channel;

and at least one of the said channels including means operable to divide the seismic trace transmitted therein into a plurality of time separated traces each having the same frequency range and means operable to apply a plurality of weights to the said time separated seismic traces

The processor may be used in processing seismic traces produced by reflection waves from horizons with different dips, being adapted

to pass seismic signals originating from one horizon and to reject seismic signals originating from the other horizon.

The processor may also be used in processing seismic traces in the form of reflection waves produced by shot stacking, being adapted to pass primary reflections and to reject ghost reflections.

The invention will be described, by way of example, with reference to the accompanying drawings of which:—

Fig. 1 is a system diagram of the invention;

Fig. 2 shows a linear seismometer array which may be used with the system of Fig. 1 and also a vectorial representation of apparent horizontal velocities of a seismic disturbance;

Fig. 3 shows a (f, K) space representation of a seismic disturbance;

Figs. 4 and 5 show the time domain operator for

$$\frac{\sin N\pi f\Delta t}{\sin \pi f\Delta t}$$

Fig. 6 shows a system for simulating both primary and multiple reflections in seismic reflection work;

Figs. 7 and 8 show the processor B (f,K) response along lines V_s and V_o, respectively, in Fig. 3;

5 Figs. 9 and 10 show the processor response along a horizontal section of Fig. 3 at two frequencies;

Figs. 11, 12 and 13 show impulse response curves for primary and multiple reflections;

10 Fig. 14 shows three seismogram records, the bottom being the input traces, the middle being the input traces processed according to the prior art and the top being the input traces processed according to this invention;

15 Fig. 15 shows a linear filter which may be used in the system of Fig. 1;

Fig. 16 shows primary and ghost reflections in shot stacking reflection seismology;

20 Fig. 17 shows a system for processing primary and ghost reflections according to this invention;

Figs. 18 and 19 show time domain operators for approximating frequency filters;

25 Figs. 20 and 21 show the processor B(f,K) response for primary and ghost reflections;

Fig. 22 shows a comparison between the de-ghosting process according to this invention (top waveform) and conventional stacking (bottom waveform).

A multi-channel system 1, . . . n, . . . N is shown in Fig. 1. The signals S₁(t)+N₁(t) . . . S_n(t)+N_n(t) . . . S_N(t)+N_N(t) are traces produced from a seismometer array or from shot stacking and may be traces stored on magnetic tape or derived directly from the seismometer outputs. Each trace is applied to a linear operator, or frequency filter, having a complex frequency response $\hat{Y}_1(f)$. . . $\hat{Y}_N(f)$. . . $\hat{Y}_N(f)$ and the outputs of the latter are then applied to summation networks 5 which produces a processor output g(t). The specification of the frequency response $\hat{Y}_n(f)$ for any filter in the multi-channel system of Fig. 1 depends on the desired seismic signal to the enhanced.

The desired seismic signal may be the processor output g(t) which is the best estimate of the received seismic signal S(t) from any particular seismometer k in the absence of an undesired signal N(t). The generalized problem of optimum linear array processing would then be: given N array elements with outputs f_n(t), to combine these outputs so as to

minimize the mean square error $\overline{E(t)^2}$ in the processor output g(t); that is, provide the best estimate in the least mean square error sense of the signal seen by the kth seismometer in the absence of the undesired signal. The best estimate of the desired signal is achieved by operating on each trace with a linear operator, or filter, having a complex

frequency response $\hat{Y}_n(f)$, and summing the filtered traces. The complex frequency response $\hat{Y}_n(f)$ for any linear filter in the solution of the stated generalized problem may be uniquely determined on the basis of the correlation functions for the desired and undesired signals as measured between all possible seismometers. In the case of no cross-correlation between desired and undesired signals in the same channel and between channels, the matrix equation for specifying $\hat{Y}_n(f)$ in terms of cross-correlation functions for the desired and undesired signals is:—

$$\{Y_i(f)\} = [S_{ii}(f) + N_{ii}(f)]^{-1} \{S_{ki}(f)\} \quad (1)$$

where

$S_{ii}(f)$ = cross-power spectrum of the desired signal between the ith and jth seismometer channels. 80

$N_{ii}(f)$ = cross-power spectrum of the undesired signal between the ith and jth seismometer channels. 85

$S_{ki}(f)$ = cross-power spectrum of the desired signal between the kth and jth seismometer channels.

$j=1$ to N , $i=1$ to N and k = the specified seismometer channel from which the desired signal is to be obtained. 90

The term "cross-power spectrum" means the Fourier transform of the cross-correlation function, the latter being $\psi_{io}(\tau)$ where:

$$\psi_{io}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f_i(t) f_o(t+\tau) dt \quad 95$$

$f_i(t)$ and $f_o(t)$ being two different time functions.

The terms along the main diagonal in matrix equation (1) are auto-power spectra as $i=j$. 100

The desired and undesired signals to be evaluated by equation (1) may be the signal S(t) [desired] and noise N(t) [undesired] where $N(t) = C(t) + R(t)$. 105

$C(t)$ = coherent noise

$R(t)$ = random noise

The specific signal and noise models to be evaluated by equation (1) are two signals distinguished by their different relative delays on the N channels plus random noise. The output of the nth seismometer is then: 110

$$f_n(t) = S(t - \tau_n) + C(t - \alpha_n) + R_n(t) \quad (2)$$

where S(t) is the desired signal to be passed C(t) is the undesired signal to be rejected called coherent noise 115

$R_n(t)$ is random noise assumed to have the same power level on all channels, the delays τ_n and α_n are unspecified as yet except that their difference $(\tau_n - \alpha_n)$ is not independent of n. 120

The power spectra are further defined as

$\phi_s(f)$ = auto-power spectrum of S(t)

$\phi_c(f)$ = auto-power spectrum of $C(t)$
 $\phi_r(f)$ = auto-power spectrum of $R_n(t)$
 then the power spectra in equation (1) become

$$5 \quad S_{ij}(f) = \begin{cases} \phi_s(f) & \text{if } i=j \\ \phi_s(f)e^{-i2\pi f(\tau_i - \tau_j)} & \text{if } i \neq j \end{cases}$$

$$N_{ij}(f) = \begin{cases} \phi_c(f) + \phi_r(f) & \text{if } i=j \\ \phi_c(f)e^{-i2\pi f(a_i - a_j)} & \text{if } i \neq j \end{cases}$$

