



Wireless Positioning System Which Operates Without Using An Explicit
Reference

Field of the invention

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The present invention relates to a method for detecting the position of an object wirelessly within an indoor environment, and to a system for detecting its own position.

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Background of Invention

Many methods determining the position of an object using a transmitter/receiver device in conjunction with at least one active/reflective device situated at a pre-determined position as a point of reference. Frequently, a system is provided
15 which is capable of monitoring its own position relative to the active/reflective device, and this system is attached to the object.

The best known of such systems is the GPS system, in which the position of a GPS receiver located on the surface of the earth is determined using 27 Earth-
20 orbiting satellites (of which 24 are in operation at any time). The distances between the receiver and at least three of the satellites are determined by measuring the travelling time of radio waves between them. Subsequently, a 3D trilateration method can be applied to determine the receiver's location on the planet (see Marshall Brain & Tom Harris, "How GPS receivers work",
25 <http://electronics.howstuffworks.com/gps.htm>). The calculations are dependent on an almanac in the receivers detailing the position of the satellites at any given time.

US patents 5977958 and 6054950 describe methods for measuring time-of-
30 arrival with ultrashort RF pulses (UWB) transmitted from the transmitter to an array of receivers. Despite the differences in technical details, both patents require devices at pre-known positions. US patent 5977958 uses 4 receivers for



its 2D scenario and US patent 6054950 needs at least 4 receivers or beacons for 3D applications (these receivers or beacons are here termed “explicit references”). There are other similar technologies which are based on angles of reflected signals instead of their time of arrival. In such technologies, multiple references are needed. The problem with this type of system is the requirement for references at pre-known positions. This increases the number of devices needed and introduces difficulty in setting up the systems in certain situations or surroundings. Furthermore, such systems require direct line of sight (LOS) between the transmitter and receivers, the performance decreases sharply in the areas where LOS is not available.

Self-positioning methods employed in robotics often use a ring of ultrasonic transducers or laser range finders to get a measurement of the respective distance from an object to a surrounding environment in each of a number of directions. A sonar/laser range image can be formed from the distances between the detector and nearby objects. This image can be compared to a known floor plan or trained database in order to find the location of the robot. However, these methods involve multiple directional sensors (e.g. a ring of ultrasonic sensors around the robot) or scanning mechanism (e.g. a scanning laser range finder) and are highly complex.

Another US patent, 6112095 proposes a method of location determination in which a transmitter transmits a radio signal and a receiver device uses an array of antennas to receive the signal both directly and along paths which include reflections (multipath signals). Different locations of the transmitter cause the set of reflected paths to be different, so that the received signals constitute a signature of the position. The determination of the location of the transmitter is based on a set of pre-calibrated signal covariance matrices corresponding to possible transmitter locations. However, the uniqueness of the signature is, in principle, not guaranteed and sometimes leads to big errors when the multipath features of one location are similar to those of other locations. Furthermore, the

teaching in this citation is meant for outdoor application, and a base-station is required for its implementation.

Summary of the invention

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The present invention aims to provide a new and useful method for determining the position of an object.

10 The invention further proposes a new and useful system which determines its own position. The system may be an object of which it is desired to monitor the position. Or it may be an separate unit which is attached to the object in order that the position of the object can be determined.

15 In general terms, the present invention proposes that a positioning system transmits wireless (normally RF) signals in all directions (e.g. all directions within a horizontal plane, or all directions in a three-dimensional space) which are reflected by the surroundings. The system receives the echoes, but without measuring which direction they come from (in this sense the transmitter and receiver operate "omni-directionally"). During a training mode, the reflected
20 signals are used to train an adaptive system, such as a set of transformation learning (TL) units which is preferably a set of neural networks, to map the varying position of the system to the characteristics of the reflected signals. During a working mode, the trained neural network is to identify the position of the system based on reflected signals.

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In a first aspect, the invention proposes a system comprising:

a transmitter for transmitting wireless signals in all directions within a range of directions, the range of directions including at least all directions in a plane;

30 a receiver for receiving echoes of the wireless signals from any direction in the range;

a processor for processing the received echoes to derive echo data signals indicative of the distance of the system to a plurality of reflective surfaces; and

an adaptive system for:

- 5 during a training stage, learning at least one association between the echo data signals and the position of the system based on training data composed of records of echo data signals and associated system position data, and
- 10 during a working stage, using the echo data and the learnt association to determine the position of the system.

In a second aspect, the invention proposes a method of determining the position of an object, the method comprising:

- 15 transmitting wireless signals from the object in all directions within a range of directions which includes at least all directions in a plane;
- receiving at the object echoes of the wireless signals from any of the range of directions;
- processing the received echoes to derive echo data signals indicative of the distance of the object to a plurality of reflective surfaces;
- 20 during a training stage, learning at least one association between the echo data signals and the position of the object based on echo data signals and associated system position data,
- during a working stage, using the echo data and the learnt association to determine the position of the object.

