What is claimed is:

1. An optimization system for transportation scheduling and inventory management of a bulk product from supply locations to demand locations comprising:

a) a mathematical model containing mathematical programming equations, wherein an objective function of the mathematical model is to minimize a cost basis of product transportation for multiple shipments over a period of time; wherein the model contains constraints that account for transportation scheduling and inventory management, and wherein the model accounts for an accumulation of bulk product at supply locations and removal of bulk products from a demand location that is not required to be constant over time;

b) a database application for data input that interfaces with the mathematical model; and,

c) a mathematical optimization solver that solves the equations provided by the mathematical model after the mathematical model receives data from the database system, whereby the optimization system provides optimized or optimal results for the data input.

2. The optimization system of claim 1 wherein the cost basis is selected from the overall cost per unit weight or volume of bulk product.

3. The optimization system of claim 1 wherein the bulk product is selected from petroleum, natural gas and liquid or gaseous intermediate and final products derived there from.

4. The optimization system of claim 1 wherein the mathematical model is in the form of a mixed integer linear programming problem.

5. The optimization system of claim 1 wherein the model accounts for flat rates, overage calculation, and demurrage costs

6. The optimization system of claim 1 wherein the transportation vessels are a heterogeneous fleet of ships.

7. The optimization system of claim 1 wherein the mathematical model comprises equations for each of the following constraints: (i) inventory balance; (ii) flow rate; (iii) logical constraints; (iv) travel time; (v) draft limit; (vi) overage calculation; (vii) demurrage or relaxed demurrage time calculation; and (viii) previously chartered vessels.

8. The optimization system of claim 7 wherein the logical constraints are comprised of equations for one or more or the following: (i) only one vessel may stop at a particular load or discharge port on any particular day; (ii) a vessel may only be at one place at one time; (iii) if a vessel does not stop at a port, then it may not have any travel legs to or from that port; (iv) a vessel may only take one leg that goes from a load port to a discharge port; (v) if a vessel stops at a load port, then there is a travel leg for leaving the port; (vi) if a vessel stops at a discharge port, then there is a travel leg for entering the port; (vii) a vessel may only stop at ports and travel between them if there is a load-to-discharge leg in the voyage.

9. The optimization system of claim 7 wherein the inventory balance constraints contain slack variables which allow the solver to terminate with a feasible solution.

10. The optimization system of claim 7 wherein the mathematical model further comprises one or more equations for a rolling time horizon.

11. The optimization system of claim 7 wherein the mathematical model further comprises one or more equations for grade segregation constraints.

12. The optimization system of claim 7 wherein the mathematical model further comprises equations for one or more of the following: bounds on the number of vessels used; minimum vessel loads; and minimum volume transported.

13. The optimization system of claim 7 wherein the mathematical model further comprises equations for a maximum cost-per-amount ratio.

14. A method for minimizing a total cost basis for the transportation scheduling and inventory management of a bulk product over a period of time comprising the steps of:

i) inputting data and/or modeling options into a database system, wherein the database system interfaces with a mathematical model;

ii) solving the mathematical model after the data and/or modeling options are input using an optimization engine to obtain a schedule for routing vessels in loading, transporting and discharging; and

optionally repeating steps i) and ii) one or more time with different data to obtain more than one schedule.

15. The optimization system of any one of claims 1 or 14 wherein the cost basis is number of vessels utilized.

What is claimed is:

1. A computer system comprising memory and a processor, the computer system being programmed to perform steps comprising:

providing a mathematical model containing equations that model transportation scheduling and inventory management of a bulk product from multiple supply locations to multiple demand locations to minimize cost per unit volume or mass of the bulk product,

the model includes a parameter for demurrage rate and constraints for minimum flow rates at the demand locations, maximum flow rates at the demand locations, and vessel draft limits, and

the model includes decision variables for: (i) the amount of bulk product to be loaded from a supply location onto a vessel, (ii) the amount of bulk product to be discharged from a vessel to a demand location, and (iii) demurrage time for a vessel;

receiving data relating to the characteristics of a fleet of vessels, supply locations, and demand locations;

applying the data to the mathematical model; and

minimizing cost per unit volume or mass of the bulk product using a mathematical optimization solver to solve the mathematical model based upon the constraints and decision variables.