The inversion of matrix equation (1) is found to be by induction

$$Y_n(f) = \frac{e^{i2\pi f\tau_n} \left[\phi_s \left(\phi_r + \phi_c \left\{ N - \sum_{k=1}^N e^{i2\pi f[(\tau_k - \tau_n) - (\tau_n - a_n)]} \right\} \right) \right]}{\phi_r(\phi_r + N\phi_s + N\phi_c) + \phi_s\phi_c \left(N(N-1) - 2 \sum_{i=k+1}^N \sum_{k=1}^N \cos 2\pi f[(\tau_k - \tau_i) - (\tau_i - a_i)] \right)} \quad (3)$$

10 $n=1, 2, \dots, N$

Considering the plane wave case where $S(t)$ and $C(t)$ propagate with apparent horizontal velocities V_s and V_c , respectively, equation (3) may be simplified.

15 Fig. 2 shows the geometry of an N element linear array having a spacing d between adjacent elements. For this example the linear array is considered to be equi-spaced although it need not be. Plane waves $S(t)$ and $C(t)$ having apparent horizontal velocities V_s and V_c , respectively, are vectorially shown propagating across the N element array. The delays τ_n and a_n now become:

$$25 \quad \begin{aligned} \tau_n &= (n-1)d/V_s \\ a_n &= (n-1)d/V_c \end{aligned}$$

For the model given [equation (2)] all that

is necessary is that the desired and undesired signals to be separated have different relative delays on the N channels of a linear array in the presence of random noise. For plane wave propagation, illustrated in Fig. 2, this is equivalent to requiring that the signals have different apparent horizontal velocities, and for point source propagation it requires spatially separated source points. However, the mode of propagation is unimportant since all that is required are signals having different relative delays on the N channels. There may be instances when the different relative delays are inconsistent with either plane wave or point source propagation, but this is immaterial according to this invention.

Equation (3) simplifies to

$$Y_n(f) = \frac{e^{i2\pi f \frac{d}{V_s}(n-1)} \left[\phi_s \left(\phi_r + \phi_c \left\{ N - e^{i2\pi f \Delta t \left(\frac{N+1}{2} - n \right)} \frac{\sin N\pi f \Delta t}{\sin \pi f \Delta t} \right\} \right) \right]}{\phi_r(\phi_r + N\phi_s + N\phi_c) + \phi_s\phi_c \left\{ N^2 - \frac{\sin^2 N\pi f \Delta t}{\sin^2 \pi f \Delta t} \right\}} \quad (4)$$

$$45 \quad \Delta t = \frac{d}{V_s} - \frac{d}{V_c}$$

The interpretation of equation (4) may be simplified if it is examined in parts.

50 First consider the case where the coherent noise power $\phi_c(f)$ is zero, and the signal $S(t)$ is to be recovered in the presence of random noise alone. The equation then becomes

$$Y_n(f) = e^{i2\pi f \frac{d}{V_s}(n-1)} \frac{\phi_s}{\phi_r + N\phi_s} \quad (5)$$

This equation (5) simply states that the signal $S(t)$ on the N traces should be lined up before

55 summing. The factor $\frac{\phi_s}{\phi_r + N\phi_s}$ is the familiar

Wiener result, and may be applied to the summed output since it is common to all $Y_n(f)$. In practice, for moderate $N \approx 10$, said factor can probably be neglected or set equal

to $\frac{\phi_s}{\phi_r + N\phi_s} \approx \frac{1}{N}$ if $\frac{N\phi_s}{\phi_r} \gg 1$. Thus, when

no coherent noise power is present, the best linear processor for an N channel array simply time shifts the outputs of the array elements to line up the signal before summing, and then a frequency filter may be applied, if desired, to the summed output to exploit any separation between the power spectra of the signal and that of the random noise.

When coherent noise power is present, simple time shifting, summing and filtering the output no longer represent the optimum processor as equation (4) shows for $\phi_c(f) \neq 0$. The term in the numerator of equation (4)

multiplied by $\phi_c(f)$ introduces a series of time shifts such that when the filtered traces are summed, the coherent noise will try to cancel

itself out. This is clear when it is recalled that said term in the numerator of equation (4) is:

$$e^{i2\pi f \Delta t \left(\frac{N+1}{2} - n \right)} \frac{\sin N\pi f \Delta t}{\sin \pi f \Delta t} = \sum_{k=1}^N e^{i2\pi f \Delta t (k-n)} \quad (6)$$

This cancellation would be perfect in the absence of random noise. The term in the denominator multiplied by $\phi_c(f)$ corrects for the distortion introduced into $S(t)$ by the cancellation process.

In order to evaluate the performance of the processor, it is convenient in the plane wave case to view the response of the array as a function of frequency f and apparent horizontal wave number K .

The signal and coherent noise may be displayed in f and K space as shown in Fig. 3.

f, K SPACE

The f, K space is the transform of three dimensional space made up of time and the X and Y coordinates of the horizontal earth plane. At each frequency it is assumed that the noise is made up of a sum of plane waves of all horizontal wavelengths and all azimuths. This is a three dimensional extension of the one dimensional Fourier analysis in which a time function is analyzed into the sum of sine waves of all periods, with appropriately weighted amplitude and phase. It is assumed that the surface of the earth in which the seismometers are planted is the horizontal plane. It is further assumed that the cross-correlation between two seismometers does not depend on their absolute locations, but only on their relative positions together with the delay time Δt between seismometers. Now let $W_i(f, K)$ be the three dimensional Fourier transform of $W_i(t, X)$, that is,

$$W_i(f, \vec{K}) = \int_{-\infty}^{+\infty} \int W_i(t, \vec{X}) e^{-i2\pi(f t - \vec{K} \cdot \vec{X})} dt d\vec{X}$$

where

$\vec{X}$ = the vector location of the i th seismometer with respect to an arbitrary origin in the horizontal plane

$\vec{K}$ = vector wave number in the horizontal plane and points in the direction of plane wave propagation

f = cycles/sec.

i = the i th seismometer channel

t = time with respect to an arbitrary time origin.