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Brief Description of The Figures

Preferred features of the invention will now be described, for the sake of illustration only, with reference to the following figures in which:

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Fig 1, which is composed of Fig 1(a) and 1(b), illustrates a system which is an embodiment of the invention, and a situation in which the system may be used in a method according to the invention;

- 5 Fig 2, which is composed of Fig 2(a) and 2(b), illustrates a simplified version of the situation of Fig 1(b) and the theory underlying it;

Fig 3 is a block diagram of a first version of a signal processing unit of the system of Fig 1;

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Fig 4 is a block diagram of a second version of a signal processing unit of the system of Fig 1;

- 15 Fig. 5, which is composed of Figs. 5(a) to 5(d), shows clustering techniques which may be used in the system of Fig. 1;

Fig 6 is a block diagram of a third version of a signal processing unit of the system of Fig 1;

- 20 Fig 7, which is composed of Fig 7(a) and 7(b), illustrates problems encountered in certain applications of the system of Fig 1(a);

Fig 8 illustrates a step of a proof contained in the Appendix;

- 25 Fig 9 illustrates another step of the proof contained in Appendix;

Fig 10 illustrates a limitation in the applications of a system as shown in Fig 1(a) in an environment which has symmetry in its shape; and

- 30 Fig 11 illustrates a limitation in the applications of a system as shown in Fig 1(b) in an environment which has parallel sides.

Detailed Description of the embodiments

Fig 1(a) is a block diagram illustrating an embodiment of the invention which
 5 comprises a wireless transceiver 11 and a signal-processing unit 12. The transceiver 11 transmits short pulses, typically RF waves, via an omni-directional antenna 16, and receives reflected signals (echoes) from the surroundings.

10 The function of the transceiver 11 is to repeatedly transmit a short pulse and then receive and digitise the echoes. The digitised echoes are then fed as inputs to the signal processing unit 12.

The antenna 16 is preferably an omni-directional antenna, which is easier to
 15 design than a directional antenna, especially for high bandwidth RF signals. Besides the antenna 16, the transceiver 11 may also include a duplexer, a pulse generation unit 111, a timing control unit 112 and an A/D converter 113. The signal processing unit 12 can be integrated to the transceiver 11, or it can be remotely stationed. The processed result may be transmitted wirelessly to
 20 other processing devices via the wireless transmitter 11.

Fig 1(b) shows how the embodiment 15 of Fig 1(a) is used in an environment which is a two-dimensional space defined by a polygon 13. The polygon 13 in Fig 1(b) may for example be the floor plan of an office in which the system 15 is
 25 located. AB, BC, CD, DE and EA stand for the walls of the office. The area 14 (which is shown as a rectangle, but may be any other shape) illustrates a working area within which a user moves the system 15 (it could be of any shape). The system 15 attempts to identify its position within the working space 14 by the antenna 16 transmitting signals and detecting and identifying the
 30 number of echoes from the sides of the polygon 13. By analysing the received echoes, the system outputs the calculated positions of the transceiver in real time 121.

Note that this situation makes the following assumptions:

- Only reflections from planar reflectors (such as walls and large items of furniture) are considered. It is assumed that reflections from point sources either do not exist or can be filtered out since their magnitude is much smaller than those from planar reflectors.
- Only direct echoes are considered. Echoes arising from multiple reflections of the signal are filtered out before the processing, using the fact that their amplitude is much smaller than those of the direct echoes.
- Only specular reflection is included. Specular reflection is the strongest component in a diffuse reflection, and other components are filtered out before further processing.

Before considering this general case, we will consider a simplified version of the problem illustrated in Fig 2(a). In this version, the polygon is replaced by a triangle in a two-dimensional space defined by co-ordinates x, y . The triangle has corners A at position $(0,0)$, B at position $(x_B,0)$, and C at position (x_C,y_C) . The distances d_{AB} , d_{AC} and d_{BC} , from the point $P(x,y)$ to the three sides of the triangle, are calculated from the TOFs (time of flight) of the reflected echoes (velocity of light $\times$ time-of-flight / 2) to those sides. Note that the system does not know *a priori* which of the three echos it receives corresponds to which side of the triangle. A vector, (d_1, d_2, d_3) , referred to here as a "vector of distances", is formed by arranging the three distance values in ascending order, such that $d_1 \leq d_2 \leq d_3$.

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It can be proven (see Appendix below) that provided the triangle is not an isosceles triangle, there is a one-to-one mapping between the vector of distances and a corresponding position in the triangle ABC. The mapping is in fact linear within each of the six sectors of the triangle ABC indicated by hashing in Fig 2(b), where point O is the "inner centre" of the triangle, i.e. the intersection of the lines which bisect each angle. In other words, in the three

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dimensional space (d_1, d_2, d_3), each of the six sectors corresponds to a different plane within the space. Each point in a given sector has a one-to-one mapping with a given point on the respective plane. In fact, these planes can be written as $l_1d_i + l_2d_j + l_3d_k = 2(\text{Area of the triangle})$, where l_1, l_2 and l_3 are the respective
 5 lengths of the sides of the triangle, and d_i, d_j and d_k are different permutations of d_1, d_2 and d_3 (d_i, d_j, d_k) depending on which of the six sectors of Fig 2(b) the point P is in. Therefore, the mapping (and its inverse) between the position of the system 14 and the vector of distances can be described by six linear
 10 functions.

It will be appreciated that the concept of the vector of distances can be generalised from the triangular space of Fig 2(a) to the general polygonal space of Fig 1(b), and it will be demonstrated in the Appendix that the presence of a one-to-one mapping between this vector of distances and locations in the
 15 polygon holds true in general cases.