2. The computer system of claim 1 wherein the bulk product is selected from petroleum, natural gas and liquid or gaseous intermediates, and final products derived there from.

3. The computer system of claim 1 wherein the bulk product is liquefied natural gas.

4. The computer system of claim 1 wherein the mathematical model is in the form of a mixed integer linear programming problem.

5. The computer system of claim 1 wherein the model accounts for flat rates, overage calculation, and demurrage costs.

6. The computer system of claim 1 wherein the transportation vessels are a heterogeneous fleet of ships.

7. The computer system of claim 1 wherein the mathematical model comprises equations for each of the following constraints: (i) inventory balances; (ii) flow rates; (iii) logical constraints; (iv) travel times; (v) draft limits; (vi) overage calculations; (vii) demurrage or relaxed demurrage time calculations; and (viii) previously chartered vessels.

8. The computer system of claim 7, wherein the supply locations are load ports and the demand locations are discharge ports, and wherein the constraints include equations for one or more of the following: (i) only one vessel may stop at a particular load or discharge port on any particular day; (ii) a vessel may only be at one place at one time; (iii) if a vessel does not stop at a port, then it may not have any travel legs to or from that port; (iv) a vessel may only take one leg that goes from a load port to a discharge port; (v) if a vessel stops at a load port, then there is a travel leg for leaving the port; (vi) if a vessel stops at a discharge port, then there is a travel leg for entering the port; (vii) a vessel may only stop at ports and travel between them if there is a load-to-discharge leg in the voyage.

9. The computer system of claim 8, wherein the supply locations are load ports and the demand locations are discharge ports, and wherein the mathematical model further comprises equations for bounds or capacities for one or more of the following continuous variables: (i) total cost; (ii) rate of flow of product from a load port to a vessel on a day; (iii) rate of flow of product from a vessel to a discharge port on a day; (iv) total flow of product from a load port to a vessel; (v) total flow of product from a vessel to a discharge port; (vi) first day a vessel starts its trip; (vii) product inventory level at a port at the end of a day; (viii) product inventory level of a vessel at the end of a day; (ix) total maximum product

inventory for a vessel; (x) last day a vessel finishes its trips; (xii) overall overage volume of a vessel; and (xii) overage volume of a vessel on a leg.

10. The computer system of claim 7 wherein the inventory balance constraints contain slack variables which allow the solver to terminate with a feasible solution.

11. The computer system of claim 7 wherein the mathematical model further comprises one or more equations for a rolling time horizon.

12. The computer system of claim 7 wherein the mathematical model further comprises one or more equations for grade segregation constraints.

13. The computer system of claim 7 wherein the mathematical model further comprises equations for one or more of the following: bounds on the number of vessels used; minimum vessel loads; and minimum volume transported.

14. The computer system of claim 7 wherein the mathematical model further comprises equations for a maximum cost-per-amount ratio.

15. The computer system of claim 7 wherein the mathematical model further comprises equations for port-revisit order and timing.

16. The computer system of claim 7 wherein the mathematical model allows one or more of the variables to be fixed.

17. The computer system of claim 7, wherein the supply locations are load ports and the demand locations are discharge ports, and wherein the received data includes one or more of the following: (i) basis amount of product for a vessel; (ii) flat rate for a leg; penalty cost or incentive for using a vessel; (iii) product lifting needs at a port on a day; (iv) demurrage days limit for a vessel; (v) world scale multiplier of a vessel; (vi) inlet draft limit