In the three dimensional situation, the motion on the surface of the horizontal earth

plane is assumed to be the sum of many horizontally moving plane waves of various frequencies, wavelengths and directions and in complex notation may be written as

$$e^{i2\pi(f t - \vec{K} \cdot \vec{X})}$$

where $\vec{K}$ is the vector wave number and has a magnitude equal to the reciprocal of wavelength λ . Thus, $W_i(f, K)$ is the response of the i th seismometer system to the unit amplitude plane wave characterized by f and K . Fig. 3 shows a vertical section of an f, K_x space plot for signals having velocities V_s and noise having velocities V_n . The K_y coordinate is not shown for simplicity. The class of all signals with velocity V forms the surface of a cone in the wave number and frequency space. For a particular seismic shot,

the direction of $\vec{K}$ will be in the direction of propagation of the seismic signal and will define a line in three dimensional f, K space. This means that the power spectra of the signal will be zero everywhere except on this line. For the directional model propagating across the array illustrated in Fig. 2, its f, K space representation defines a line V_s and V_n .

It can be seen from Fig. 3 that the frequency and wave number spectra for the signal V_s and noise V_n overlap. Therefore, filtering applied on either basis (f or K) will not be satisfactory in enhancing the signal, for example, and suppressing the noise. Filtering in f, K space, however, is capable of discriminating between the signal and noise. The optimum linear processor shown in Fig. 1 where the complex frequency response $Y_n(f)$ is determined according to equation (1) for solving the stated generalized problem has the capability of separating the signal V_s and the noise V_n on a velocity basis. The processor having an overall response $B(f, K)$ passes the desired signal V_s and rejects the undesired noise V_n .

Thus, signal power is distributed along a line in f and K space with slope V_s and the coherent noise power along a line with slope V_n . If relative amplitude response is represented as a surface above the f and K plane, perfect separation requires that the response surface pass through unity above the signal line V_s and touch zero along the coherent noise line V_n .

PROCESSOR RESPONSE $B(f, K)$

To see how well the optimum processor achieves this goal of perfect separation, the processor response as a function of f and K is calculated. This is accomplished by evaluating the processor response for a signal traveling with an arbitrary velocity $V = \frac{f}{K}$.

The relative amplitude response is then

After substituting for $Y_n(f)$ from equation (4) we have

$$B(f, K) = \sum_{n=1}^N Y_n(f) e^{-i2\pi K X_n} \quad (7)$$

$$\text{where } X_n = (n-1)d \quad 10$$

$$\text{and } K(n-1)d = \frac{(n-1)d}{V} f$$

$$e^{i2\pi d \left(\frac{N-1}{2} \left(\frac{f}{V_s} - K \right) \right)} \left[(\phi_s \phi_r + N \phi_s \phi_c) \left(\frac{\sin N\pi d \left(\frac{f}{V_s} - K \right)}{\sin \pi d \left(\frac{f}{V_s} - K \right)} \right) - \phi_s \phi_c \frac{\sin N\pi d \left(\frac{f}{V_c} - K \right)}{\sin \pi d \left(\frac{f}{V_c} - K \right)} \frac{\sin N\pi f \Delta t}{\sin \pi f \Delta t} \right]$$

$$B(f, K) = \frac{\phi_r(\phi_r + N\phi_s + N\phi_c) + \phi_s \phi_c \left\{ N^2 - \frac{\sin^2 N\pi f \Delta t}{\sin^2 \pi f \Delta t} \right\}}{\sin^2 \pi f \Delta t} \quad (8)$$

Clearly, if $K = \frac{f}{V_s}$ or $\frac{f}{V_c}$ the response

tends to unity or zero, respectively, in the limit $\phi_r \rightarrow 0$. Of course, these are the responses desired. However, random noise downgrades the performance from the ideal as Figs. 7, 8, 9 and 10 show for the response along the lines V_s , V_c , $f\Delta t = .2$ and $f\Delta t = .125$, respectively. These curves were calculated for a twelve channel array with the velocities and element spacing given by

$$\left. \begin{array}{l} \frac{d}{V_s} = .1 \text{ sec.} \\ \frac{d}{V_c} = .075 \text{ sec.} \end{array} \right\} \text{ or } \Delta t = \frac{d}{V_s} - \frac{d}{V_c} = .025 \text{ sec.}$$

Therefore, Figs. 9 and 10 show the processor response $B(f, K)$ along the line $f = 8 \frac{\text{cycles}}{\text{sec.}}$ and $5 \frac{\text{cycles}}{\text{sec.}}$ respectively in Fig.

3. The power spectra were taken as white with $\phi_s = \phi_c$ and $\frac{\phi_s}{\phi_r} = 10$. In other words, no frequency separation was exploited in this example.

The separation of $S(t)$ and $C(t)$ was effected purely on a velocity basis. The performance of the processor is not significantly effected by the presence of this amount of random noise. This is of course dependent upon the number of channels in the array. Had the example been computed for a two element array, the same level of random noise would

have produced a larger departure from the ideal response.

The performance of the processor breaks down at zero frequency and multiples of

$$f = \left[\frac{d}{V_s} - \frac{d}{V_c} \right]^{-1} \quad (9)$$

This is readily understood when it is recalled that the processor is essentially a velocity filter. At zero frequency all velocities look alike, as Fig. 3 shows, and the processor cannot distinguish between zero frequency in the signal spectrum and the coherent noise spectrum. The same is true for frequencies given by equation (9) which have periods that are multiples of the relative delay between signal and coherent noise on adjacent channels.

Thus the processor response $B(f, K)$ given by equation (8) is a function of frequency and wave number which follows the velocities V_s and V_c . This type of response cannot be achieved with conventional array processing such as frequency independent weight or band pass filtering applied to the channels before summation. Additionally, $Y_n(f)$ based on the least mean square error criterion ensures minimum signal distortion with a given signal and noise model. Wide band signals such as signal and noise with overlapping frequency and wave number spectra, therefore, are capable of being separated on a velocity basis according to this invention.