Methods which are embodiments of the invention include a training step in which training data is generated by locating the system 15 successively in different training positions (x_i, y_i) by digitising reflected signals (the variable i
 20 labels the items of training data). The dotted lines 14 of Fig 1(b) represent the working space within which the training positions are be arbitrarily chosen. Due to the phenomenon illustrated in Fig 2(b) that the vector of distances shows discontinuous changes at the edges of sectors, the training data will tend to be clustered, such that (in the case of the environment of Fig 2(a)) it is composed
 25 of points on the six planes in the (d_1, d_2, d_3) space. The training data, i.e. pre-known coordinates (x_i, y_i) and the digitised echoes received at these positions, is used to learn the mapping between the location of the system 15 and the vector of distances. Then there is a working step in which the system 15 is located in a unknown position (x_t, y_t), and its position is determined using the
 30 vector of distances and the mapping.

Fig 3 shows the construction of the signal processing unit 12 in a first version of the embodiment of Fig 1. The training mode workflow is represented by dotted arrows and the working mode workflow is represented by solid arrows. The system 15 includes a switch (not shown) for switching between these two processing modes.

In the training mode, the reflected waveforms 21a received by the signal processing unit 12 are first pre-processed in a pre-processing unit 21. The unit 21 performs a pre-processing procedure comprising steps of sampling and digitising the received signals, averaging among signals of several emission-reception cycles to reduce noise, and optionally other steps (e.g. filtering) to further remove noise or distortions. The pre-processed waveforms 22a are then fed into a neural network 22. The training data also includes the 2D coordinates of each training point (x_i, y_i) , and these 2D coordinates 22c are also fed into the neural networks 22. The neural network coefficients of the neural network 22 are adjusted (using conventional neural network algorithms) to reduce the sum over the training points of an error function which measures the difference between the known positions (x_i, y_i) and the output 24 of the neural network 22 when the waveforms 22a are input to the neural network 22.

When so trained, the system 15 in the working mode can determine its position (x_t, y_t) based on echoes. Signals are again sent out by the system 15, and the reflections are again picked up and fed as signals 21b into the pre-processing unit 21. The pre-processed signal 22b is then fed into the neural network 22 which now determines the position (x_t, y_t) of the transmitter based on the mapping and outputs a position signal 24.

The workflow of a second version of the signal processing unit 12 of the system of Fig 6 is shown in Fig 4. The training workflow is represented by the dotted arrows while the working workflow is represented by the solid arrows. During the training, as in the version of the signal processing unit 12 shown in Fig 3, the reflected signals 41a are fed into the pre-processing unit 41 to remove noise

and signal distortions. The signals 42a output by the unit 41 are then fed into a unit 42 to calculate vectors of distances 43a. The vectors of distances 43a are then fed into a neural network 43, along with the coordinates 44 of the present training point (x_i, y_i) . The neural network coefficients of the neural network 43 are
 5 adjusted (using conventional neural network algorithms) to reduce the sum over the training points of an error function which measures the difference between the known positions (x_i, y_i) and the output 45 of the neural network 43 when the vectors 43a are input to the neural network 43.

10 In the working mode, the reflected signals 41b are pre-processed in the pre-processing unit 41. The resultant signals 42b are then fed into the unit 42 for calculating the vector of distances 43b, which are subsequently fed into the neural network 43 which determines the 2D position (x_t, y_t) of the system as output 45.

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The neural networks of the first and second variants are preferably multi-layer feed-forward (FF) or radial basis functions (RBF) networks.

Whereas the first and second variants of the signal processing do not take any
 20 account of the importance of the sectors illustrated in Fig 2(b), a third variant of the signal processing unit 12 of Fig 1 includes means for clustering the vectors of distances.

The clustering of the training data into six different planes can be done by a
 25 Hough transform (see Digital Image Processing, page 432-438, Rafael C. Gonzalez, Richard E. Woods and Addison Wesley, 1993). A Hough Transform is frequently used in image processing to link points by determining whether they lie on a curve of specified shape. This method transforms the description of the curve in the image space into the parameter space to reduce the
 30 computation load. The detailed procedure can be found in normal image processing textbooks. Experimentally, it has been found that a Hough Transform is able very accurately to classify vectors of distances obtained in a

set-up as shown Fig 2(a) into the 6 planes. The Hough Transformation method, however, may not converge if requires the surroundings are not closed polygons.

- 5 Other than by a Hough transform, the clustering may be done by self-organising networks. Self-organising networks detect regularities and correlations in the inputs and accordingly adapt responses.

There are two common structures of self-organising networks, competitive
 10 networks and self-organising maps. These techniques use entities referred to as “neurons”, but these are not to be confused with the neurons used in the neural networks 22, 43 used in Figs. 3 and 4. Competitive networks learn to recognise groups of similar input vectors. Self-organising maps learn to recognise groups of similar input vectors in such a way that neurons physically
 15 near each other in the neuron layer respond to similar input vectors. In both cases, the state of the networks are defined by “neural network coefficients”.

In a first form of learning using self-organising maps, the neural network coefficients are adjusted to recognise groups of similar distance-location
 20 vectors. A “distance-location vector” is made up of the vector of distances concatenated with the vector of the location (i.e. the training positions (x_i, y_i)). This distance-location vector provides more information for the clustering of the training data.