of a vessel at a loading port; (vii) outlet draft limit of a vessel at a loading port; (viii) inlet draft limit of a vessel at a discharge port; (ix) minimum flow rate at a port; (x) maximum flow rate at a port; (xi) minimum inventory at a port on a day; (xii) maximum inventory at a port on a day; (xiii) maximum amount of product on a vessel; (xiv) maximum volume of a vessel; (xv) minimum number of tons of product to be transported; (xvi) lower bound on the number of vessels used; (xvii) upper bound on the number of vessels used; (xviii) number of days of discharge port offset; (xix) total number of days in time horizon; (xx) overage rate of a vessel; (xxi) minimum percent full base volume for a vessel; (xxii) product amount produced at a port each day; (xxiii) maximum cost per volume ratio; (xxiv) start day for demurrage calculation for a chartered vessel; (xxv) travel time for a leg; (xxvi) time window initial day for availability of product load/discharge at a third party port; (xxvii) time window final day for availability of product load/discharge at a third party port; (xxviii) time window initial day for availability at a port; (xxix) time window final day for availability at a port; (xxx) time window initial day for availability of a vessel; (xxxi) time window final day for availability of a vessel; and (xxxii) initial inventory of a port.

18. The computer system of claim 1, wherein the bulk product is a liquid bulk product.

19. The computer system of claim 18, wherein the bulk product is crude oil.

20. The computer system of claim 18, wherein the bulk product is vacuum gas oil.

21. The computer system of claim 18, wherein the bulk product is selected from the group of gasoline, kerosene, and aviation fuel.

22. The computer system of claim 18, wherein the bulk product is gas oils.

23. The computer system of claim 18, wherein the bulk product is intermediate refinery products.

24. The computer system of claim 1, wherein the cost basis is the number of vessels utilized.

25. The computer system of claim 1, wherein the decision variables for the amount of bulk product to be loaded or discharged from a vessel designates the location and time of the loading or discharge.

26. The computer system of claim 25, wherein the constraints include constraints relating to inventory balances and the rate at which bulk product can be loaded or discharged.

27. A method for minimizing a total cost basis for the transportation scheduling and inventory management of a bulk product over a period of time, the method comprising the steps of:

providing a mathematical model containing equations that model transportation scheduling and inventory management of a bulk product from multiple supply locations to multiple demand locations to minimize cost per unit volume or mass of the bulk product,

the model includes a parameter for demurrage rate and constraints for minimum flow rates at the demand locations, maximum flow rates at the demand locations, and vessel draft limits; and

the model includes decision variables for: (i) the amount of bulk product to be loaded from a supply location onto a vessel, (ii) the amount of bulk product to be discharged from a vessel to a demand location, and (iii) demurrage time for a vessel;

receiving data relating to the characteristics of a fleet of vessels, supply locations, and demand locations;

applying the data to the mathematical model;

minimizing a total cost basis for the transportation scheduling and inventory management of a bulk product over a period of time by minimizing the cost per unit volume or mass of the bulk product by solving the mathematical model by a computer system based upon the constraints and decision variables;

determining a transportation plan based on the solution to the mathematical model; and

loading bulk product onto a vessel, discharging bulk product from a vessel, or both, according to the transportation plan.

28. The method of claim 27 wherein the step of solving the model is performed by an optimization engine in an hour or less.

29. The method of claim 28 wherein the step of solving the model is performed by an optimization engine in 15 minutes or less.

30. The method of claim 29 wherein the mathematical model comprises equations for the following constraints: (i) inventory balances; (ii) flow rates; (iv) travel times; (v) draft limits; (vi) overage calculations; (vii) demurrage or relaxed demurrage time calculations; and (viii) previously chartered vessels.

31. The method of claim 28 wherein the step of solving the model is performed by an optimization engine in 5 minutes or less.

32. The method of claim 28 wherein the step of solving the model is performed by an optimization engine in 10 seconds or less.

33. The method of claim 27, wherein the decision variables for the amount of bulk product to be loaded or discharged from a vessel designates the location and time of the loading or discharge.

34. The method of claim 33, wherein the constraints include constraints relating to inventory balances and the rate at which bulk product can be loaded or discharged.