IMPLEMENTATION

The information required to implement this processor is the number of channels N , detector or point source spacing d , velocities V_s and V_c , and the correlation functions for the desired and undesired signals. In practice, V_s ,

V_e and the correlation functions may be obtained from seismic data derived from the location, or obtained from theoretical considerations appropriate to the particular problem. In practice, V_s and V_e may not be known

5 *a priori*, but recorded data will usually give equivalent information in terms of relative delay times for signal and noise on adjacent channels.

10 When no frequency separation can be exploited (the frequency spectra for the desired and undesired signals substantially overlap) and the various power spectra $\phi(f)$ are assumed to be substantially equal or differ by a constant, then the weighting and time delay factors described by equation (4) may be applied to the multi-traces as follows:

(1) the n^{th} trace is static time shifted by

$\frac{d}{V_s}(n-1)$ seconds, to give what is designated n_1^{th} trace

20 (2) the n_1^{th} trace is then multiplied by the constant term $(\phi_s\phi_r + N\phi_s\phi_e)$ giving n_2^{th} trace which is temporarily stored in a

$$45 \left[\phi_r(\phi_r + N\phi_s + N\phi_e) + \phi_s\phi_e \left\{ N^2 - \frac{\sin^2 N\pi f \Delta t}{\sin^2 \pi f \Delta t} \right\}^{-1} \right]$$

yielding the desired output $g(t)$.

The denominator of the filters $Y_n(f)$ is the same for all channels and need be applied only once to the processor output trace.

50 $Y_n(f)$ may be inverted to the time domain $Y_n(t)$ by machine computation or analytically.

An example of a digital technique for processing N traces stored on magnetic tape would be to store the above information (1) through (7) on magnetic tape and program a digital computer to process the N traces according to steps (1) through (7). Or, alternatively, an analog technique may be employed using sample point operators.

60 When exploitation of frequency separation is possible, the power spectra ϕ_s , ϕ_e , ϕ_r would be used in equations (3) or (4) and the entire expression computed as a function of frequency, inverted to the time domain $Y_n(t)$ and embodied as a sample point operator on the n^{th} channel.

70 Fig. 15 illustrates a sample point operator comprising a limited length delay line 6, sample points $a, b, c, \dots x$, weighting factors $a_1, a_2, \dots a_x$ and summation network 7.

75 The time domain sample operator shown in Fig. 15 is the approximated frequency filter $Y_n(f)$ whose input is connected to the n^{th} trace and whose output is connected to summation network 5 shown in Fig. 1.

Assuming the spacing between sample points $a, b, c, \dots x$ to be given by τ , the

suitable delay network or on magnetic tape.

(3) the original n^{th} trace is now static time shifted by $\left(\frac{d}{V_s} - \frac{d}{V_e}\right) \left(\frac{N+1}{2} - n\right)$

seconds relative to the previous shift, giving n_3^{th} trace.

(4) the n_3^{th} trace is multiplied by the constant $(-\phi_s\phi_e)$ giving n_4^{th} trace.

(5) the n_4^{th} trace is controlled with the $\sin N\pi f \Delta t$

time domain operator for $\frac{\sin \pi f \Delta t}{\sin N\pi f \Delta t}$

which is shown in Figs. 4 and 5 for N even and odd respectively giving n_5^{th} trace.

(6) the two traces derive from the n^{th} trace, that is the n_2^{th} and n_5^{th} trace, are then added.

(7) this is repeated for all $1 \dots n \dots N$ traces and the outputs are all summed. The resulting summed trace is convolved with the time domain operator for the common denominator.

impulse time response of the i^{th} filter is given by:

$$80 Y_i(t) = \sum_{n=0}^N a_n \delta(t + n\tau - \rho\tau)$$

where $\delta(t)$ = DIRAC Delta function

$\rho\tau$ = the reference time between the first sample point a and the output time of the filter.

85 Given the complex frequency response for any filter $Y_n(f)$, $Y_n(t)$ may be derived and the weights a_n calculated from the above equation.

Therefore, the n^{th} trace is sampled every τ millisecond, weighted by a_n and summed to

approximate filter response $Y_n(f)$.

90 The delay line 6 may be a series of electrical circuits having the appropriate time delay between sample points, or alternatively, the n^{th} trace may be stored on magnetic tape and driven passed reproducing heads spaced τ milliseconds apart.

APPLICATION

The processing according to this invention has general utility in both seismology and seismic prospecting where it is desired to separate arrivals with different apparent horizontal velocities characterized by two signals having different relative delays on the N channels across the record.

In earthquake seismology, the far field 105

source or plane wave approximation is usually well met. Overlapping arrivals appear on the earthquake seismogram as propagating with different apparent horizontal velocities may be separated by processing according to this invention. A single linear array at a station may be used where the direction of the seismic signal is known, or several linear arrays may be used to obtain good directional coverage.

In seismic prospecting, the processing according to this invention has utility in reflection and refraction work. Examples for processing different traces from the same and different shots in reflection work are given.

PRIMARY AND MULTIPLE SEPARATION

Sometimes the reflection seismogram is complicated by interference of two reflections from horizons with different dips. for example, the interference of multiple reflections from a shallow dipping horizon with primary reflections from a deeper flat horizon, or vice versa. The processing technique according to this invention may be used to pass one dip reflection and reject the other. Another and more important problem which may be solved is the separation of multiples and primaries when no dip difference exists.

When differential normal move out exists between primary and multiple reflections because of a velocity increase with depth, it may be exploited according to this invention. Normal move out difference between primary and multiple events simply means that the delays in the primary and multiple signals across the record are not the same, or that the primary and multiple signals have a different apparent horizontal velocity across the linear N element array. The relative different delays will not generally be consistent with a plane wave approximation since these are near field sources. This, however, does not effect the performance of the processor. All that is necessary are the proper delays τ_n and z_n in equation (3) for the primary and multiple signals respectively.

Both synthetic and actual field data were processed according to this invention for evaluation purposes. The synthetic example consisted of passing a wavelet through two filters with appropriate delays to simulate both primary and multiple events, and summing the filtered outputs. This is shown schematically in Fig. 6. The filters Y_A and Y_B were designed to pass events with no relative delay ($t_1=t_2$) and to reject events with four milliseconds of delay ($t_2-t_1=.004$ sec).