- 25 Fig 5(a) illustrates a process of clustering based on distance-location vectors. Vectors of distances 46a are extracted from training data, and are fed into a unit 47 along with the corresponding vector of location 46b. The unit 47 concatenates the vectors of distances and the 2D coordinates into the distance-location vectors. The distance-location vectors are then fed into a self-
 30 organising network 48 which clusters the distance-location vectors into a pre-set number (N) of groups, based on the regularities in them. The output 49 of the unit 48 whenever a given distance-location vector is input to it, is a the number

in the range 1 to N indicative of which group the vector of distances corresponds to.

In a triangle such as that in Fig 2, the self-organising networks 48 can cluster
 5 the input vectors into 6 groups corresponding to 6 sectors of the triangle divided by the bi-sectors. Unlike a clustering method using the Hough transform, this method does not require the surroundings to be a closed polygon in shape and can be applied to more general scenarios.

10 An alternative to clustering using the distance-location vectors is shown in Fig. 5(b) and Fig. 5(c). This method of clustering is based on an assumption that, within sections of an enclosed area, there is a linear transformation between the vector of distances and the 2D coordinates. By using at least 3 neighbouring training points, which do not lie on a same line, the linear transformation
 15 between the 2D coordinates of a position and a vector of distances of the position can be estimated. If these three neighbouring points happen to all lie in the same section in the enclosed area, the linear transformation for the section can be correctly estimated. The linear transformations between the distance vector and the 2D coordinates are estimated for every N neighbouring training
 20 points (the selection of the points affects the success of the estimations). The estimation of the linear transformation may be done by the least-square method. Fig 5(b) shows a lattice-like group of training points where the distance vectors and the 2D coordinates of the darkened point and the 4 points nearest to it are used to estimate a linear transformation. This localised transformation
 25 is assigned to the point of interest, i.e. the darkened center point.

Fig 5(c) is a block diagram of a clustering process based on locally estimated linear transformation. Vectors of distances 491a extracted from the training data are fed, together with location vectors 491b, into a unit 492 for working out the
 30 linear transformation. The resultant matrix is then converted into a vector and then fed into a self-organising network 493, by which they are clustered into groups. The output 494 of the unit 493 whenever a given linear transformation

is input to it is a the number in the range 1 to N indicative of which group the vector of distances corresponds to.

Conceivably, if the points shown in Fig. 5(b) do not all lie within a same section,
 5 i.e. they do not all belong to one cluster, so an inaccurate estimation of the linear transformation will result. However, if the training data is well distributed, such that there are many more correct estimations than inaccurate ones, the self-organising network 493 can be used to accurately cluster the linear transformations into a pre-set number of groups (six in the case of a triangle).
 10 This method does not have the disadvantage of requiring the surroundings to be closed-polygonal in shape, unlike the Hough transform.

As described above, the self-organising networks 48, 493 learn to output a signal indicative of a cluster whenever an appropriately processed portion of the
 15 training data (e.g. a distance-location vector or linear transformation) is input to them. The output of the self-organising networks is useful because it can train another unit, a "classifying unit", to recognise associations between the training data and these clusters.

20 This process is shown in Fig. 5(d), in which, during the training phase, an adaptive model 496 known as an LVQ (learning vector quantization) network is taught to classify input vectors of distance 495a into the clusters derived by the clustering units of Figs. 5(a) or 5(c). During the training phase, the LQV network 496 is simultaneously fed (as shown by the dotted lines) with the vectors of
 25 distance 495a and the corresponding indices 49, 494 obtained from the clustering units (these indices are shown as 497 in Fig. 5(d)).

During the working phase, the unit 496 is receives only the vectors of distances (shown by a solid arrow), and operates based only on them to identify the
 30 group, and outputting the number 498 in the range 1 to N indicative of which group the vector of distances corresponds to. Again, this method does not require the surrounding to be a closed polygon.

Referring now to Fig 6, a third variant of the signal processing unit 12 of Fig 1 is shown. The work flow in the training mode is shown in dotted arrows and the work flow in the working mode is shown in solid arrows. Again, in the training mode, signals are transmitted to the surroundings by the system 15 and signals reflected by planar surfaces are received by the transceiver 11 of the system 15. The output 71a of the transceiver is fed into a pre-processing unit 71 to remove noise and signal distortions. The filtered signals 72a are then fed into a unit 72 to calculate the vector of distances, and the output 73a is fed into a clustering/classifying unit 73 to be clustered into planes. The clustering/classifying unit 73 may employ the Hough Transformation. Alternatively, it may include a clustering unit 731 such as shown in Fig. 5(a) or 5(c) (implementing a clustering algorithm, such as those described above) and a classification unit 732 (as shown in Fig. 5(d)). As mentioned above, the clustering unit 731 during the learning phase receives both the location data (from database 75) and the vector of distances, and it outputs an index representing a cluster. This is used during the training phase to train the clustering unit 732 to recognise clusters based on vectors of distances only. During the working phase, the clustering unit 732 is not used, and the classification unit recognises vectors of distances without any input from the clustering unit 731.

The system further includes a number of neural networks 741, 742, 743. Note that the neural networks 741, 742, 743 may be replaced by any other form of adaptive transformation unit (an example is given below). Furthermore, although three neural networks are shown, the number is not limited to three (and is in general many more). According to the number of clusters which the clustering unit 731 identifies, an equal number of the TL learning units are activated and one TL learning unit is associated with each respective cluster.

In the training mode, the output 78 of the clustering unit 731 is also fed to two selection units 75a and 733. The selection unit 733 passes the vector of distances (which it receives from the classification unit 731) to the one of the TL units 741, 742, 743 identified by the signal 78. Simultaneously, the selection unit 75a passes the corresponding location data (x_i, y_i) from the database 75 to the TL unit identified by the signal 78. The coefficients of that TL unit are adjusted (using conventional algorithms) to reduce the sum over the training points of an error function which measures the difference between the known positions (x_i, y_i) and the output 76a of that TL unit when the signal 74a is input to the TL unit.

In the working mode, reflected signals 71b are firstly fed into the pre-processing unit 71. The filtered signal 72b is then fed into the unit 72 to obtain the vector of distances. Next, the vector of distances is fed into the clustering/classifying unit 73 and is classified as belonging to one of the clusters, by the classification unit 732 which outputs a result 76b. The selection unit 733 feeds the vector of distances 74a (which it obtained from the classification unit 732) to the TL unit 741, 742, 743... identified by the signal 76b. The unit 76 receives the outputs of all the TL units and the signal 76b indicating which cluster the vector of distances belongs to. The unit 76 outputs a signal 77 which is the output of the TL unit associated with that cluster.

In the case that the TL units 741, 742, 743 are neural networks, they may be very simple: preferably simple 2-neuron linear feed-forward structures (i.e. a two layer structure having an input layer and an output layer, with no hidden layer). This is simply because the distance from a point to a line is a linear function of x-y coordinates. For example, in the configuration of Fig 2(a), the distances to AB, BC and AC are:

$$\begin{aligned}
d_{AB} &= y \\
d_{BC} &= \frac{(y_C x - (x_C - x_B)y - x_B y_C)}{\sqrt{y_C^2 + (x_C - x_B)^2}} \\
d_{AC} &= \frac{(y_C x - x_C y)}{\sqrt{x_C^2 + y_C^2}}
\end{aligned}$$

Note that many variants of the system of Fig 6 are possible within the scope of the invention. In particular, as mentioned above, the TL units need not be neural networks, but may instead be units which perform a least squares estimation of linear transformation 741, 742, 743. The transformation between the vectors of distances and the 2D coordinates obtained by estimation based on least squares is represented mathematically below. If the mapping between the vectors of distances and the 2D coordinates for one of the clustered training points is written as (for one point with sub-index i):

$$\begin{bmatrix} d_{1i} \\ d_{2i} \\ \dots \\ d_{Ni} \end{bmatrix} = A \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix} \quad (1)$$

Then for all the training data with sub-index $i = 1$ to M within this same cluster:

$$\begin{bmatrix} d_{11} & d_{12} & \dots & d_{1M} \\ d_{21} & d_{22} & \dots & d_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N1} & d_{N2} & \dots & d_{NM} \end{bmatrix} = A \begin{bmatrix} x_1 & x_2 & \dots & x_M \\ y_1 & y_2 & \dots & y_M \\ 1 & 1 & \dots & 1 \end{bmatrix} \quad (2)$$

As long as $M \geq 3$ and not all the points fall on one straight line, the transformation matrix A can be estimated in least-square sense as:

$$A = D \cdot \text{pseudo-inverse}(X) \quad (3)$$

where $D = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1M} \\ d_{21} & d_{22} & \cdots & d_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ d_{N1} & d_{N2} & \cdots & d_{NM} \end{bmatrix}$ and $X = \begin{bmatrix} x_1 & x_2 & \cdots & x_M \\ y_1 & y_2 & \cdots & y_M \\ 1 & 1 & \cdots & 1 \end{bmatrix}$

The various embodiments of the invention have been described above with
 5 reference to the situation of Fig. 2, but their implementation in the situation of
 Fig. 1(b) is straightforward. Although the input from a polygonal environment to
 the neural networks will comprise more than three elements making up a vector
 of distances, e.g. $(d_1, d_2, d_3, \dots, d_N)$, the clustering will then be to find 2D planes in
 the N-dimensional space of distances. The Hough Transform is still applicable.
 10 If the number of sides in the environment is unknown, a pre-set number of
 planes can be assumed. The structure of the neural network remains the same
 simple 2-neuron linear feed-forward structure. Furthermore, the situations of Fig
 1(b) and Fig 2(a) are two-dimensional. However, the embodiments are readily
 extended to real life, 3D situations, since the distance from a point (x_p, y_p, z_p) to
 15 a plane $Ax + By + Cz + D = 0$ is

$$\frac{|Ax_p + By_p + Cz_p + D|}{\sqrt{A^2 + B^2 + C^2}},$$

which is again linear. The linear mapping will be the same within 3D sections
 divided by the bisector planes in 3D spaces. The clustering step will then be to
 20 find a 3D "hyper plane" within a space of at least 4 dimensions (the Hough
 Transform method is also applicable in 3D space). The neural networks,
 however, will have 3 neurons instead of 2.

Note that the embodiments above do have some limitations. For example,
 25 considering the environment shown in Fig 7(a) which is an obtuse triangle, there
 may be locations where not every element in a vector of distances can be
 measured, because the direct specular reflections from one of the sides cannot
 be received. Fig 7(b) is a graph of the intensity of the received signal against

time when the system 15 is in one of the shaded areas of Fig 7(a), and contains only two peaks. For this reason, the embodiments do not work within the shaded areas of the obtuse triangle in Fig 7(a) because the one-to-one mapping pre-requisite is not valid. In those regions, any pair of points that are symmetric to the bisector of $\angle A$ or $\angle B$ give the same vector of distances. However, this will not influence the validity of the method within the un-shaded area of the triangle where reflections from all three sides of the triangle can be obtained. This situation also can be generalised to a more complex space defined by more than three sides. Whether the work area 14 chosen by the user of the system is workable or not can be found out by looking for changes in the number of echoes during the training session.