In addition, the random noise power level was assumed to be 0.1 of the primary or multiple power level. The same wavelet was used for $S(t)$ and $C(t)$, the primary and multiple, respectively.

The results are shown in Figs. 11, 12, and 13. Fig. 11 shows the impulse response of filters Y_A and Y_B . In addition the impulse

response for primaries is shown having been obtained by adding the response for Y_A and Y_B . The response for multiples was obtained by first delaying Y_A with respect to Y_B by .004 sec. and then adding. Fig. 12 shows the input wavelet and the input wavelet convolved with the response for primaries and the response for multiples. The primary response reproduced the waveform with no discernible distortion and less than -2db attenuation. The multiple response attenuated the wavelet by about -15db. This is in good agreement with the predicted response for the 2 channel processor for the frequency band of the wavelet.

Fig. 13 again shows the response for primary and multiple of $(t_2-t_1)=4$ milliseconds, as well as for events with relative delays of two and six milliseconds. These are included to demonstrate the capability of the processor to handle events for which it was not specifically designed. At two milliseconds of delay the event is neither a primary nor multiple but lies between, and the processor compromises by attenuating it one half that which it would attenuate a four millisecond "true" multiple. At six milliseconds of delay the event is clearly not a primary and the attenuation is of the same order as for "true" multiples.

The significant point is that the response of the processor is not particularly sensitive to departures from the signal and noise model for which it was designed. This is an important practical consideration as real data will not conform exactly to a given model, and the above suggests that small departures from the model will result in small departures from the predicted response of the processor.

As another example, the two channel processor was applied to field data. In Fig. 14, the records show traces taken from an area where multiple reflections represent the dominant geophysical exploration problem. The bottom record shows the traces with no processing. The middle record shows the traces from a different shot processed according to the skip mix method. The top record shows the traces from another shot processed according to this invention. The primaries at about 1.25 sec. on the three records are lined up in Fig. 14. The group of reflections occurring at about 0.75-1.0 sec. on the record represent true primary reflections. The first multiple reflections from the horizon begin at about 1.5 sec. and continue down the record masking weaker primaries arriving later than 1.5 sec.

The multiple generating horizon is essentially flat, and the resulting primaries line up across the record after normal move-out correction. The multiples at 1.5 sec. and beyond clearly show some residual move-out which serves as the basis for applying processing according to this invention to distinguish be-

5 between primaries and multiples. The traces, bottom record Fig. 14, were first processed in a conventional manner by the skip mix method adding selected pairs of traces 1 and 6, 2 and 7, etc. This process reinforces pri-
 10 maries that line up and partially cancels multiples which have a relative displacement of about 7 milliseconds between added pairs. The result is shown in the middle record of Fig. 14. As expected, primary reflections came through well but multiple attenuation is only a few decibels.

15 Optimum linear processing according to this invention was applied to the same pairs of traces and the result is shown in the top record of Fig. 14. The primaries again came through with little or no distortion, but now the multiples at 1.5 sec. and beyond are attenuated 10–15 db relative to their strength
 20 on the original record.

PRIMARY AND GHOST SEPARATION

25 Another example of the utility of this invention is ghost suppression as applied to shot stacking in reflection seismology. The ghost problem is illustrated in Fig. 16 which shows a seismometer reflection spread receiving energy from two shot points A and B at different depths. The ghost energy is shown by the dashed line and is that part of the
 30 initial explosive energy that travels upward from the shot. The primary energy travels downward and is shown by the solid line. From each reflecting horizon, a primary reflection and shortly afterwards a ghost reflection will arrive at seismometer n. The ghost

energy often complicates the seismic record, and if interpreted as a primary would result in a false interpretation of the subsurface layering. Therefore, ghost energy is objectionable and classified as seismic noise. 40

As a result of two or more shots at each spread location, there are two or more seismograms respectively corresponding to each shot (A and B) produced by each seismometer n. Each seismogram from seismometer n contains a primary and ghost reflection from the sub-
 45 surface layering. The time delay between primary and ghost for any given reflector (layer) differs between the two or more seismograms because of the difference in shot depth for A and B. Thus the primary and ghost reflections on each seismograph may be treated as coherent events with different relative
 50 delays on the seismographs. Processing by applying the appropriate filters $Y_A(f)$ and $Y_B(f)$ respectively to trace A (seismogram A) and trace B (seismogram B) and then summing enhances the primaries and suppressed the ghosts. 55

If traces A and B from seismometer n are time shifted so that the primaries on each trace line up, deghosting is accomplished as follows: 60

Referring to Fig. 17, $S(t)$ is the primary signal with no time delay between channels A and B. $C(t)$ represents ghost energy with time delay Δt between channels and $R(t)$ is random noise on each channel. The filters $Y_A(f)$, $Y_B(f)$ to produce $g(t)$ which is the best estimate of $S(t)$ in the least mean square error
 65 sense are 70

$$Y_A(f) = \frac{(\phi_s \phi_r + \phi_s \phi_c) - \phi_s \phi_c e^{-i2\pi f \Delta t}}{\phi_r(\phi_r + 2\phi_s + 2\phi_c) + 2\phi_s \phi_c - 2\phi_s \phi_c \cos 2\pi f \Delta t} \quad (10)$$

$$Y_B(f) = Y_A^*(f)$$

75 Assuming substantial overlap in the power spectra

$$\phi_s(f) = \phi_c(f)$$

$$\phi_r(f) = 0.1 \phi_s(f)$$

then

$$Y_A(f) = \frac{1.1 - e^{-i2\pi f \Delta t}}{2.41 - 2 \cos 2\pi f \Delta t} \quad (11)$$

$$80 \quad Y_B(f) = \frac{1.1 - e^{+i2\pi f \Delta t}}{2.41 - 2 \cos 2\pi f \Delta t} \quad (12)$$