It was noted above that the one-to-one mapping property for the environment of Fig 2(a) is only valid if the triangle is not an isosceles triangle. More generally, if the surroundings of the system 15 are a polygon with more than three sides (as in Fig 1(b)), the one-to-one property of the mapping remains true only if there is no symmetry in the polygon (an example of a symmetrical polygon is an equilateral pentagon), for example if it has no parallel sides like a rectangle. However, even in these cases, within a certain sub-area, this one-to-one mapping property may still be valid.

Finally, although it is theoretically proven that any one-to-one mapping can be learnt by a neural network (e.g. RBF) to high levels of accuracy, given a large enough number of neurons and training data, in reality there is always a limit on the complexity of the neural network and the acceptable amount of training data. These factors may limit the performance of the embodiment of Fig. 5, but the embodiment of Fig. 6 is more resistant to this problem.

Although the invention has been described above with reference to the above embodiments, many other embodiments are possible within the scope of the invention, as will be clear to a skilled reader. For example, it will be obvious to a

skilled man that the neural networks used in the embodiments are only examples of the broader scope of transformation learning technologies.

Appendix I

Statement:

“Given an arbitrary acute triangle which is not isosceles and a point within the triangle, the distance vector is defined as the distances from the point to the three sides of the triangle in ascending order, there is a one-to-one mapping between the point position and the distance vector”

(1) It is obvious that given a point, there is only one vector of distances.

10

(2) Suppose that there is a distance vector (d_1, d_2, d_3) with $d_1 \leq d_2 \leq d_3$, it has to be proven that there is only one point within a triangle which gives this distance vector.

15 Assume that there are two points (x_1, y_1) and (x_2, y_2) which give the same distance vector (d_1, d_2, d_3) . If the distance to the three sides AB, BC and AC to the first point are labelled d_{AB1} , d_{AC1} and d_{BC1} respectively, and the distances to from the three sides to the second point are labelled d_{AB2} , d_{AC2} and d_{BC2} , without loss of generality, there are only three possibilities:

$$20 \quad (a) \begin{cases} d_{AB1} = d_{AB2} \\ d_{AC1} = d_{AC2} \\ d_{BC1} = d_{BC2} \end{cases}; \quad (b) \begin{cases} d_{AB1} = d_{AB2} \\ d_{AC1} = d_{BC2} \\ d_{BC1} = d_{AC2} \end{cases} \quad \text{and} \quad (c) \begin{cases} d_{AB1} = d_{BC2} \\ d_{AC1} = d_{AB2} \\ d_{BC1} = d_{AC2} \end{cases}.$$

For case (a) it is easy to prove the two points will be identical as shown in Fig 8.

If two lines are drawn parallel to AB and AC, with distance $d_{AB1} = d_{AB2}$ and $d_{AC1} = d_{AC2}$ to them respectively, both points must lie on these two lines concurrently. Since there will be only one intersection for two straight lines, the uniqueness is proved (see Fig 8).

25

For case (b) the situation is shown in Fig 9, where $FG \parallel AB$, $PD \perp AC$, $PE \perp BC$, $P'D' \perp BC$, $P'E' \perp AC$ and $PD = P'D'$, $PE = P'E'$. It follows that $PD \parallel P'E'$ and $PE \parallel P'D'$, and therefore $\frac{FP}{FP'} = \frac{PD}{P'E'} = \frac{P'D'}{PE} = \frac{GP'}{GP}$. Since $FP' = FP + PP'$ and $GP = GP' + PP'$, it can be further deduced that $FP = GP'$. Then together with $PD = P'D'$, we obtain $\triangle PFD \cong \triangle P'GD'$. As a result $\angle DFP = \angle D'GP'$. Since $FG \parallel AB$, $\angle DFP = \angle A$ and $\angle D'GP' = \angle B$, it is shown that the two points do not coincide only when the triangle is an isosceles one. On the other hand, if this condition is satisfied, it means that there is existing symmetry in the geometry of the triangle and there are at least two different points with same vector of distances.

Case (c) can be proven by calculation of the three distances. Without loss of generality, the triangle and the coordinate systems is aligned as shown in Fig. 9. Then the distance from a point (x, y) to the three sides AB, BC and AC, respectively, is:

$$\begin{aligned} d_{AB} &= y \\ d_{BC} &= \frac{|y_C x - (x_C - x_B)y - x_B y_C|}{\sqrt{y_C^2 + (x_C - x_B)^2}} \\ d_{AC} &= \frac{|y_C x - x_C y|}{\sqrt{x_C^2 + y_C^2}} \end{aligned}$$

and following equations must hold.