The time domain operators $h_A(t)$ and $h_B(t)$ for filters $Y_A(f)$ and $Y_B(f)$ may be applied to traces A and B in either of two ways. One, convolve trace A with $h_A(t)$, convolve trace B with $h_B(t)$, and then sum to produce $g(t)$. Or, 85 two, convolve trace A with the time domain

operator $h_A'(t)$ for the numerator of equation (11), convolve trace B with the time domain operator $h_B'(t)$ for the numerator of equation (12), then sum, and convolve the sum with the time domain operator $h_c'(t)$ for the common denominator of equations (11) and (12) to produce $g(t)$. 90

The time domain operators $h_A(t)$, $h_B(t)$, $h_A'(t)$, $h_B'(t)$, $h_c'(t)$ obtained by inverting equations (11) and (12) to the time domain are shown in Figs. 18 and 19. 95

The operator points shown in Figs. 18 and 19. are correct when Δt is an integer multiple of the sample period τ (time between points on the delay line). For one millisecond data (sample period) Δt may take any integer value; however for 2 millisecond data Δt must be 2, 4, 6 . . . or $2N$ milliseconds. The table below shows the time domain operator points for $\Delta t = k\tau$ where k = integer 100 105

τ = sample period in milliseconds
 $\Delta t = \frac{d}{V_s} - \frac{d}{V_c}$ or in this case the time

delay between ghosts on channels A and B since the primaries are lined up.

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TABLE 1
TIME DOMAIN OPERATOR POINTS

Case I		Case II			
Time ms.	$h_A(t)$	$h_B(t)$	$h_A'(t)$	$h_B'(t)$	$h_C'(t)$
10					
-10 Δt	.0007796	-.001064	0	0	.001371
-9 Δt	.001474	-.002012	0	0	.002440
-8 Δt	.002730	-.003742	0	0	.004783
-7 Δt	.005135	-.007033	0	0	.008975
15					
-6 Δt	.009649	-.01322	0	0	.01719
-5 Δt	.01808	-.02478	0	0	.03191
-4 Δt	.03397	-.04655	0	0	.05981
-3 Δt	.06377	-.08738	0	0	.1124
-2 Δt	.1197	-.1640	0	0	.2111
20					
-1 Δt	.2247	-.3079	0	-1.0	.3961
0	.4219	+.4219	1.1000	1.1	.7437
1 Δt	-.3079	.2247	-1.0000	0	.3961
2 Δt	-.1640	.1197	0	0	.2111
3 Δt	-.08738	.06377	0	0	.1124
25					
4 Δt	-.04655	.03397	0	0	.05981
5 Δt	-.02478	.01808	0	0	.03191
6 Δt	-.01322	.009649	0	0	.01719
7 Δt	-.007033	.005135	0	0	.008975
8 Δt	-.003742	.002730	0	0	.004783
30					
9 Δt	-.002012	.001474	0	0	.002440
10 Δt	-.001064	.0007796	0	0	.001371

The weights specified by Table I are applied to the sample point operator, for example, that shown in Fig. 15 as follows:

35 Assuming

$\Delta t = 10$ milliseconds

$\tau = 1$ millisecond

then the center sample point, that is, the sample point between end sample points (designated zero millisecond), and every tenth sample point on delay line 6 are weighted according to Table I and summed by network 7. If $\tau = 2$ ms, the center sample point and every fifth sample point are weighted according to Table I and summed. If $\Delta t \neq k\tau$, then the sample point operators, Figs. 18 and 19, will in general have non-zero values at all sample points rather than at just multiples of Δt . For one millisecond data Δt can be rounded off to the nearest integer millisecond, and the operators in Figs. 18 and 19 apply. For two millisecond data, however, it may be desirable or mandatory in some instances to have other than an even number milliseconds of Δt . In these cases the operators, equations (11) and (12), must be inverted for the particular Δt as the results in Table I will not apply.

60 Figs. 20 and 21 show the processor response $B(f, K)$ for the given primary and ghost reflection models, respectively. It is desirable to center the power spectrum of the ghost

energy in the strong reject region between $0.2 \leq f\Delta t \leq 0.8$. This may be accomplished by the proper choice of Δt through proper depth spacing of shots A and B.

To further illustrate the deghosting process according to this invention, Fig. 22 shows a comparison with conventional stacking (straight summing) on a synthetic example.

The filter points $h_A(t)$ and $h_B(t)$ were used with $\Delta t = .010$ sec. The difference in the two procedures is apparent from the ghost residual.

In summary, the optimum linear processing method described herein operates on multi-channel sources $f_n(t)$ derived from a detector array or from shot stacking, to enhance an important frequency band. This processing is capable of synthesizing a velocity filter for discriminating between desired and undesired signals having overlapping frequency and wave number spectra over the entire frequency spectra of interest. Each trace applied to a channel is viewed as a signal source having an output,

$$f_n(t) = S(t - \tau_n) + C(t - \alpha_n) + R_n(t)$$

where τ_n and α_n define different relative delays on the N channels. Said multi-source outputs are processed prior to summation by relative frequency dependent weights and time delays according to multi-channel Wiener least mean square error criterion to produce

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a sum output $g(t)$ which is the best estimate of the desired signal using linear operations.

The examples given in the solution of matrix equation (1) have been for a one dimensional seismometer array; however, it is to be understood that equation (1) may be applied to two or three dimensional arrays for describing the complex frequency filters to be used prior to summation.

WHAT WE CLAIM IS:—

1. A method of processing seismological and seismic prospecting data for separating desired and undesired signals having different apparent velocities and overlapping frequency and wave number spectra, including the steps of:

- (a) generating a plurality of different seismic signal traces
- (b) modifying the said plurality of seismic traces each in a manner different from

$$[S_1(t) + N_1(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

- (b) modifying the said N traces by relatively weighting and time delaying them as a function of frequency, the modifying operator $Y_i(f)$ for the i th trace being given by the matrix equation:

$$\{Y_i(f)\} = [S_{ii}(f) + N_{ii}(f)]^{-1} \{S_{ki}(f)\}$$

where $S_{ii}(f)$ and $N_{ii}(f)$ are the cross-power spectra for the desired and undesired signals, respectively, between the i th and j th seismometer channels; $S_{ki}(f)$ is the cross-power spectrum of the desired signal between the k th and j th seismometer channels, $i=1$ to N , $j=1$ to N and k =the specified seismometer channel from which the desired signal is to be obtained, where $N>1$; and

- (c) summing the said modified traces to produce a trace which is the best estimate of the said desired signal in the least mean square error sense.