(i) For (x_1, y_1) :

$$\left[\frac{x_B * y_C}{BC} - \frac{y_C}{AC} \right] x_1 + \left[\frac{AC * BC - x_B * (x_C - x_B)}{BC^2} + \frac{x_C}{AC} \right] y_1 - \frac{x_B * y_C * (x_B + BC)}{BC^2} = 0 \quad \text{--}$$

(1)

(ii) For (x_2, y_2) :

$$\begin{cases} y_2 = \frac{y_C * x_1 - (x_C - x_B) * y_1 - x_B * y_C}{BC} \\ y_C * x_2 - x_C * y_2 = y_1 * AC \end{cases} \text{-----}$$

(2)

Equation (1) describes a line that (x_1, y_1) must fall on given case (c) and
 5 equation (2) describes (x_2, y_2) if (x_1, y_1) is given in case (c). It can be proved
 using algebra that it is impossible for both points to be within the triangle.

By cases (a), (b) and (c), the uniqueness of a point corresponding to a vector of
 distances (d_1, d_2, d_3) is proved, given that the triangle is not isosceles.

10 Combining (1) and (2) the statement is proved.

For the polygon with 4 sides, similar reasoning can divide the possibility of
 following cases (where AB, BC, CD and DA are the four sides):

a) At least two distances to two respective sides are equal, such as

$$\begin{cases} d_{AB1} = d_{AB2} \\ d_{BC1} = d_{BC2} \\ d_{CD1} = d_{DA2} \\ d_{DA1} = d_{CD2} \end{cases}.$$

15

b) Only one distance to a side are equal, including

$$\begin{cases} d_{AB1} = d_{AB2} \\ d_{BC1} = d_{CD2} \\ d_{CD1} = d_{DA2} \\ d_{DA1} = d_{BC2} \end{cases} \text{ and } \begin{cases} d_{AB1} = d_{AB2} \\ d_{BC1} = d_{DA2} \\ d_{CD1} = d_{BC2} \\ d_{DA1} = d_{CD2} \end{cases}$$

c) Two vector-of-distances' elements are of totally different permutation, including:

$$\begin{aligned}
 & \left\{ \begin{array}{l} d_{AB1} = d_{BC2} \\ d_{BC1} = d_{CD2} \\ d_{CD1} = d_{DA2} \\ d_{DA1} = d_{AB2} \end{array} \right\}, \left\{ \begin{array}{l} d_{AB1} = d_{BC2} \\ d_{BC1} = d_{AB2} \\ d_{CD1} = d_{DA2} \\ d_{DA1} = d_{CD2} \end{array} \right\}, \left\{ \begin{array}{l} d_{AB1} = d_{BC2} \\ d_{BC1} = d_{DA2} \\ d_{CD1} = d_{AB2} \\ d_{DA1} = d_{CD2} \end{array} \right\}, \\
 & \left\{ \begin{array}{l} d_{AB1} = d_{CD2} \\ d_{BC1} = d_{AB2} \\ d_{CD1} = d_{DA2} \\ d_{DA1} = d_{BC2} \end{array} \right\}, \left\{ \begin{array}{l} d_{AB1} = d_{CD2} \\ d_{BC1} = d_{DA2} \\ d_{CD1} = d_{AB2} \\ d_{DA1} = d_{BC2} \end{array} \right\}, \\
 & \left\{ \begin{array}{l} d_{AB1} = d_{CD2} \\ d_{BC1} = d_{AB2} \\ d_{CD1} = d_{BC2} \\ d_{DA1} = d_{AB2} \end{array} \right\}, \left\{ \begin{array}{l} d_{AB1} = d_{DA2} \\ d_{BC1} = d_{AB2} \\ d_{CD1} = d_{BC2} \\ d_{DA1} = d_{CD2} \end{array} \right\}, \left\{ \begin{array}{l} d_{AB1} = d_{DA2} \\ d_{BC1} = d_{CD2} \\ d_{CD1} = d_{AB2} \\ d_{DA1} = d_{BC2} \end{array} \right\} \quad \text{and} \quad \left\{ \begin{array}{l} d_{AB1} = d_{DA2} \\ d_{BC1} = d_{CD2} \\ d_{CD1} = d_{BC2} \\ d_{DA1} = d_{AB2} \end{array} \right\}
 \end{aligned}$$

For case (a) the uniqueness of the vector of distance can be proven in the same manner as for a triangle.

- 10 For cases (b) and (c), it can be proven by calculation that there is at most only a pair of vectors that satisfy any of the cases. Even when a solution for two different points exists, these two points are either identical (in case b) or cannot be within the polygon at the same time (as in case c), given that there is no symmetry in the polygon (e.g. Fig 10, shows an example of a pair of problematic points) and there are no parallel sides (e.g. Fig 11 shows an example of pair of problematic points).

For a polygon with more than 4 sides, i.e. $N > 4$, the condition for two points in the polygon corresponding to the same vector of distance can be written as:

$$M_{N \times 3} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} = M'_{N \times 3} \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix}$$

where M stands for the matrix with each row the function of the line of each side with coordinates of x and y normalised; M' is made up of rows of M with a certain permutation.

If M and M' are written as: $(M_1 \mid M_2)$ and $(M'_1 \mid M'_2)$, the equation

can then be further re-written as:

$$(M_1 - M'_1)_{N \times 4} \begin{pmatrix} x \\ y \\ x' \\ y' \end{pmatrix} = (M_2 - M'_2)_{N \times 1} \text{ or } A \begin{pmatrix} x \\ y \\ x' \\ y' \end{pmatrix} = B.$$

- 5 If $N > 5$, there will be no solution to this equation unless the according rows in A and B are proportional. This is impossible based on the definition of M and M' and the assumption that there is no symmetry in the polygon and no parallel sides.