3. The method of processing seismological and seismic prospecting data as defined in claim 2, wherein the said seismic traces have at least two components $S(t-[n-1]t_1) + C(t-[n-1]t_2)$ in the presence of random noise $R_n(t)$, (n being the number of the trace; t_1 being the time delay between consecutive channels for the desired signals and t_2 being the time delay between consecutive channels

any other trace by relatively weighting and time shifting the traces as a function of desired signal frequency, the time shifts and weights for one of the said traces being dependent upon the undesired signal in at least one other of the said traces to which weights and time shifts are applied,

- (c) and thereafter summing all of the said modified traces to pass the said desired signal and suppress the said undesired signal.

2. A method of processing seismological and seismic prospecting data from seismometer channels for separating desired and undesired signals having different apparent velocities and overlapping frequency and wave number spectra, including the steps of:

- (a) generating a plurality N of different seismic signal traces,

for the undesired signals), one component of each trace having a relative delay $[n-1]t_1$ different from the other component $[n-1]t_2$ including the steps of

- (1) static time shifting the n th trace by $t_1(n-1)$ seconds,

- (2) multiplying the said time shifted trace by $(\phi_s \phi_r + \phi_s \phi_c N)$ wherein ϕ_s , ϕ_c and ϕ_r are the autopower spectra of the said two components and the said random noise respectively;

- (3) storing the said multiplied trace;

- (4) static time shifting the n th trace by $\frac{N+1}{2}(t_1-t_2) - n$ seconds relative to $t_1(n-1)$,

- (5) multiplying the said last mentioned time shifted trace by $(-\phi_s \phi_c)$,

- (6) convolving the said last mentioned trace with the time domain operator for $\frac{\sin N\pi f(t_1-t_2)}{\sin \pi f(t_1-t_2)}$,

- (7) summing the said stored multiplied trace and the said convolved trace,

- (8) summing all of the modified summed N traces, and

- (9) convolving the sum of said last mentioned modified traces with the time domain operator for

$$\left[\phi_r(\phi_r + N\phi_s + N\phi_c) + \phi_s \phi_c \left\{ N^2 - \frac{\sin^2 N\pi f(t_1-t_2)}{\sin^2 \pi f(t_1-t_2)} \right\} \right]^{-1}$$

where f =frequency in cycles/sec.

4. A method of processing seismological and seismic prospecting data from seismo-

meter channels for separating desired and undesired signals, including the steps of:

- (a) generating a plurality of different seismic signal traces



a sum output $g(t)$ which is the best estimate of the desired signal using linear operations.

The examples given in the solution of matrix equation (1) have been for a one dimensional seismometer array; however, it is to be understood that equation (1) may be applied to two or three dimensional arrays for describing the complex frequency filters to be used prior to summation.

WHAT WE CLAIM IS:—

1. A method of processing seismological and seismic prospecting data for separating desired and undesired signals having different apparent velocities and overlapping frequency and wave number spectra, including the steps of:

- (a) generating a plurality of different seismic signal traces
- (b) modifying the said plurality of seismic traces each in a manner different from

$$[S_1(t) + N_1(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

- (b) modifying the said N traces by relatively weighting and time delaying them as a function of frequency, the modifying operator $Y_i(f)$ for the i th trace being given by the matrix equation:

$$\{Y_i(f)\} = [S_{ij}(f) + N_{ij}(f)]^{-1} \{S_{ki}(f)\}$$

where $S_{ij}(f)$ and $N_{ij}(f)$ are the cross-power spectra for the desired and undesired signals, respectively, between the i th and j th seismometer channels; $S_{ki}(f)$ is the cross-power spectrum of the desired signal between the k th and j th seismometer channels, $i=1$ to N , $j=1$ to N and k =the specified seismometer channel from which the desired signal is to be obtained, where $N>1$; and

- (c) summing the said modified traces to produce a trace which is the best estimate of the said desired signal in the least mean square error sense.

3. The method of processing seismological and seismic prospecting data as defined in claim 2, wherein the said seismic traces have at least two components $S(t-[n-1]t_1) + C(t-[n-1]t_2)$ in the presence of random noise $R_n(t)$, (n being the number of the trace; t_1 being the time delay between consecutive channels for the desired signals and t_2 being the time delay between consecutive channels

any other trace by relatively weighting and time shifting the traces as a function of desired signal frequency, the time shifts and weights for one of the said traces being dependent upon the undesired signal in at least one other of the said traces to which weights and time shifts are applied,

- (c) and thereafter summing all of the said modified traces to pass the said desired signal and suppress the said undesired signal.

2. A method of processing seismological and seismic prospecting data from seismometer channels for separating desired and undesired signals having different apparent velocities and overlapping frequency and wave number spectra, including the steps of:

- (a) generating a plurality N of different seismic signal traces,

for the undesired signals), one component of each trace having a relative delay $[n-1]t_1$ different from the other component $[n-1]t_2$ including the steps of

- (1) static time shifting the n th trace by $t_1(n-1)$ seconds,
- (2) multiplying the said time shifted trace by $(\phi_s \phi_r + \phi_s \phi_e N)$ wherein ϕ_s , ϕ_e and ϕ_r are the autopower spectra of the said two components and the said random noise respectively;
- (3) storing the said multiplied trace;
- (4) static time shifting the n th trace by $\frac{N+1}{2}(t_1-t_2) - n$ seconds relative to $t_1(n-1)$,

- (5) multiplying the said last mentioned time shifted trace by $(-\phi_s \phi_e)$,
- (6) convolving the said last mentioned trace with the time domain operator for

$$\frac{\sin \pi f(t_1 - t_2)}{\sin \pi f(t_1 - t_2)},$$

- (7) summing the said stored multiplied trace and the said convolved trace,
- (8) summing all of the modified summed N traces, and
- (9) convolving the sum of said last mentioned modified traces with the time domain operator for

$$\left[\phi_r(\phi_r + N\phi_s + N\phi_e) + \phi_s \phi_e \left\{ N^2 - \frac{\sin^2 N\pi f(t_1 - t_2)}{\sin^2 \pi f(t_1 - t_2)} \right\} \right]^{-1}$$

where f =frequency in cycles/sec.