Claims

1. A system comprising:
 - a transmitter for transmitting wireless signals in all directions within a
5 range of directions, the range of directions including at least all directions in a
plane;
 - a receiver for receiving echoes of the wireless signals from any direction
in the range;
 - a processor for processing the received echoes to derive echo data
10 signals indicative of the distance of the system to a plurality of reflective
surfaces; and
 - an adaptive system for:
 - during a training stage, learning at least one association between the
echo data signals and the position of the system based on training data
15 composed of records of echo data signals and associated system position data,
and
 - during a working stage, using the echo data signals and the learnt
association to determine the position of the system.
- 20 2. A system according to claim 1 in which the processor includes a distance
vector determination unit which provides the echo data signals as a series of
numbers which indicate the distances from the system to each of a number of
respective reflective surfaces.
- 25 3. A system according to claim 1 or claim 2 which includes a clustering and
classifying unit, the clustering and classifying unit being arranged:
 - during the training stage, to partition the echo data signals of the training
data into clusters, the adaptive system learning an association for each cluster
between the echo data signals of that cluster and the associated positions of
30 the system; and

during the working stage, to determine which cluster the echo data signals belong to, whereby the adaptive system determines the position of the system using the learnt association of the corresponding cluster.

5 4. A system according to claim 3 in which the clustering and classifying unit includes a clustering unit adapted during the training stage to derive a cluster index indicative of a cluster associated with the echo data signals, and a classifying unit arranged during the training stage to learn associations between the echo data signals and the cluster indices and during the working phase to
10 output a cluster index based on the echo data signals.

5. A system according to claim 4 in which the classifying unit is arranged to employ a learning vector quantization algorithm.

15 6. A system according to claim 5 in which the clustering unit during the training stage is arranged to take as input a vector comprising the echo data signals and the associated positions.

7. A system according to claim 5 in which the clustering unit during the
20 training stage is arranged to take as input a vector comprising a linear transformation derived from the echo data signals and associated positions.

8. A system according to any preceding claim in which the adaptive system includes at least one neural network.

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9. A system according to claim 8 when dependent upon claim 3 in which the adaptive system includes a neural network for each cluster, the neural network being trained using training data for the corresponding cluster to learn the association for echo data signals of the corresponding cluster.

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10. A system according to claim 9 in which each neural network is a two-layer network.

11. A system according to claim 10 in which each neural network performs a linear function of its inputs.
- 5 12. A system according to claim 3 in which the adaptive system includes for each cluster a respective transformation unit adapted during the training stage to derive a respective linear transformation of its inputs.
- 10 13. A system according to claim 12 in which the transformation units derive the linear transformation by a least squares algorithm based on (i) vectors of distances as an input and (ii) positions associated with the echo data signals as outputs.
- 15 14. A system according to any preceding claim in which the signals are RF wireless signals.
15. A system according to claim 14 in which the transmitter is an omni-directional antenna.
- 20 16. A method of determining the position of an object, the method comprising:
- transmitting wireless signals from the object in all directions within a range of directions which includes at least all directions in a plane;
 - receiving at the object echoes of the wireless signals from any of the
 - 25 range of directions;
 - processing the received echoes to derive echo data signals indicative of the distance of the object to a plurality of reflective surfaces;
 - during a training stage, learning at least one association between the echo data signals and the position of the system based on echo data signals
 - 30 and associated object position data,
 - during a working stage, using the echo data and the learnt association to determine the position of the object.

17. A method according to claim 16 in which the echo data signals indicate the distances from the object to each of a number of reflective surfaces.

- 5 18. A method according to claim 16 or claim 17 further including:
during the training stage, partitioning echo data signals of the training data into clusters, the adaptive system learning an association for each cluster between the echo data signals of that cluster and the associated positions of the object; and
10 during the working stage, determining which cluster the echo data signals belong to, whereby the adaptive system determines the position of the object using the learnt association of the corresponding cluster.

19. A method according to claim 18 in which:
15 during the training stage a clustering unit derives a cluster index indicative of a cluster associated with the echo data signals, and a classifying unit learns associations between the echo data signals and the cluster indices, and
during the working phase to output a cluster index based on the echo
20 data signals.

20. A method according to claim 19 in which the classifying unit employs a learning vector quantization algorithm.

- 25 21. A method according to claim 20 in which the clustering unit takes as input a vector comprising the echo data signals and the associated positions.

22. A method according to claim 20 in which the clustering unit takes as input a vector comprising a linear transformation derived from the echo data
30 signals and associated positions.

23. A method according to any of claims 16 to 22 in which the learning is at least partly performed by at least one neural network.

24. A method according to claim 23 when dependent upon claim 18 which
5 employs a neural network for each cluster, the neural network being trained using training data for the corresponding cluster to learn the association for echo data signals of the corresponding cluster.

25. A method according to claim 24 in which each neural network is a two
10 layer network.

26. A method according to claim 25 in which each neural network performs a linear function of its inputs.

15 27. A method according to claim 18 in which the adaptive system includes for each cluster a respective transformation unit which during the training stage derives a respective linear transformation of its inputs.

28. A system according to claim 27 in which the transformation units derive
20 the linear transformation by a least squares algorithm based on (i) vectors of distances as an input and (ii) positions associated with the echo data signals as outputs.

29 A system according to any of claims 16 to 28 in which the range of
25 directions is all directions within a three-dimensional space.