4. A method of processing seismological and seismic prospecting data from seismo-

meter channels for separating desired and undesired signals, including the steps of:

- (a) generating a plurality of different seismic signal traces



$$[S_1(t) + N_1(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

- (b) modifying each of the said seismic traces by applying relative weight and time shifts to the said traces, the modifying operator $Y_i(f)$ for the i th trace being given by the matrix equation:

$$\{Y_i(f)\} = [S_{ij}(f) + N_{ij}(f)]^{-1} \{S_{kj}(f)\}$$

where $S_{ij}(f)$ and $N_{ij}(f)$ are the cross-power spectra for the desired and undesired signals between the i th and j th channels, $S_{kj}(f)$ is the cross-power spectra of the desired signal between the k th and j th channels, $i=1$ to N , $j=1$ to N , and k =the specified channel from which the desired signal is to be obtained;

- (c) dividing the seismic trace in at least one of the said channels into a plurality of time separated seismic traces each having the same frequency range, and applying a plurality of weights to the said time separated seismic traces; and
(d) summing the said modified traces to pass the said desired signal and to reject the said undesired signal.

5. A processor for seismological and seismic prospecting data for separating desired and undesired signals, including a plurality of means operable to receive different seismic signal traces

$$[S_1(t) + N_1(t), \dots, S_n(t) + N_n(t), \dots, S_N(t) + N_N(t)],$$

- filters having complex frequency responses $Y_1(f), Y_n(f), \dots, Y_N(f)$ connected in respective transmission channels (1, . . . n . . . N) operable to feed the respective seismic traces to signal summing means, the said filters being operable to apply relative weights and time shifts to the seismic traces in the said transmission channels and having complex frequency responses given by the matrix equation:

$$\{Y_i(f)\} = [S_{ij}(f) + N_{ij}(f)]^{-1} \{S_{kj}(f)\}$$

- where $S_{ij}(f)$ and $N_{ij}(f)$ are the cross-power spectra for the desired and undesired signals between the i th and j th channels, $S_{kj}(f)$ is the cross-power spectrum of the desired signal between the k th and j th channels, $i=1$ to N , $j=1$ to N , and k =the specified channel from

which the desired signal is to be obtained;

and at least one of the said channels including means operable to divide the seismic trace transmitted therein into a plurality of time separated traces each having the same frequency range and means operable to apply a plurality of weights to the said time separated seismic traces.

6. A processor according to claim 5, for operating on seismic traces such that the seismic trace in each transmission channel is represented by:

$$f_n(t) = S(t - \tau_n) + C(t - \alpha_n) + R_n(t)$$

where $S(t)$ is the desired signal to be passed; $C(t)$ is the undesired signal to be rejected; $R_n(t)$ is random noise; and wherein the frequency dependent filters are arranged to apply relative weights and time shifts in accordance with

$$Y_n(f) = \frac{e^{i2\pi f \tau_n} \left[\phi_s \left(\phi_r + \phi_c \left\{ N - \sum_{k=1}^N e^{i2\pi f[(\tau_k - \alpha_k) - (\tau_n - \alpha_n)]} \right\} \right) \right]}{\phi_r(\phi_r + N\phi_s + N\phi_c) + \phi_s \phi_c \left(N(N-1) - 2 \sum_{i=k+1}^N \sum_{k=1}^n \cos 2\pi f[(\tau_k - \alpha_k) - (\tau_i - \alpha_i)] \right)}$$

where $n=1, 2, \dots, N$

- $\phi_s(f)$ =auto-power spectrum of $S(t)$
 $\phi_c(f)$ =auto-power spectrum of $C(t)$
 $\phi_r(f)$ =auto-power spectrum of $R_n(t)$
 $\tau_n = (n-1)d/V_s$
 $\alpha_n = (n-1)d/V_c$
 d =spacing between adjacent seismometer locations or source locations (shot stacking)
 V_s =apparent horizontal velocity of $S(t)$
 V_c =apparent horizontal velocity of $C(t)$

7. A processor according to claim 6, in which the said weights and time shifts of the frequency dependent filters $Y_n(f)$

- (a) time shift the n th seismic trace by $t_1(n-1)$ seconds,
 (b) multiply said time shifted trace by $(\phi_s \phi_r + \phi_s \phi_c N)$,
 (c) time shift said n th seismic trace by $\frac{N+1}{2} (t_1 - t_2) - n$ seconds relative to $[t_1(n-1)]$,
 (d) multiply said last mentioned time



shifted trace by $(-\phi_s\phi_e)$,
 (e) convolve said last mentioned multiplied trace with the time domain operator for $\sin N\pi f(t_1-t_2)$; and

- 5 (f) sum the convolved trace with the first mentioned multiplied trace, and including means connected to apply the summed N traces to the said sum-

$$\left[\phi_r(\phi_r + N\phi_s + N\phi_e) + \phi_s\phi_e \right] \left\{ N^2 - \frac{\sin^2 N\pi f\Delta t}{\sin^2 \pi f\Delta t} \right\}^{-1}$$

$$\text{where } \Delta t = \frac{d}{V_s} - \frac{d}{V_e}.$$

- 20 9. A processor according to any one of claims 5—8, for processing seismic traces produced by reflection waves from horizons with different dips, and adapted to pass seismic signals originating from one horizon and
 25 to reject seismic signals originating from the other horizon.
 10. A processor according to any one of claims 5—8, for processing seismic traces in the form of reflection waves produced by
 30 shot stacking, and adapted to pass primary reflections and to reject ghost reflections.

ming means, where t_1 is the time delay between consecutive channels for desired signals and t_2 is the time delay between consecutive channels for undesired signals. 10

8. A processor according to claim 7, including means operable to convolve the output of the said summing means with the line domain operator for 15

11. A processor for seismological and seismic prospecting data, substantially as described herein with reference to and as illustrated by the accompanying drawings. 35

12. A method of processing seismological and seismic data, substantially as described herein with reference to and as illustrated by the accompanying drawings.

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